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Bivariate Frequency Analysis of Hydrological Drought Using Copula: A Case Study of Northern Iraq

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ABSTRACT

In this research work, copula-based methodology is adopted to analyze the hydrological drought frequency. Standardized Runoff Index SRI was calculated using monthly-streamflow data for 50 years of two gauging stations in the northern region of Iraq. The drought duration and severity were extracted using run theory. Three Archimedean family and Gaussian copulas were used and compared to select the most appropriate copula model for bivariate frequency analysis of hydrological-drought characteristics. The dependence between drought duration and drought severity was estimated by Pearson's, Spearman's rho and Kendall's tau correlations. Various probability distributions were utilized to determine the best fit marginal distributions for drought characteristic variables based on the Kolmogorov-Smirnov and Chi-squared statistics. Uni-variate and joint return periods were estimated and compared. Generally, the results indicate that Archimedean copulas performed better than the Gaussian copulas. Exponential and Weibull distributions are the best fit for drought duration and severity, respectively, except for drought severity in case of the 9-month time scale at Lesser Zab region, where lognormal distribution was chosen. The current study can give helpful information for drought-risk assessment and water-resource management under climate change.

KEYWORDS: Hydrological drought, Copula, SRI, Greater Zab river, Lesser Zab river, Bivariate return periods.

INTRODUCTION

Drought is considered a significant natural hazard that threatens human life, environment, agriculture and economy. Drought monitoring is very important for water-resource planning and management (Mishra and Singh, 2010). Therefore, its risk assessment requires accurate modeling. In general, drought is the deficit of water resources. Droughts can be classified into hydrological, meteorological, agricultural and socio-economic categories (Wilhite and Glantz, 1985). Recently, many techniques are developed for monitoring, assessing and analyzing drought and determining its characteristics. Drought indices are considered the most widely used among these techniques (Li et al., 2015; Qin et al., 2021). Various indices have been applied and new ones developed to detect droughts; for example, Standardized Runoff Index (SRI) and Standardized Precipitation Index (SPI).

Uni-variate analysis cannot produce a complete assessment of hydrological events that are characterized by a set of interdependent variables (Mirakbari et al., 2010; Masina et al., 2015). A multi-variate frequency analysis of the hydrological events with two or more variables is necessary to fully understand the phenomenon (Grimaldi and Serinaldi, 2006; Chebana and Ouarada, 2011). Drought is a multi-variate phenomenon characterized by multiple variables, such as magnitude, duration, intensity and severity, obtained from the drought indices. Drought duration and severity are important variables in the field of drought frequency analysis (Yusof et al., 2013; Abdulla and Malkawi,

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2020; EskandariPour and Soltaniniab, 2022). Studying each variable separately could guide to uncertainty in the assessment of drought risk. Bivariate drought analysis is required for more reliable analysis, because it takes into account the dependence between the variables.

Copulas have been used successfully in previous studies for multi-variate analysis in various fields of water resources and hydrology (Ashkar and Aucoin, 2011; Bezak et al., 2014; Chen et al., 2015). Song and Singh (2010) applied different copulas for multi-variate analysis of drought. The copula method performed better than the generalized least squares regression method for bivariate rainfall frequency analysis (Sherly et al., 2015). Ayantobo et al. (2019) developed a multi-dimensional copula approach using drought characteristics extracted from the standardized precipitation evapotranspiration index. Asif Khan et al. (2021) examined the suitability of various multi-dimensional copula for modeling drought events based on two meteorological drought indices.

Recently, there has been a growing interest in multi-variate frequency modeling of hydrological events; for example, flood events (Sraj et al., 2015; Duan et al., 2016). For drought events, multi-variate frequency modeling of drought parameters has been applied by many researchers (Pontes Filho et al., 2020; Asif Khan et al., 2021). Wang et al. (2021) analyzed the frequency of drought intensity and duration adopting the theory of run and copula function.

Iraq is considered as an arid and semi-arid region (Al-Ansari, 2016). Drought is one of the main challenges facing Iraq, which leads to a decrease in streamflow rates, a reduction in groundwater levels and agricultural land and an increase in dust storms.

Streamflow is the most important water resource in Iraq for various purposes, such as drinking water, irrigation and hydroelectric power generation. After reviewing the relevant literature, it was found that there are no detailed studies on bivariate hydrological drought frequency analysis in Iraq, which contains several important rivers. In order to use a more effective method for assessment of drought risks, the present study aimed to analyze the bivariate hydrological drought frequency at Greater Zab and Lesser Zab rivers' basins in northern Iraq based on two-dimensional copula methodology. The procedure includes the estimation of SRI values at 9-and 12-month timescales, which is used to determine the drought duration and severity through run theory. Also, the study aimed at selecting the suitable marginal distributions for drought characteristics separately, employing the Archimedean family and Gaussian copulas to build joint distributions of drought characteristics and estimate uni-variate and bivariate return periods of drought characteristics.

Study Area and Collected Data

The study area is located in the northern region of Iraq, which is considered a semi-arid region. Greater Zab and Lesser Zab rivers are two main tributaries of the Tigris river in Iraq, as shown in Figure 1. Geographically, the two river basins are located between latitudes of 35° 21' N to 38° 06' N and longitudes of 43° 25' E to 46° 03' E (Hasan and Saeed, 2020). Greater Zab river catchment area covers about 26750 km^2 and the mainstream river length is approximately 370 km, joining the Tigris river south of Mosul city. Lesser Zab river catchment area covers about 18735 km^2 and the length of the mainstream river is approximately 418 km, joining the Tigris river north of Baiji city.

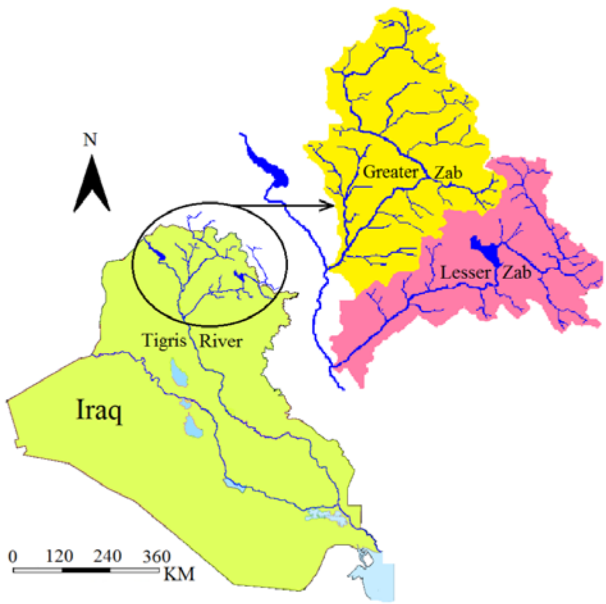

Figure (1): Study area and river systems in north Iraq

The average monthly temperature is about 38°C during the summer season. The mean annual rainfall ranges between 550 mm in mountain zones and 350 mm towards the southwest. Most of rainfall occurs during November to May. Rainfall is inversely related to air temperature.

The considered rivers are the main water sources for water supply to many cities and towns, such as Erbil and Kirkuk, hydroelectric-power generation as Dokan dam and lots of irrigation projects, such as Kirkuk irrigation project. The high streamflow season of the hydrological regime of the Greater Zab and Lesser Zab occurs during winter and spring seasons (December to May) due to heavy rainfall and the period of low streamflow occurs during the summer and autumn seasons (June to November). In the current study, the monthly streamflow data for 50 years (1960-2010) of Dokan station at Lesser Zab river and Eski-Kelek station at Greater Zab river was used. The data is collected from Iraqi Meteorological Organization and Seismology and USGS data series no.540 (Saleh, 2010). Table 1 shows the statistical analysis of the streamflow data records for the selected rivers.

Table 1. Statistical analysis of the streamflow data for the selected stations

Station	Record length (year)	Average monthly flow (m3/s)	Maximum monthly flow (m3/s)	Minimum monthly flow (m3/s)	Standard deviation	Coefficient of variation	Variance
Greater Zab	50	382.7	1781.0	33.8	313.8	82.0	98482
Lesser Zab	50	187.2	1569.1	8.7	194.1	103.7	37676

THEORETICAL BACKGROUND

Standardized Runoff Index

SRI presented by Shukla and Wood (2008) is a very simple, widely used hydrological drought index which only requires monthly streamflow data and is based on the SPI calculation procedure. SRI is used to analyze and monitor droughts over time series.

Standardized indices can be derived for different time scales (Golian et al., 2015). A 1-month time scale is suitable for assessing meteorological drought, 3-month and 6-month time scales are preferred for assessing agricultural drought, while more than 6-month time scales are preferred for assessing hydrological drought (Yaseen et al., 2021; Hou et al., 2021). In the present study, 9-month and 12-month time scales of SRI index are used for hydrological-drought assessment. Since the SRI is a standardized index, it is subject to the same categories of drought severity of SPI given by McKee et al. (1993), as shown in Table 2.

Table 2. Drought classification of SRI

SRI value	Drought categories
Index ≥ 0.0	No drought
0.0 > Index > -1.0	Mild drought
-1 ≥ Index > -1.5	Moderate drought
-1.5 ≥ Index > -2.0	Severe drought
Index ≤ -2.0	Extreme drought

Estimation of Drought-event Characteristics

A drought event is usually known as the period in which a particular drought index is consistently under a specific threshold (Shiau, 2006; Mishra and Singh, 2010). In the current study, drought events are onset when the monthly SRI time-series values are consistently lower than zero. Therefore, negative values of SRI denote dry conditions.

Drought events can be described by several characteristics. Duration and severity characteristics are usually more highly correlated than others, so that in many studies, drought events are mainly described by these two characteristics. The considered drought duration and severity are obtained from the monthly time series of drought indices based on applying the run theory, which is used to characterize drought (Wu et al., 2019). The run theory is defined as the portions which of a drought index value which are higher or lower than a specified threshold level (Yevjevich, 1969; Almeida and Barbosa, 2020). The considered drought characteristics are defined as described below.

Drought duration D is the number of successive months where drought-index value persists under the threshold level of 0 (i.e., negative SRI). Since the specific drought index SRI is defined monthly, the shortest drought duration is one month. Drought severity S is the absolute cumulative value of SRI below the threshold over the drought duration. The sample size of drought duration (D1, D2, ..., Dn) equals the sample size of drought severity (S1, S2, ..., Sn). Figure 2 shows the definition of drought events relative to a threshold level of 0 based on the run theory. As shown, S_1 is the most severe drought event and the second drought event has the longest duration D_2.

Duration and severity of drought are considered the most mutually correlated characteristics, which satisfies the concept of copula to build the bivariate distribution function (Dai et al., 2020). In the current study, bivariate drought characteristics were extracted from the monthly SRI time-series data for the period (1960-2010) and used for drought frequency analysis.

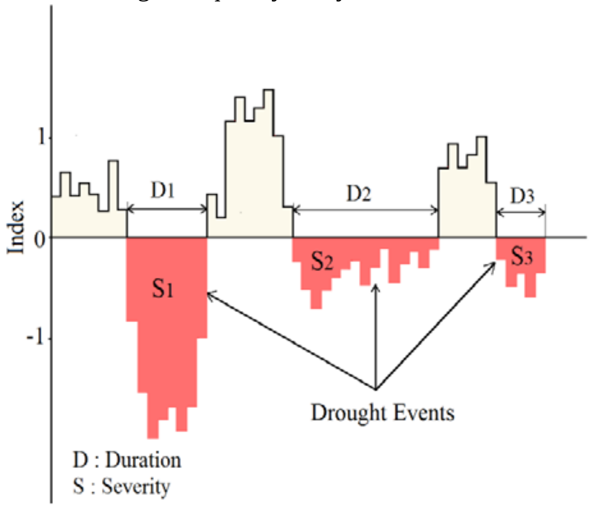

Figure (2): Drought-event definition diagram based on the run theory

Concept of Copula

The copula function demonstrates the dependence structure of multiple random variables by assigning the suitable marginal distributions separately. Sklar (1959) first defined the concept of copula as a joint distribution function built by mixing the marginal distributions of two or more random variables. The copula or joint cumulative distribution function (joint CDF) $F_{X,Y}(x, y)$ of duration = X and severity = Y variables with marginal distribution functions $u=F_X(x)$ and $v=F_Y(y)$ is expressed as follows (Nelsen, 2006; Qin et al., 2021):

$$F_{X,Y}(x,y) = P(X \leq x, Y \leq y) = C\big(F_X(x), F_Y(y)\big) \quad (1)$$

where C is the copula function, X is the hydrological drought duration and Y is the hydrological drought severity.

The related joint probability density function (joint PDF) $f_{X,Y}(x,y)$ of X and Y variables is the partial derivative of joint CDF as follows (Volpi and Fiori, 2012; Bezak et al., 2014):

$$f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y} \quad (2)$$

The first step of the copula method starts by determining the strength of dependence between the studied variables using statistical-correlation (dependence) criteria. Drought duration and drought severity are determined by the same physical phenomenon; they are obviously mutually correlated. To confirm this, Pearson's (r), Spearman's rho (ρ) and Kendall's tau (τ) statistical tools were applied to test the correlation between duration and severity variables. These three correlation coefficient values vary between +1 and -1. The +ve correlation coefficient values indicate a positive relationship, while the -ve correlation coefficient values indicate a negative relationship.

In the second step of copula, after examining the correlation between the modelled variables, an appropriate marginal distribution must be chosen for each variable individually using the Kolmogorov-Smirnov (K-S) and Chi-squared (X^2) criteria at the commonly used significance level α=0.05. An advantage of the copula approach is that each variable will have its own distribution function, because they may have different marginal distributions. Four marginal distributions are proposed to fit drought severity and drought duration, which are: exponential, gamma, lognormal and Weibull distributions. The proposed probability distributions are widely used in hydrology (Wu et al., 2015; Hu et al., 2020; Hasan, 2020). The maximum-likelihood method is utilized to calculate the parameters of the marginal distribution.

Finally, the two-variate Archimedean copulas including Clayton, Frank and Gumbel, as well as the Gaussian copulas are utilized as candidate functions to determine the best-fit model for drought characteristics. The Archimedean copulas are broadly used in hydrological and water-resource analyses, due to their construction simplicity, strong representation and the possibility to be used even if the correlation between the variables is negative (Zhang and Singh, 2006; Tsakiris et al., 2015). Bayesian information criteria (BIC), root mean square error (RMSE) and Nash–Sutcliffe efficiency (NSE) performance measures were used as goodness-of-fit tests to identify the most suitable function among the candidate copulas. They are more accurate than graphical diagnostics (Chen and Guo, 2019). The statistical expressions of RMSE, BIC and NSE are as follows:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(Pe_i - P_i)^2} \quad (3)$$

$$BIC = N\ln(MSE) + m\ln(N) \quad (4)$$

$$NSE = 1 - \frac{\sum_{i=1}^{N}(Pe_i - P_i)^2}{\sum_{i=1}^{N}(Pe_i - \overline{Pe})^2} \quad (5)$$

where *Pei* is the empirical probability, *Pi* is the theoretical probability, $\overline{Pe}$ is the mean of empirical values, *N* is the sample size, *m* is the parameters' number. *MSE* is the mean square error. The copula with the minimum values of RMSE and BIC and maximum value of NSE is the optimal copula function.

The following step of the bivariate drought frequency analysis is to use the optimal copula function for calculating the bivariate return periods of drought events. Hydrological-event return periods are widely used and represent a useful approach in water-resource management, especially return periods of drought events (Shiau and Shen, 2001; Serinaldi et al., 2009). Conventionally, uni-variate return periods are determined by fitting a marginal distribution function to hydrologic data series, such as extreme hydrologic events including annual maximum discharge and drought-event variables (Li and Zheng, 2016, Chen and Guo, 2019). Several probability distribution functions have been used by many researchers to analyze uni-variate return periods for extreme hydrological events (Du et al., 2015; Kuswanto et al., 2021).

The uni-variate return periods of drought duration T_X and severity T_Y are calculated by the formulae expressed by Shiau and Shen (2001) and Shiau (2006) as:

$$T_X = \frac{N}{n(1 - F_X(x))} \quad (6)$$

$$T_Y = \frac{N}{n(1 - F_Y(y))} \quad (7)$$

where *N* is the sample size, *n* is the drought-event number in *N*, $F_X(x)$ and $F_Y(y)$ are the marginal distribution functions of hydrological-rought duration and severity, respectively.

Bivariate return period is important to describe multi-variate hydrological events (Yue and Rasmussen, 2002). The bivariate return period $T_{(x,y)}$ of an event (X, Y) with joint CDF of $F_{X,Y}(x, y)$ in (X>x, or Y>y) cases can be expressed as (Shiau, 2003; Volpi and Fiori, 2012):

$$T_{(x,y)} = \frac{N}{n(1 - F_{X,Y}(x,y))} = \frac{N}{n(1 - C(u,v))} \quad (8)$$

RESULTS AND DISCUSSION

Dependence Analysis between Duration and Severity

Before constructing the copula function, it is necessary to test the dependence between the pairs of considered random variables. The well-known Pearson's (r), Spearman's rho (ρ) and Kendall's tau (τ) correlation coefficients are used to examine the dependence between duration and severity variables. The calculated values of (r), (τ) and (ρ) at Greater Zab GZ and Lesser Zab LZ regions are shown in Table 3.

Table 3. Values of correlation coefficients for drought characteristics

Region		Pearson		Kendall		Spearman	
		(r)	p-value	(τ)	p-value	(ρ)	p-value
GZ	SRI-9	0.9405	0.000	0.8964	0.000	0.9757	0.000
	SRI-12	0.9185	0.000	0.9225	0.000	0.9821	0.000
LZ	SRI-9	0.9264	0.000	0.8418	0.000	0.9541	0.000
	SRI-12	0.9129	0.000	0.8519	0.000	0.9573	0.000

Table 3 reveals that the correlation-coefficient values between duration and severity were positive and significant at α = 0.05 significance level. The significant and high positive correlation between duration D and severity S indicates that bivariate modeling of drought characteristics is feasible and constructing copula functions are appropriate for drought frequency analysis.

Estimation of Appropriate Marginal Distributions

The suggested probability distributions for fitting marginal distributions to drought-event characteristics include the exponential, gamma, lognormal and Weibull distributions. The parameters of the marginal distributions are calculated using the maximum-likelihood method. Goodness-of-fit tests of Kolmogorov-Smirnov (K-S) and Chi-squared (X^2) are used to find the most appropriate distribution for duration and severity data. The selected marginal distributions for individual drought characteristic variables are presented in Table 4.

Table 4. Most appropriate marginal distribution functions selected for drought characteristics

Region		Duration	Severity
GZ	SRI-9	Exponential	Weibull
	SRI-12	Exponential	Weibull
LZ	SRI-9	Exponential	Lognormal
	SRI-12	Exponential	Weibull

The best marginal distribution for different drought characteristic is not necessarily the same. The statistical test results showed that the exponential distribution is the best for modeling the drought duration in all cases. At the same time, Weibull distribution was the best function for the drought severity in most cases, except for SRI-9 in the Lesser Zab river, where the lognormal distribution was the best fit distribution.

Estimation of Appropriate Copula Models

The proposed bivariate copulas were applied to all pairs of duration and severity values. More than one copula function can be utilized to structure the dependence between duration and severity characteristics, but the best function is always adopted. In the current work, in order to determine the best-fit function (among the proposed Clayton, Gumbel, Frank and Gaussian copulas) to connect drought duration and severity, RMSE, BIC and NSE goodness-of-fit tests are adopted. Table 5 compares the values of goodness-of-fit tests of 9-month and 12-month timescales for the considered copulas. For each case, the copula function with the lowest values of RMSE and BIC and the highest value of NSE is the best one.

Table 5. Comparison of statistical test results of considered copulas

Region		Copula	RMSE	BIC	NSE
GZ	SRI-9	Clayton	0.163	-76.72	0.983
		Frank	**0.084**	**-105.89**	**0.995**
		Gumbel	0.112	-93.23	0.992
		Gaussian	0.132	-86.00	0.989
	SRI-12	Clayton	**0.054**	**-84.85**	**0.997**
		Frank	0.064	-79.75	0.995
		Gumbel	0.079	-73.44	0.993
		Gaussian	0.070	-77.06	0.995
LZ	SRI-9	Clayton	0.162	-87.78	0.985
		Frank	**0.089**	**-117.73**	**0.995**
		Gumbel	0.101	-111.41	0.994
		Gaussian	0.104	-109.94	0.993
	SRI-12	Clayton	0.123	-80.82	0.989
		Frank	0.092	-92.44	0.994
		Gumbel	**0.083**	**-96.56**	**0.995**
		Gaussian	0.093	-92.01	0.994

Bolded is the best copula function.

Generally, Archimedean copulas showed better performances than Gaussian ones based on the results of RMSE, BIC and NSE tests. According to Table 5, the Frank copula was selected as the best-fit copula function for drought variables with SRI-9 in both GZ and LZ regions, since it has the lowest values of RMSE and BIC and the highest value of NSE. The Clayton and Gumbel copula functions are more properly fitted with the drought variables extracted from SRI-12 in the GZ region and the LZ region, respectively.

To conduct bivariate frequency analysis of droughts, the most appropriate copula shown in Table 5 was selected for each case.

To realize the probable performance of the dependency structure between drought severity and duration, Figure 3 shows the joint CDF and joint PDF of the best copulas including Frank for SRI-9 and Clayton for SRI-12 in the GZ region, Frank for SRI-9 and Gumbel for SRI-12 in the LZ region.

The joint PDF clearly reveals the main features of the dependency structure (Jovanovic et al., 2018). As shown in Figure 3, the adopted copulas have different behaviors at the upper tail point (1, 1) and the lower tail point (0, 0). Frank copula shows a positive linear dependence and a weak tail dependence, while Clayton copula catches a lower tail dependence, while Gumbel copula catches an upper tail dependence and a very weak lower tail dependence between duration and severity.

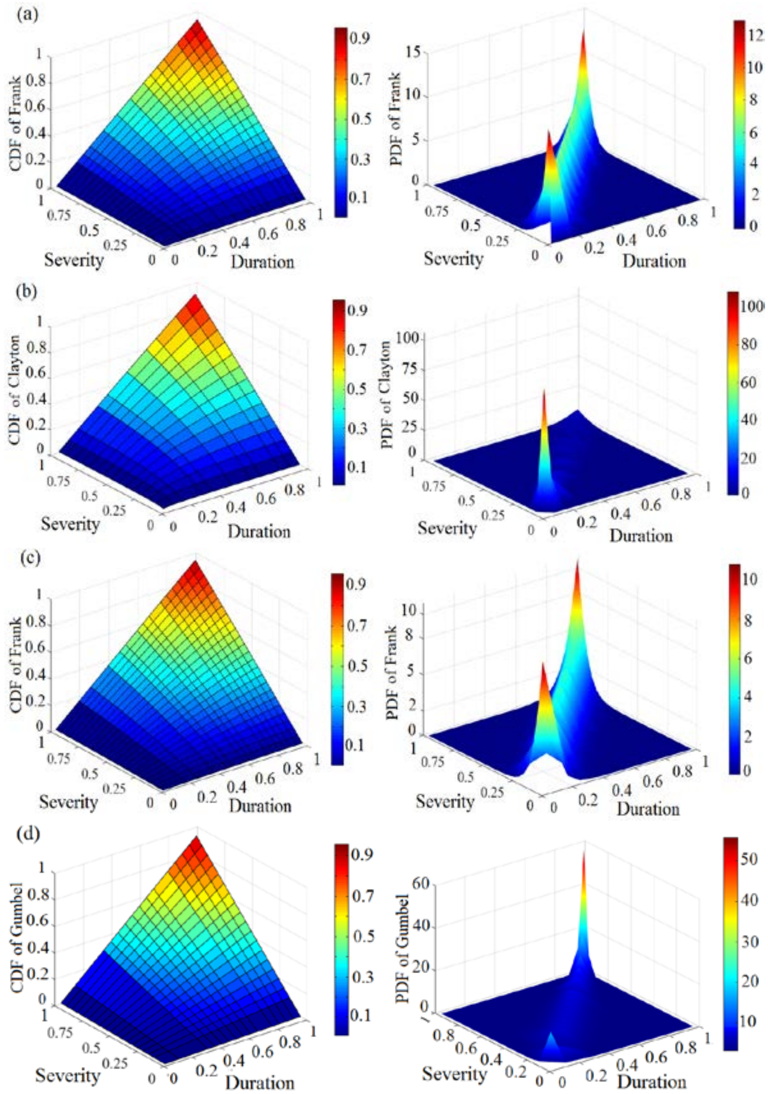

Figure (3): Joint CDF and Joint PDF of drought duration and severity; a: GZ (SRI-9), b: GZ (SRI-12), c: LZ (SRI-9) and d: LZ (SRI-12)

Joint Probability of Drought Duration and Severity

Joint probabilities of drought events are essential for drought management (Mirabbasi et al., 2012). According to the best copula chosen for each case shown in Table 5, the bivariate joint probability P(X≤x, Y≤y) of drought duration X and severity Y is computed. Figure 4 shows the contour lines of joint cumulative probabilities of drought events in the GZ and LZ regions, for the 9-month and 12-month time scales using the same copulas described above.

Figure 4 denotes the probabilities of two characteristics (duration and severity) occurring simultaneously. To determine the bivariate joint probability of occurrence of drought characteristics, for example P(D ≤ 16 , S≤ 9.8) for 12-month time scale in the GZ region is 0.53. This will help determine the drought-event frequency. As shown, most drought events (scatter points) for all cases have low probability levels and only a few points (duration- severity pairs) have probability levels greater than 0.90, which indicates more frequent drought events (with medium-to-short duration-severity) and higher return periods of severe drought events (high duration-severity).

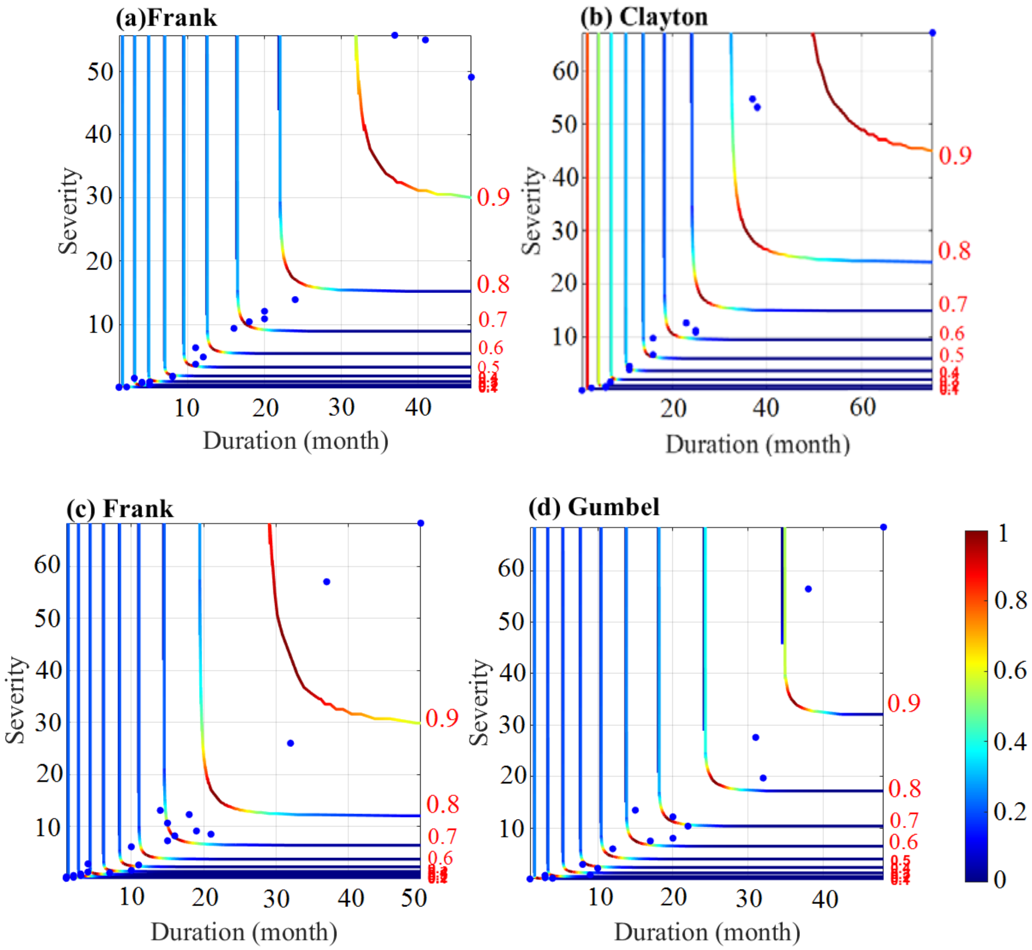

Figure (4): Contours of joint cumulative probabilities for drought events based on best fitted copulas; a: GZ (SRI-9), b: GZ (SRI-12), c: LZ (SRI-9) and d: LZ (SRI-12)

Bivariate Return Period

The uni-variate return period is related to one-variate probabilities and joint return period is related to joint probability. The bivariate (joint) return period of drought duration and severity was calculated adopting the most appropriate copula function selected in each considered region under 9-and 12-month time scales. The univariate return periods of duration and severity are obtained using the most appropriate uni-variate marginal distributions and they are computed separately using Eq. 6 and Eq. 7. The joint return periods of duration and severity are computed by Eq. 8. Simultaneous exceedence event ($X \geq x$, $Y \geq y$) is of interest when focusing on return periods.

Alike to the bivariate joint probability, the bivariate joint return period can be displayed using contour plots. Figure 5 presents the contour lines of the drought characteristics' joint return periods. As shown, all drought events (scatter points) for the adopted cases have return periods of less than 25 years. For example, the joint return period of the highest-degree drought event (longest duration 75 month and highest severity 67.18) (X ≥ 75, Y ≥ 67.18) in the GZ region for a 12-months' time scale is approximately 19 years.

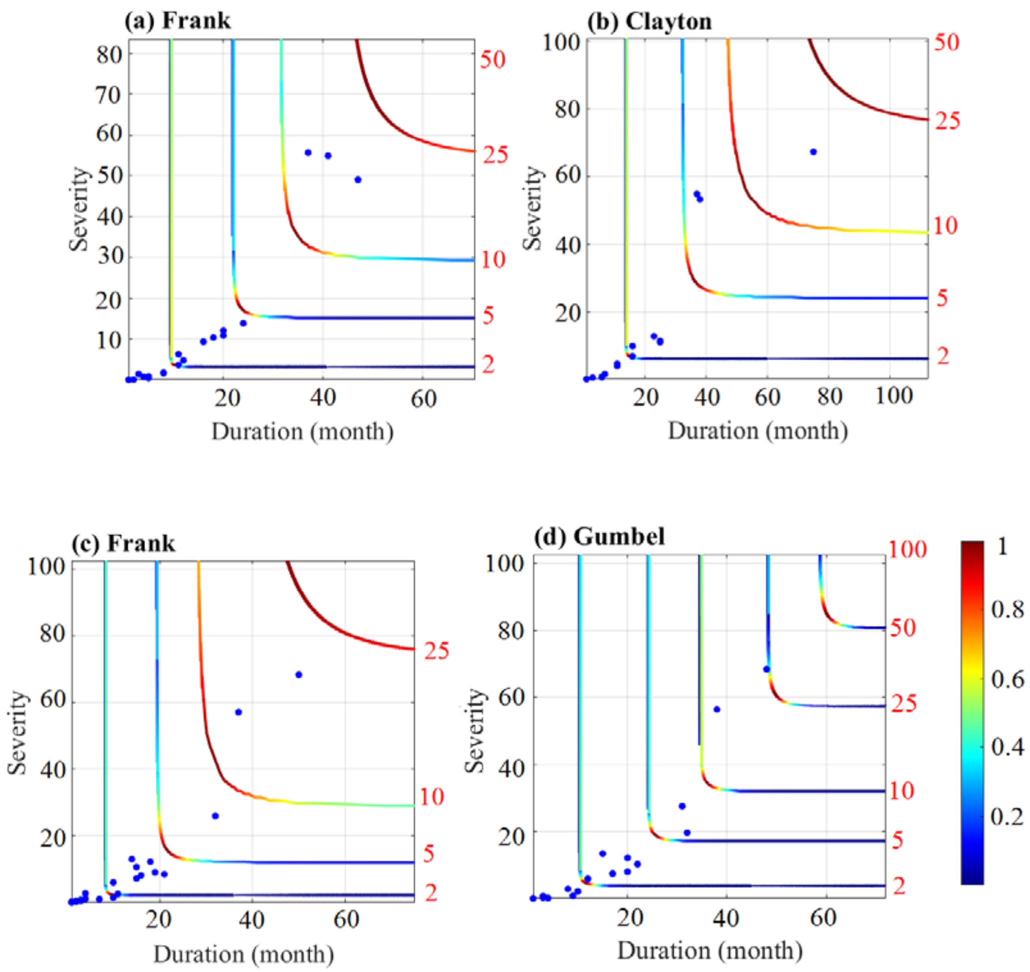

Figure (5): Contours of joint return period for drought events adopting best-fit copulas; a: GZ (SRI-9), b: GZ (SRI-12), c: LZ (SRI-9) and d: LZ (SRI-12)

From Figure 5, one can obtain the expected joint return period of a particular drought event directly by affording its duration and severity. For example, let us consider a drought event with the duration of 10 months and a severity of 12. The corresponding joint return period (D ≥10 or S ≥12) in case of 12-month time scale at the LZ region is approximately 2.2 years. Generally, the joint return periods increase with increasing values of drought duration and severity.

The uni-variate return periods determined by separate duration d and severity s and their corresponding bivariate joint return periods are compared. It is noticed that the joint return period of drought events is less than the corresponding uni-variate return period. For example, in the GZ region for 12-month time scale and 10-year uni-variate return period, the value of drought duration d was 46 months and the value of drought severity s was 42. The corresponding joint return period of (D≥46 or S≥42) is 7 years. This leads to that the uni-variate frequency analysis does not offer sufficient information about the drought risks associated with two correlated variables. Therefore, bivariate frequency analysis is adopted to overcome the underestimation of drought risks (i.e., the return period is long) when uni-variate frequency analysis is considered.

CONCLUSIONS

The current study highlights the significance of considering the drought duration and severity variables for a reliable bivariate drought frequency analysis based on the copula approach. Drought variables were defined by the standardized run-off index based on the run theory. Two hydrological stations at Greater Zab and Lesser Zab rivers in northern Iraq were studied. The main conclusions of the current study are as follows:

According to the results of SRI index, hydrological drought events became more serious in the last two decades. For the GZ region, the longest duration and highest severity of drought events were 75 months and 67.2, respectively. For the LZ region, the longest duration and highest severity were 48 months and 68.5, respectively.

Pearson's(r) Spearman's rho and Kendall's tau statistical test results showed that the dependence between duration and severity is statistically highly significant for all cases. Hence, bivariate drought frequency analysis employing the copula method is feasible.

Exponential, gamma, lognormal and Weibull distributions are utilized to fit drought duration and severity data series. Exponential distribution is best fit with drought-duration datasets for all cases and Weibull distribution is best fit with drought-severity datasets, except for the 9-month time scale in the LZ region, where the lognormal distribution was selected as the best distribution.

Three Archimedean and Gaussian copulas are applied to construct the joint bivariate distributions. Based on RMSE, BIC and NSE goodness-of-fit tests, Frank copula provides the most suitable function for data extracted from a 9-month time scale in the GZ and LZ regions, while the Clayton and the Gumbel copulas were selected as the most appropriate functions for data extracted from a 12-month time scale at GZ and LZ, respectively. In the copula method, the choice of marginal distributions is not restricted, since it allows linking of different uni-variate marginal distributions. This characterized the copula as a flexible and useful tool in multi-variate analysis of drought variables.

As shown in our illustrative examples, the bivariate return periods are shorter than the uni-variate return periods of drought variables. It is very reliable to calculate the bivariate return period of drought events, because the actual drought situations may be underestimated (long return periods) if only the uni-variate return period is taken into account.

Most of the drought events for the adopted cases have short joint return periods of less than 5 years. This study provides important information for effective drought-risk assessment and water resource planning in the study region. The current study is the first attempt to discover the effect of two hydrological drought variables simultaneously by applying the copula approach in the considered river regions.

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Conflict of Interest

The authors declare no conflict of interest.

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