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An Analytical Solution of Frost Heaving Pressure for Cold-region Tunnel Considering Freeze-thaw Cycles and *in-situ* Stress

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ABSTRACT

The damage caused by the frost heaving pressure on the surrounding rocks and lining structure of cold-region tunnels is always common, which can seriously threaten the safety and stability for cold-region tunnels. Although many achievements of frost heave pressure model have been obtained, two factors have been often ignored, which are *in-situ* stress and freeze-thaw cycles. Therefore, the calculation mechanical model of cold-region tunnels is established and the expression of frost heaving pressure considering frost heaving effect and *in-situ* stress is derived based on the elastic theory. The relationship between the elastic modulus of surrounding rocks and the number of freeze-thaw cycles was fitted by experimental data and the calculation formula of frost heaving rate of rocks considering their porosity change caused by freeze-thaw cycles is derived. Based on that, the calculation method of frost heaving pressure considering *in-situ* stress and freeze-thaw cycles is proposed. The example analysis results show that frost heaving ratio and frost heaving pressure gradually increase with freeze-thaw cycles, which are eventually subjected to a steady value. Simultaneously, the frost heaving pressure acting on lining increases with *in-situ* stress for tunnels in cold regions and some effective insulation measures should be applied to prevent frost damage.

KEYWORDS: Frost heaving pressure, Cold-region tunnels, Freeze-thaw cycles, Analytical solution, Zero-frost heave displacement.

INTRODUCTION

According to statistics, the total frozen soil accounts for about 50% of the total land area around the world, while the total frozen soil accounts for about 80% of the total land in China (Ma and Wang, 2014; He et al., 2022). Consequently, numerous engineering constructions were established in cold regions, such as tunnels, subgrades, pipelines and bridges (Wang and Zhou, 2018). For cold-region tunnels, excavation and frost heaving have great influence on the safety and stability of the tunnel. The frost heaving pressure is produced when the surrounding rocks are subjected to frost heaving, where lining cracking, wall hanging ice and other frozen damages can be caused (Rempel, 2007; Mimouni et al., 2014). Meanwhile, the surrounding rocks will be damaged under the condition of repeated freeze and thaw process, which seriously affects the mechanical properties of surrounding rocks (Altindag et al., 2004; Hori and Morihiro, 1998). Therefore, it is extremely important to research the frost heaving pressure of surrounding rocks after multiple freeze-thaw cycles.

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At present, abundant research achievements on frost heaving pressure of cold-region tunnels have been obtained, including theoretical calculations (Lai et al., 2002a; Gao et al., 2012; Huang et al., 2015; Feng et al., 2017, 2019; Liu et al., 2018; Lv et al., 2019), laboratory experiments (Yan et al., 2017; Yuan et al., 2021), numerical simulations (Lai et al., 1998, 2005, 2009; Zhang et al., 2002a, 2002b, 2004; Tan et al., 2013; Zheng et al., 2015); and *in-situ* monitoring (Zhang et al., 2018; Yan et al., 2020). The theoretical calculation of frost heaving pressure can provide the most basic and significant reference for the design of cold-region tunnels. Lai et al. (1998) used Laplace transform to derive the viscoelastic solution of frost heaving pressure for cold-region tunnels. Gao et al. (2012) applied the continuity Equation and semi-analytical method to derive the analytical solutions of frost heaving pressure for cold-region tunnels. Feng et al. (2014) derived the elastoplastic solutions of the frost heaving pressure for cold-region tunnels based on the uniform frost heaving characteristics of surrounding rocks and Mohr-Coulomb criterion. Feng et al. (2017) used the complex function and elastic theory for deducing analytical solutions of frost heaving pressure for cold-region tunnels with the condition of unequal compression. Lv et al. (2018) and Xia et al. (2019) used a test to prove that the frost heaving of rocks for cold-region tunnels along the radial direction of the tunnel is unidirectional freezing and verified that this frost heaving of rocks is transverse isotropic. Liu et al. (2018) deduced the elastoplastic solutions of the frost heaving pressure for cold-region tunnels considering transverse isotropic frost heave, support strength and support timeliness. Feng et al. (2019) proposed the elastoplastic solutions of the frost heaving pressure for cold-region tunnels on the basis of the Drucker-Prager criteria and transverse isotropic frost heave. Lv et al. (2019) proposed the condition Equation to decide the elastic or plastic condition for surrounding rocks after frost heaving and derived the elastoplastic solutions of the frost heaving pressure for cold-region tunnels considering transverse isotropic frost heaving and Mohr–Coulomb criterion. Guo et al. (2017) used the generalized Kelvin model and Laplace transform to derive the viscoelastic solutions of frost heaving pressure. The aforementioned research results for frost heaving pressure of cold-region tunnels only consider its single frozen state, while the repeated freeze and thaw condition of surrounding rocks is neglected. Additionally, the mechanical properties of surrounding rocks can be affected by freeze-thaw cycles, which is extremely disadvantageous to the stability and safety of tunnels in cold regions (Dalila and Mebare, 2022).

Therefore, it is necessary that the analytical solution of the frost heaving pressure for cold-region tunnels after freeze-thaw cycles should be explored. Liu et al. (2019, 2020) deduced the elastic models for calculating the frost heaving pressure of surrounding rocks for cold-region tunnels, considering the common influence of decrease of elastic modulus and increase of porosity after freeze-thaw cycles. Zhang et al. (2020) used the complex variable function to derive elastic analytical solutions of frost heaving pressure for cold-region tunnels considering freeze-thaw cycles and the anisotropic features of surrounding rocks. Li and Chen (2020) applied a complex variable function to obtain a method for calculating the frost heaving pressure of non-circular tunnels in seasonal frozen-soil zones. However, the calculation methods above for frost heaving pressure of surrounding rocks considered reeze-thaw cycles, but the influence of *in-situ* stress field was ignored and the computing method of frost heaving rate for surrounding rocks after multiple freeze-thaw cycles also lacked.

Based on this, the calculation model of frost heaving pressure for cold-region tunnels is established and the analytical solution of frost heaving pressure considering the influence of frost heaving and *in-situ* stress is derived using the elastic theory. Then, a formula for calculating the frost heaving rate considering the variation of rock porosity after *N* freeze-thaw cycles is deduced based on the test data. Finally, a method for calculating the frost heaving pressure considering *in-situ* stress and freeze-thaw cycles is proposed. The research results can supplement the calculation methods of frost heaving pressure under the condition of freeze-thaw cycles.

Basic Assumptions

In order to derive the analytical solutions of frost heaving pressure for cold-region tunnels considering freeze-thaw cycles and *in-situ* stress, the following assumptions are made:

(1) The lining and surrounding rocks are considered to be homogeneous, continuous and isotropic and the tunnel is supported immediately after excavation.

(2) The longitudinal length of the tunnel is much bigger than its cross-sectional size and the tunnel can be simplified as a plane strain issue.

(3) The tunnel is subjected to hydrostatic pressure and the influence of gravity on lining and surrounding rocks is ignored.

Analysis of Frost Heaving Pressure of Surrounding rocks for Cold-region Tunnels

Mechanical Calculation Model

The calculation model of frost heaving pressure for cold-region tunnels is shown in Fig.1, which consists of three zones; namely, lining zone (zone I, $r_L \geq r \geq r_a$), frozen zone (zone II, $r_f \geq r \geq r_L$) and unfrozen zone (zone III, $r \geq r_f$). As shown in Fig.1, r_a is the inside radius of lining; r_L is the outside radius of lining; r_f is the radius of frozen zone; σ_f is the radial stress acting on the interface between lining zone and frozen zone; σ_h is the radial stress acting on the interface between frozen zone and unfrozen zone; P_0 is the *in-situ* stress.

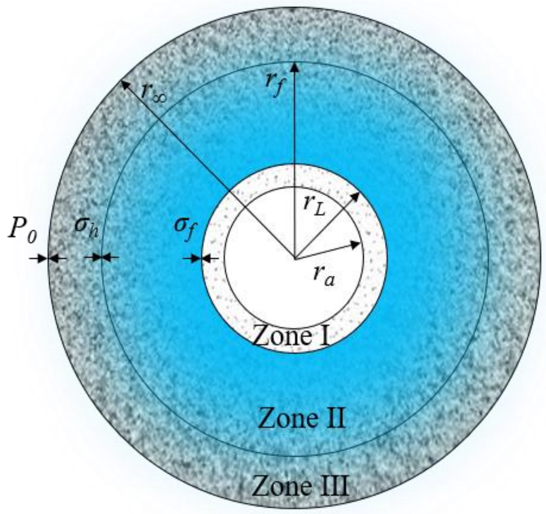

Figure (1): Calculation model of frost heaving pressure for cold-region tunnels

Under the plane axisymmetric condition, the stress of surrounding rocks within all zones satisfies the following equilibrium equation:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{1}$$

where σ_r and σ_θ are the radial stress and tangential stress, respectively. r is the radial dimension of the cold-region tunnel.

All zones conform with the geometric equations:

$$\varepsilon_r = \frac{du_r}{dr} \tag{2}$$

$$\varepsilon_\theta = \frac{u_r}{r} \tag{3}$$

where u_r is the radial displacement the of cold-region tunnel and ε_r and ε_θ are the radial strain and tangential strain, respectively.

The elastic constitutive equations of lining are:

$$\varepsilon_r^I = \frac{1-\mu_L^2}{E_L}\left(\Delta\sigma_r - \frac{\mu_L}{1-\mu_L}\Delta\sigma_\theta\right) \tag{4}$$

$$\varepsilon_\theta^I = \frac{1-\mu_L^2}{E_L}\left(\Delta\sigma_\theta - \frac{\mu_L}{1-\mu_L}\Delta\sigma_r\right) \tag{5}$$

where ε_r^I and ε_θ^I are the radial strain and tangential strain of lining, respectively. E_L and μ_L are the lining elastic modulus and Poisson's ratio, respectively. $\Delta\sigma_r = \sigma_r - P_0$ and $\Delta\sigma_\theta = \sigma_\theta - P$.

The elastic constitutive equations of surrounding rocks of the unfrozen zone are:

$$\varepsilon_r^{III} = \frac{1-\mu_s^2}{E_s}\left(\Delta\sigma_r - \frac{\mu_s}{1-\mu_s}\Delta\sigma_\theta\right) \tag{6}$$

$$\varepsilon_\theta^{III} = \frac{1-\mu_s^2}{E_s}\left(\Delta\sigma_\theta - \frac{\mu_s}{1-\mu_s}\Delta\sigma_r\right) \tag{7}$$

where ε_r^{III} and ε_θ^{III} are the radial strain and tangential strain of unfrozen surrounding rocks, respectively. E_s and μ_s are the elastic modulus and Poisson's ratio of unfrozen surrounding rocks, respectively.

Based on the literatures (Liu et al., 2018; Lv et al., 2019), the elastic constitutive equations of surrounding rocks in the frozen zone are:

$$\varepsilon_r^{II} = \frac{1-\mu_f^2}{E_f}\left(\Delta\sigma_r - \frac{\mu_f}{1-\mu_f}\Delta\sigma_\theta\right) - (1+\mu_f)\varepsilon_0 \tag{8}$$

$$\varepsilon_\theta^{II} = \frac{1-\mu_f^2}{E_f}\left(\Delta\sigma_\theta - \frac{\mu_f}{1-\mu_f}\Delta\sigma_r\right) + (1+\mu_f)\varepsilon_0\left(\frac{r_0-r}{r}\right) \tag{9}$$

where ε_r^{II} and ε_θ^{II} are the radial strain and tangential strain of frozen surrounding rocks, respectively. E_f and μ_f are the elastic modulus and Poisson's ratio of frozen surrounding rocks, respectively. r_0 is the radius of zero-frost heaving displacement. ε_0 is the linear strain caused by frost heaving, which is equal to 1/3 times the volumetric strain of surrounding rocks ε_V based on the basic assumption (1).

Xia et al. (2013) proposed a modified formula for calculating the frost heaving rate ε_V of saturated rocks considering the constraint of water ice phase change, as follows:

$$\varepsilon_V = 2.17\%\eta n \tag{10}$$

where η is the influence coefficient of water-heat transfer. η is 158.46% when the rocks are sensitive to frost heaving, otherwise η is 1 and n is the rock porosity ratio.

Analytical Solution for Zone I

Zone I is regarded as a thick-walled cylinder, that is affected by frost heaving pressure σ_f. On the basis of the elastic theory (Xu, 2013), the stress of zone I can be expressed as:

$$\sigma_r^I = \frac{1-(r_a/r)^2}{1-(r_a/r_L)^2}\sigma_f \tag{11}$$

$$\sigma_\theta^I = \frac{1+(r_a/r)^2}{1-(r_a/r_L)^2}\sigma_f \tag{12}$$

Substituting Equations (11) and (12) into Equations (4) and (5), the strain of the lining can be obtained as:

$$\varepsilon_r^I = \frac{1+\mu_L}{E_L}\left[\frac{(1-2\mu_L)-(r_a/r)^2}{r_L^2-r_a^2}\right]\sigma_f r_L^2 \tag{13}$$

$$\varepsilon_\theta^I = \frac{1+\mu_L}{E_L}\left[\frac{(1-2\mu_L)+(r_a/r)^2}{r_L^2-r_a^2}\right]\sigma_f r_L^2 \tag{14}$$

Substituting Equation (14) into Equation (3), the radial displacement of the lining is written as:

$$u_r^I = \frac{(1+\mu_L)r}{E_L}\left[\frac{(1-2\mu_L)+(r_a/r)^2}{(r_L{}^2-r_a{}^2)}\right]\sigma_f r_L{}^2 \tag{15}$$

Analytical Solution for Zone II

According to the analytical solution for zone I, simultaneously, the inside and outside edges of zone II are subjected to frost heaving pressures σ_f and σ_h, respectively. Therefore, the stress of zone II can be written as:

$$\sigma_r^{II} = \frac{1-(r_f/r)^2}{1-(r_f/r_L)^2}\sigma_f + \frac{1-(r_L/r)^2}{1-(r_L/r_f)^2}\sigma_h \tag{16}$$

$$\sigma_\theta^{II} = \frac{1+(r_f/r)^2}{1-(r_f/r_L)^2}\sigma_f + \frac{1+(r_L/r)^2}{1-(r_L/r_f)^2}\sigma_h \tag{17}$$

Substituting Equations (16) and (17) into Equations (8) and (9), the strain of zone II is written as:

$$\varepsilon_r^{II} = \frac{1+\mu_f}{E_f}\left\{\frac{\left[2\mu_f-1+(r_f/r)^2\right]\sigma_f r_L^2+\left[1-2\mu_f-(r_L/r)^2\right]\sigma_h r_f^2}{r_f^2-r_L^2} - P_0(1-2\mu_f)\right\} - (1+\mu_f)\frac{\varepsilon_V}{3} \tag{18}$$

$$\varepsilon_\theta^{II} = \frac{1+\mu_f}{E_f}\left\{\frac{\left[2\mu_f-1-(r_f/r)^2\right]\sigma_f r_L^2+\left[1-2\mu_f+(r_L/r)^2\right]\sigma_h r_f^2}{r_f^2-r_L^2} - P_0(1-2\mu_f)\right\} + (1+\mu_f)\frac{\varepsilon_V}{3}\left(\frac{r_0-r}{r}\right) \tag{19}$$

Substituting Equation (19) into Equation (3), the radial displacement of zone II can be obtained as:

$$u_r^{II} = \frac{1+\mu_f}{E_f}\left\{\frac{\left[2\mu_f-1-(r_f/r)^2\right]\sigma_f r_L^2+\left[1-2\mu_f+(r_L/r)^2\right]\sigma_h r_f^2}{r_f^2-r_L^2} - P_0(1-2\mu_f)\right\} r + (1+\mu_f)\frac{\varepsilon_V}{3}(r_0-r). \tag{20}$$

Analytical Solution for Zone III

In addition, zone III is only affected by frost heaving pressure σ_h and the stress of zone III can be expressed as:

$$\sigma_r^{III} = P_0 - (P_0-\sigma_h)\left(\frac{r_f}{r}\right)^2 \tag{21}$$

$$\sigma_\theta^{III} = P_0 + (P_0-\sigma_h)\left(\frac{r_f}{r}\right)^2 \tag{22}$$

Substituting Equations (21) and (22) into Equations (6) and (7), the strain of zone III can be obtained as:

$$\varepsilon_r^{III} = \frac{(1+\mu_s)(\sigma_h-P_0)}{E_s}\left(\frac{r_f}{r}\right)^2 \tag{23}$$

$$\varepsilon_\theta^{III} = \frac{(1+\mu_s)(P_0-\sigma_h)}{E_s}\left(\frac{r_f}{r}\right)^2 \tag{24}$$

Substituting Equation (24) into Equation (3), the radial displacement of zone III is written as:

$$u_r^{III} = \frac{(1+\mu_s)(P_0-\sigma_h)}{E_s}\frac{r_f^2}{r} \tag{25}$$

Solution Steps

In Fig. 1, the calculation model for frost heaving pressure has two interfaces, where the continuous boundary equations of displacement and stress are satisfied at the two interfaces, shown as follows:

$$u_r^I = u_r^{II} \quad (r = r_L) \tag{26}$$

$$\sigma_r^I = \sigma_r^{II} \quad (r = r_L) \tag{27}$$

$$u_r^{II} = u_r^{III} \quad (r = r_f) \tag{28}$$

$$\sigma_r^{II} = \sigma_r^{III} \quad (r = r_f) \tag{29}$$

Equation (11) and Equation (16) obviously satisfy the continuous boundary Equation (27) and Equation (16) and Equation (21) obviously satisfy the continuous boundary Equation (29). Equations (15), (20) and (25) are substituted into Equation (26) and Equation (28) and the implicit equations of the radial stress σ_f and σ_h in the frozen zone are obtained as:

$$\sigma_h = \frac{A_1 B_1 + \frac{2(1-\mu_f{}^2) r_L^2}{E_f (r_f^2 - r_L^2)} B_2}{A_1 A_2 + \left[\frac{2(1-\mu_f{}^2)}{E_f (r_f^2 - r_L^2)}\right]^2 r_L^2 r_f^2} \tag{30}$$

$$\sigma_f = \frac{1}{A_1}\left[\frac{(1+\mu_f)(1-2\mu_f)}{E_f} P_0 - \frac{\varepsilon_V}{3}(1+\mu_f)\left(\frac{r_0 - r_L}{r_L}\right) - \frac{2(1-\mu_f{}^2) r_f^2}{E_f (r_f^2 - r_L^2)} \sigma_h\right] \tag{31}$$

where

$A_1 = \frac{(1+\mu_f)\left[(2\mu_f - 1) r_L^2 - r_f^2\right]}{E_f (r_f^2 - r_L^2)} - \frac{(1+\mu_L)\left[(1-2\mu_L) r_L^2 + r_a^2\right]}{E_L (r_L^2 - r_a^2)}$;

$A_2 = \frac{(1+\mu_s)}{E_s} + \frac{(1+\mu_f)\left[(1-2\mu_f) r_f^2 + r_L^2\right]}{E_f (r_f^2 - r_L^2)}$; $\quad B_1 = \frac{(1+\mu_s)}{E_s} P_0 + \frac{(1+\mu_f)(1-2\mu_f)}{E_f} P_0 - \frac{\varepsilon_V}{3}(1+\mu_f)\left(\frac{r_0 - r_f}{r_f}\right)$;

$B_2 = \frac{(1+\mu_f)(1-2\mu_f)}{E_f} P_0 - \frac{\varepsilon_V}{3}(1+\mu_f)\left(\frac{r_0 - r_L}{r_L}\right)$.

Therefore, the frost heaving pressures of cold-region tunnels are written as:

$$\Delta\sigma_f = \sigma_f - \sigma_{f0}$$

$$\Delta\sigma_h = \sigma_h - \sigma_{h0}$$

where $\Delta\sigma_f$ and $\Delta\sigma_h$ are the frost heaving pressures acting on the inside and outside edges of the frozen zone, respectively. σ_{f0} and σ_{h0} are the initial radial stresses acting on the inside and outside edges of the frozen zone before the frost heaving for surrounding rocks, respectively.

Parameter Analysis

From Equation (30) and Equation (31), the frost heaving pressures $\Delta\sigma_f$ and $\Delta\sigma_h$ are not only connected with the geometric size of the tunnel (the inside radius r_a of lining, the outside radius r_L of the lining and the outside radius r_f of the frozen zone), but are also connected with the mechanical parameters of the lining (the lining elastic modulus and Poisson's ratio (E_L,μ_L)) and the surrounding rocks (the elastic modulus and Poisson's ratio of the frozen zone (E_f,μ_f) and the unfrozen zone (E_s,μ_s), respectively). Simultaneously, the frost heaving pressures $\Delta\sigma_f$ and $\Delta\sigma_h$ are related to the frost heaving rate ε_V, the radius r_0 of zero-frost heaving displacement and *in-situ* stress P_0. The relevant calculation parameters of cold-region tunnels are listed in Table 1 (Feng et al., 2017, 2017). The initial linear frost heaving ratio ε_0 of surrounding rocks is 0.0055, radius r_0 of zero-frost heaving displacement is assumed to be 6.0 m and *in-situ* stress P_0 is assumed to be 2.0MPa. Thus, the single-factor analysis method is adopted to obtain the parameter sensitive analysis results of frost heaving pressures $\Delta\sigma_f$ and $\Delta\sigma_h$ of cold-region tunnels.

Table 1. The calculation parameters of cold-region tunnels

r_a / m	r_L / m	r_f / m	E_L / GPa	μ_L	E_f / GPa	μ_f	E_s / GPa	μ_s
4.5	5.0	7.0	28	0.16	7.8	0.35	4.6	0.35

Influence of Linear Frost Heaving Ratio

The variation law of frost heaving pressures $\Delta\sigma_f$ and $\Delta\sigma_h$ of surrounding rocks with the linear frost heaving ratio ε_0 is shown in Fig. 2. When the value of linear frost heaving ratio ε_0 gradually increases from 0 to 0.05%, 0.1%, 0.25% and 0.55%, the frost heaving pressure $\Delta\sigma_f$ increases from 0MPa to 0.387MPa, 0.773MPa, 1.934MPa and 4.254MPa, while the frost heaving pressure $\Delta\sigma_h$ increases from 0MPa to 0.323MPa, 0.645MPa, 1.613MPa and 3.549MPa. It is indicated that the frost heaving pressures $\Delta\sigma_f$ and $\Delta\sigma_h$ increase linearly with the increase of the linear frost heaving ratio ε_0. Meanwhile, it can be observed from Equation (10) that the calculation result of frost heaving ratio of surrounding rocks is related to their porosity ratio. The larger the porosity ratio is, the greater is the frost heaving ratio and the larger is the volume expansion of surrounding rocks resulting from water-ice phase change. Therefore, it is necessary to effectively control the porosity ratio and water migration of surrounding rocks for decreasing the frost heaving pressure.

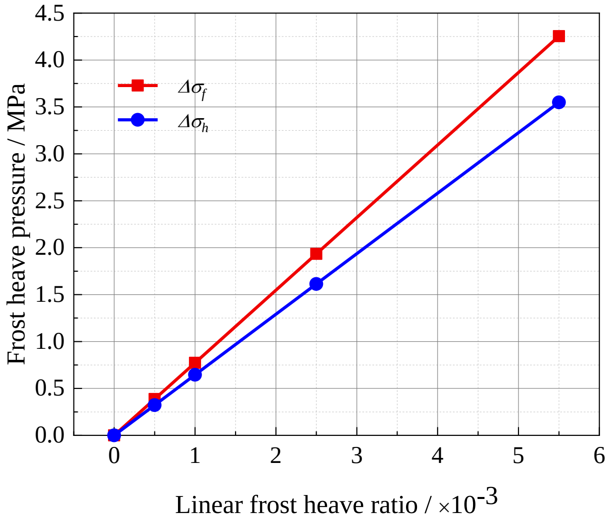

Figure (2): Variation of the frost heaving pressure with the linear frost heaving ratio of surrounding rocks

Influence of Radius of Zero-frost Heaving Displacement

As shown in Fig. 3, the variation law of frost heaving pressures $\Delta\sigma_f$ and $\Delta\sigma_h$ of surrounding rocks with the radius r_0 of zero-frost heaving displacement is obtained. As the value of r_0 gradually increases from 5.5m to 5.75m, 6.0m, 6.25m and 6.5m, the frost heaving pressure $\Delta\sigma_f$ increases from 3.665MPa to 3.959MPa, 4.254MPa, 4.549MPa and 4.849MPa, while the frost heaving pressure $\Delta\sigma_h$ decreases from 4.226MPa to 3.887MPa, 3.549MPa, 3.211MPa and 2.873MPa. It is indicated that frost heaving pressure $\Delta\sigma_f$ decreases and $\Delta\sigma_h$ increases with the increase of the radius r_0 of zero-frost heaving displacement, which means that the radius r_0 of zero-frost heaving displacement has an important effect on the frost heaving pressures $\Delta\sigma_f$ and $\Delta\sigma_h$. This is because the frozen zone expands in both directions toward the inside and outside edges and the increase of the radius r_0 of zero-frost heaving displacement can promote the frost heaving effect and the frost heaving pressure of the outer edge of the frozen zone.

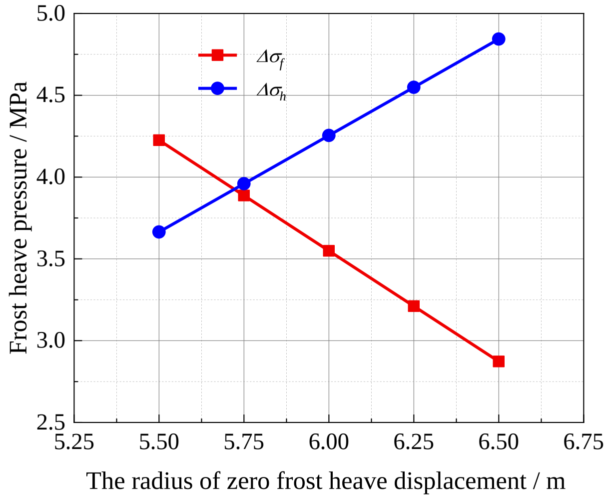

Figure (3): Variation of the frost heaving pressure with the radius of zero-frost heaving displacement

Influence of *in-situ* Stress

The variation law of frost heaving pressures $\Delta\sigma_f$ and $\Delta\sigma_h$ of surrounding rocks with the *in-situ* stress P_0 is shown in Fig. 4. When the value of r_0 gradually increases from 0MPa to 1.0MP, 1.5MPa, 2.0MPa, 2.5MPa and 3.0MP, the frost heaving pressure $\Delta\sigma_f$ increases from 3.443MPa to 3.848MPa, 4.051MPa, 4.254MPa, 4.457MPa and 4.660MPa, while the frost heaving pressure $\Delta\sigma_h$ increases from 2.030MPa to 2.789MPa, 3.169MPa, 3.549MPa, 3.929MPa and 4.309MPa. It is manifested that the frost heaving pressures σ_f and σ_h increase with the increase of *in-situ* stress P_0, which can be proved by Equations (30) and (31).

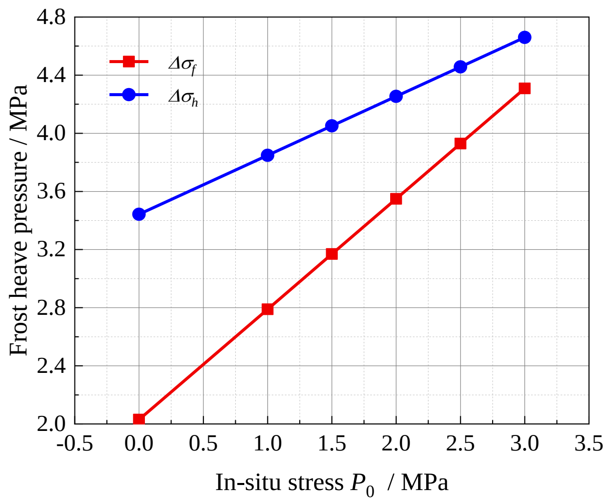

Figure (4): Variation of the frost heaving pressure with the *in-situ* stress

Analytical Solution of Frost Heaving Pressure Considering Freeze-thaw Cycles

Parameters of Surrounding Rocks under freeze-thaw Cycles

The analytical solution of frost heaving pressure for cold-region tunnels within frozen condition has been obtained, where the elastic modulus of surrounding rocks increases under the condition of frozen state. However, the surrounding rocks of cold-region tunnels undergo several freeze-thaw cycles, where the freeze-thaw damage of surrounding rocks is caused by freeze-thaw cycles, resulting in variations in mechanical parameters of surrounding rocks, such as the decrease of elastic modulus and the increase of volume frost heaving ratio of surrounding rocks. Additionally, it's supposed that freeze-thaw cycles only affect the mechanical parameters of frozen surrounding rocks. Therefore, the mechanical parameters of the surrounding rocks in the frozen zone are replaced by those of the surrounding rocks after *N* freeze-thaw cycles; namely, the analytical solutions of frost heaving pressure of surrounding rocks after *N* freeze-thaw cycles are obtained.

The influence of the number of freeze-thaw cycles is considered and frost heave pressures $\sigma_{f,N}$ and $\sigma_{h,N}$of surrounding rocks after *N* freeze-thaw cycles can be obtained by Equations (32) and (33).

$$\sigma_{h,N} = \frac{A_1 B_1 + \frac{2\left(1-\mu_f{}^2\right) r_L^2}{E_{f,N}\left(r_f^2 - r_L^2\right)} B_2}{A_1 A_2 + \left[\frac{2\left(1-\mu_f{}^2\right)}{E_{f,N}\left(r_f^2 - r_L^2\right)}\right]^2 r_L^2 r_f^2} \tag{32}$$

$$\sigma_{f,N} = \frac{1}{A_1}\left[\frac{(1+\mu_f)(1-2\mu_f)}{E_{f,N}} P_0 - \frac{\varepsilon_{V,N}}{3}\left(1+\mu_f\right)\left(\frac{r_0 - r_L}{r_L}\right) - \frac{2\left(1-\mu_f{}^2\right) r_f^2}{E_{f,N}\left(r_f^2 - r_L^2\right)} \sigma_{h,N}\right] \tag{33}$$

where $E_{f,N}$ and $\varepsilon_{V,N}$ are the elastic modulus and the volume frost heaving ratio of surrounding rocks after *N* freeze-thaw cycles, respectively.

Therefore, the frost heaving pressure of surrounding rocks after *N* freeze-thaw cycles can be written as:

$$\Delta\sigma_{f,N} = \sigma_{f,N} - \sigma_{f0,N} \tag{34}$$

$$\Delta\sigma_{h,N} = \sigma_{h,N} - \sigma_{h0,N} \tag{35}$$

where $\Delta\sigma_{f,N}$ and $\Delta\sigma_{\boxtimes,N}$ are the frost heave pressures acting on the inside and outside edges of the frozen zone after *N* freeze-thaw cycles, respectively, while $\sigma_{f0,N}$ and $\sigma_{\boxtimes 0,N}$ are the initial radial stresses acting on the inside and outside edges of the frozen zone before the frost heaving of surrounding rocks, respectively.

The Elastic Modulus of Surrounding Rocks

According to the research result of Tan et al. (2013), Galongla tunnel of Tibet is taken as the research object. The elastic modulus of surrounding rocks after *N* freeze-thaw cycles was measured by rock compression test, which can be fitted into a function of the number of freeze-thaw cycles, as shown in Fig. 5.

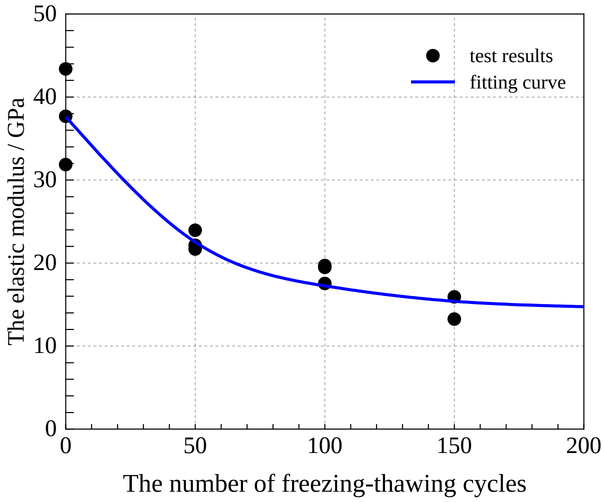

Figure (5): Relation between elastic modulus and the number of freeze-thaw cycles

According to Fig. 5, the elastic modulus of surrounding rocks decreases with freeze-thaw cycles and the fitting function relationship between the elastic modulus of surrounding rocks and the number of freeze-thaw cycles is as follows:

$$E_{f,N} = 14.40 + 23.23 e^{-0.021N} \tag{36}$$

Frost Heaving Ratio of Surrounding Rocks

The elastic modulus of the rocks decreases with the freeze-thaw cycles, the rock porosity increases and the frost heaving ratio of surrounding rocks calculated by Equation (10) increases significantly as the freeze-thaw cycles increase. Based on the damage-mechanics theory, the relationship between rock-damage variables and elastic modulus after *N* freeze-thaw cycles is as follows:

$$D_N = 1 - \frac{E_{f,N}}{E_0} = 1 - \frac{\rho_N V_{PN}^2}{\rho_0 V_P} = 1 - \frac{(1-n_N)V_{PN}^2}{(1-n_0)V_P} \quad (37)$$

where ρ_0 and ρ_N are the initial rock density and the rock density after Nfreeze-thaw cycles, respectively. V_P and V_{PN} are the initial longitudinal wave velocity and the longitudinal wave velocity after N freeze-thaw cycles, respectively. n_0 and n_N are the initial rock porosity ratio and the rock porosity ratio after N freeze-thaw cycles, respectively.

Gardner et al. (1974) obtained the relationship between rock density and the longitudinal wave velocity as follows:

$$\rho = kV_P^{1/4} \quad (38)$$

where k is constant.

Substituting Equation (38) into Equation (37), the rock void porosity n_N after N freeze-thaw cycles is:

$$n_N = 1 - (1 - n_0)\left(\frac{E_N}{E_0}\right)^{\frac{1}{9}}. \quad (39)$$

Substituting Equation (39) into Equation (10), the rock frost heave ratio ε_V after N freeze-thaw cycles is:

$$\varepsilon_V = 2.17\%\eta\left(1 - (1 - n_0)\left(\frac{E_N}{E_0}\right)^{\frac{1}{9}}\right). \quad (40)$$

To prove the rationality of the proposed calculation formula of frost heaving ratio, the frost heaving property of surrounding rocks and the corresponding range of frost heaving ratio were determined based on the existing literature (Xia et al., 2013). Combined with the existing literatures (Liu et al., 2019; 2020), comparative analysis was performed with the calculation results of rock frost heaving ratio established in this study. In addition, the comparative calculation results are shown in Table 2 and the variation law for rock frost heaving ratio under the condition of N freeze-thaw cycles is shown in Fig. 6.

Table 2. Comparison of calculation results of frost heaving ratio considering freeze-thaw cycles

Literature	Number of freeze-thaw cycles	Calculation formula of porosity	Porosity	Calculation formula of frost heaving ratio	frost heaving ratio ($\times 10^{-3}$)	Range of frost heaving rate (×10-3)	Frost heaving feature
Liu (2019)	0 50 100 150	$1-(1-n_0)\left(\frac{E_{f,N}}{E_s}\right)$	0.0067 0.4054 0.5012 0.6150	$2.17\%\eta\left[1-(1-n_0)\left(\frac{E_{f,N}}{E_s}\right)\right]$	0.1454 8.7981 10.8768 13.3450	<1.3 8.0~16.0 8.0~16.0 8.0~16.0	no strong strong strong
Liu (2020)	0 50 100 150	$\frac{1}{s}ln\left(\frac{E_{f,N}}{E_s}\right)+n_0$	0.0067 0.0409 0.0526 0.0699	$2.17\%\eta\left[\frac{1}{s}ln\left(\frac{E_{f,N}}{E_s}\right)+n_0\right]$	0.1454 0.8878 1.1420 1.5164	<1.3 <1.3 <1.3 1.3~4.7	no no no weak
This study	0 50 100 150	$1-(1-n_0)\left(\frac{E_{f,N}}{E_s}\right)^{1/9}$	0.0067 0.0617 0.0799 0.1060	$2.17\%\eta\left[1-(1-n_0)\left(\frac{E_{f,N}}{E_s}\right)^{1/9}\right]$	0.1454 1.3401 1.7337 2.2997	<1.3 1.3~4.7 1.3~4.7 1.3~4.7	no weak weak weak

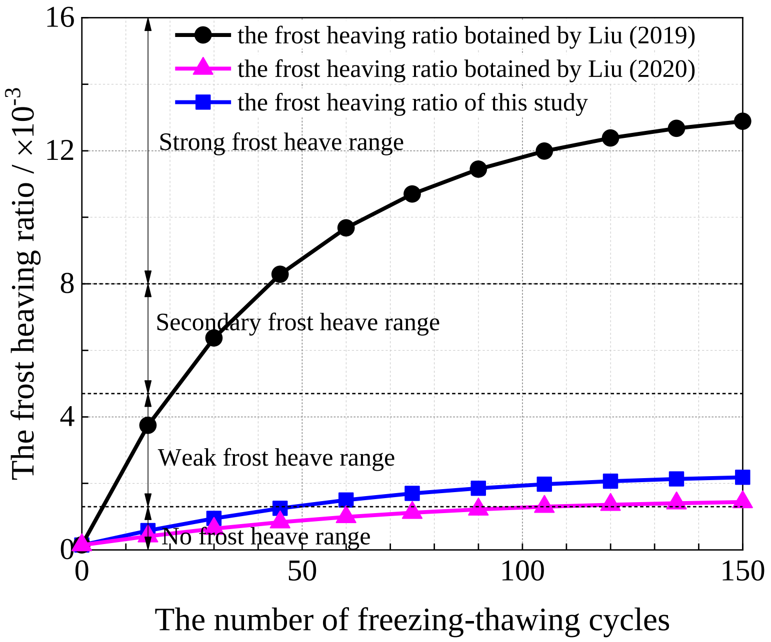

Figure (6): Relation between frost heaving ratio and the number of freeze-thaw cycles

From Fig. 6, with the increase of freeze-thaw cycles, rock porosity and frost heaving ratio gradually increase and the change trend slows down gradually, which indicates that the porosity expansion speed of surrounding rocks slows down gradually. Meanwhile, the comparative variation law of rock frost heaving ratio is shown in Table 3. The calculation result obtained by Liu et al. (2019) is too large due to the assumption of $V_P = V_{PN}$ and the calculation result obtained by Liu et al. (2020) is too small by considering the influence of the fitted coefficient s. However, the calculation formula of frost heaving ratio established in this study takes into account the change of V_{PN} and is not affected by the fitted coefficient s. Therefore, with the increase of freeze-thaw cycles, the value of frost heaving ratio and the variation trend of frost heaving feature obtained in this study are more reasonable.

Example Analysis

To study the variation law of frost heaving pressure of surrounding rocks with the freeze-thaw cycles, Galongla tunnel of Tibet was taken as the research object (Tan et al., 2013). The tunnel size model is shown in Fig. 1 and its calculation parameters are shown in Table 3.

Table 3. The calculation parameters of cold-region tunnels

r_a/m	r_L/m	r_f/m	E_L/GPa	μ_L	$E_{f,0}$/GPa	μ_f	E_s/GPa	μ_s	P_0 /MPa
3.0	3.6	5.0	10	0.3	37.64	0.25	37.64	0.25	2.0

It has been shown that frost heaving pressure of surrounding rocks is not only connected with the tunnel size and mechanical parameters, but also with the frost heaving rate ε_V, the radius r_0 of zero-frost heaving displacement and the *in-situ* stress P_0. Therefore, the influences of three factors on frost heaving pressure are analyzed in the following content.

Radius of Zero-frost Heaving Displacement

To obtain the radius of zero-frost heaving displacement with the freeze-thaw cycles, the frost heaving effect of rocks is only considered. Therefore, the calculation model of frost heaving is shown in Fig. 7.

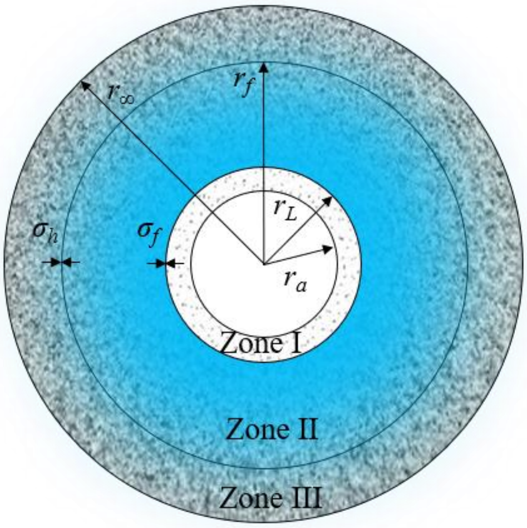

Figure (7): Calculation model with only considering frost heaving effect

The volume expansion of surrounding rocks occurs at the frozen zone and the deformation directions in the inside and outside edges for the frozen zone are opposite. The displacement at $r = r_L$ is u_1 and the displacement at $r = r_f$ is $-u_2$. There is a position in the frozen zone where the frost heaving displacement is 0 and the radius of zero-frost heaving displacement is r_0 (see Fig. 8).

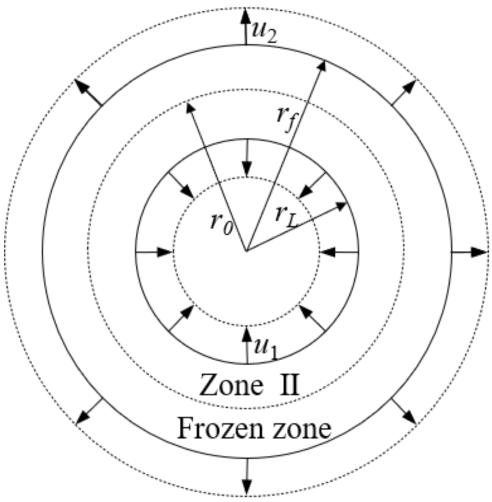

Figure (8): Deformation direction of frost heaving displacement in the frozen zone

It is assumed that frost heaving displacement is linear and the radius of zero-frost heaving displacement is obtained as follow:

$$\frac{r_f-r_0}{r_0-r_L} = -\frac{u_2}{u_1} \tag{41}$$

According to Equation (20), the displacement at the inside and outside edges of the frozen zone can be expressed as:

$$u_1 = \frac{1+\mu_f}{E_f}\left\{\frac{\left[2\mu_f-1-(r_f/r_L)^2\right]\sigma_f r_L^2+[2-2\mu_f]\sigma_h r_f^2}{r_f^2-r_L^2}\right\} r_L + (1+\mu_f)\frac{\varepsilon_V}{3}(r_0-r_f); \tag{42}$$

$$u_2 = \frac{1+\mu_f}{E_f}\left\{\frac{[2\mu_f-2]\sigma_f r_L^2+\left[1-2\mu_f+(r_L/r_f)^2\right]\sigma_h r_f^2}{r_f^2-r_L^2}\right\} r_f + (1+\mu_f)\frac{\varepsilon_V}{3}(r_0-r_f). \tag{43}$$

Therefore, the frost heaving pressure with only considering frost heave can be obtained by Equations (30) and (31).

$$\sigma'_h = \frac{A'_1 B'_1+\frac{2(1-\mu_f^2)r_L^2}{E_f(r_f^2-r_L^2)}B'_2}{A'_1 A'_2+\left[\frac{2(1-\mu_f^2)}{E_f(r_f^2-r_L^2)}\right]^2 r_L^2 r_f^2} \tag{44}$$

$$\sigma'_f = \frac{1}{A'_1}\left[-\frac{\varepsilon_V}{3}(1+\mu_f)\left(\frac{r_0-r_L}{r_L}\right)-\frac{2(1-\mu_f^2)r_f^2}{E_f(r_f^2-r_L^2)}\sigma'_h\right] \tag{45}$$

where

$$A'_1 = \frac{(1+\mu_f)\left[(2\mu_f-1)r_L^2-r_f^2\right]}{E_f(r_f^2-r_L^2)} - \frac{(1+\mu_L)\left[(1-2\mu_L)r_L^2+r_a^2\right]}{E_L(r_L^2-r_a^2)};$$

$$A'_2 = \frac{(1+\mu_s)}{E_s} + \frac{(1+\mu_f)\left[(1-2\mu_f)r_f^2+r_L^2\right]}{E_f(r_f^2-r_L^2)};$$

$$B'_1 = -\frac{\varepsilon_V}{3}(1+\mu_f)\left(\frac{r_0-r_f}{r_f}\right);$$

$$B'_2 = -\frac{\varepsilon_V}{3}(1+\mu_f)\left(\frac{r_0-r_L}{r_L}\right).$$

Considering the influence of freeze-thaw cycles, the solution steps for calculating the radius of zero-frost heaving displacement are as follows:

(1) r_0 is assumed to be $\frac{r_L+r_f}{2}$ and the frost heaving pressures $\sigma'_{h,0}$ and $\sigma'_{f,0}$ can be obtained by substituting r_0, ε_V and E_f into Equations (44) and (45).

(2) The displacements at the inside and outside edges of the frozen zone are obtained by substituting $\sigma'_{h,0}$ and $\sigma'_{f,0}$ into equations (42) and (43) and the new radius

r_0 of zero-frost heaving displacement is obtained by Equation (41).

(3) According to solution steps (1) and (2), the newly obtained r_0 is repeatedly computed until the difference between the n^{th} result and the $(n+1)^{th}$ result is less than 0.001; namely, $|r_0(n+1)-r_0(n)|<0.001$. Meanwhile, $r_0(n+1)$ is considered as the final result for the radius of zero-frost heaving displacement.

(4) The radius $r_{0,N}$ of zero-frost heaving displacement at the N^{th} freeze-thaw cycles can be solved by substituting $\varepsilon_{v,N}$ and $E_{f,N}$ into solution steps (1) ~ (3).

The detailed solution process of the radius of zero-frost heaving displacement is shown in Fig. 9.

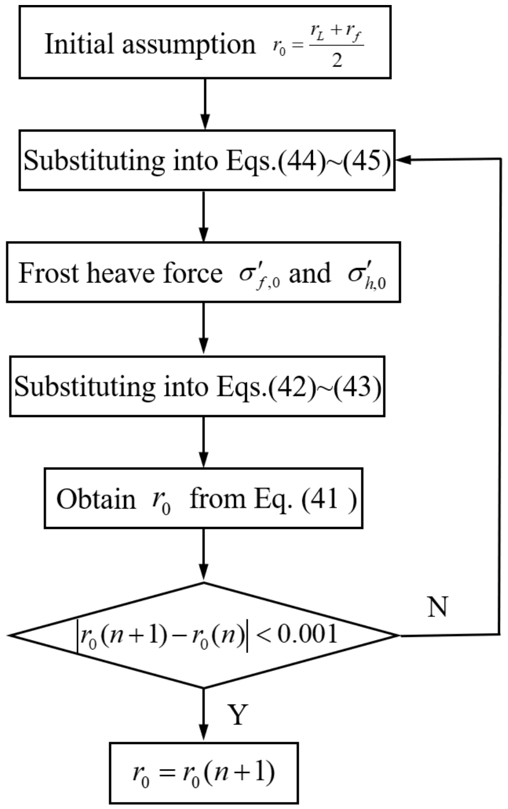

Figure (9): Solution process of radius of zero-frost heaving displacement

As shown in Fig. 9, the radius r_0 of zero-frost heaving displacement is solved by using calculation software under different freeze-thaw cycles and the calculation result is shown in Table. 4. It is clear that the radius r_0 of zero-frost heaving displacement is stable at 4.110m with the freeze-thaw cycles. Therefore, r_0 is taken as 4.110m in the following content.

Table 4. The radius of zero-frost heaving displacement with the freeze-thaw cycles

The number of freeze-thaw cycles	**0**	**50**	**100**	**150**
Radius of zero-frost heaving displacement/m	4.125	4.108	4.107	4.109

Frost Heaving Pressure of Surrounding Rocks

The frost heaving pressures $\Delta\sigma_{f,N}$ and $\Delta\sigma_{h,N}$ after N freeze-thaw cycles are written as Equations (34) and (35). The *in-situ* stress is assumed to be 2MPa, the radius r_0 of zero-frost heaving displacement is 4.110 m and the frost heaving ratio $\varepsilon_{V,N}$ is obtained from Table 2. Therefore, the variation law curve of frost heaving pressures $\Delta\sigma_{f,N}$ and $\Delta\sigma_{h,N}$ of surrounding rocks with the freeze-thaw cycles is verified by calculation case and shown in Fig. 10. It is found that the frost heaving pressures $\Delta\sigma_f$ and $\Delta\sigma_h$ increase as the number of freeze-thaw cycles increases. The frost heaving pressure $\Delta\sigma_{f,N}$ increases from 0 to 0.695MPa after 200 freeze-thaw cycles and gradually reaches a steady value. Moreover, the frost heaving pressure $\Delta\sigma_{h,N}$ increases from 0 to 0.925MPa after 200 freeze-thaw cycles and gradually tends to be stable. This is because the rock void porosity and the rock frost heaving ratio increase gradually. With the increase in the number of freeze-thaw cycles, the expansion ratio of rock pores slows down gradually and the rock damage and the rock void porosity gradually become stable.

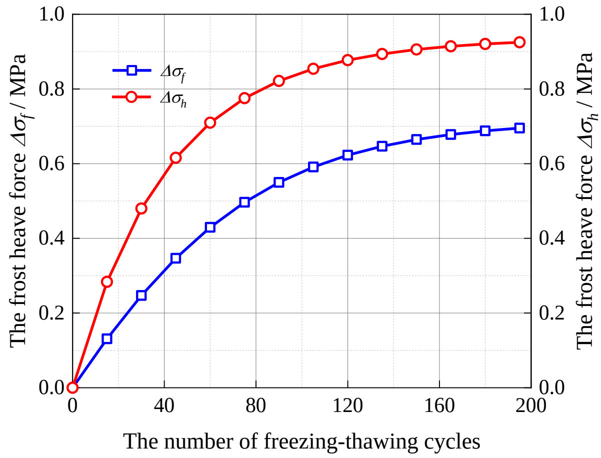

Figure (10): Variation of frost heaving pressure with the number of freeze-thaw cycles

In order to obtain the variation law of frost heaving pressure affected by the *in-situ* stress P_0 and the number of freeze-thaw cycles, the analysis results of frost heaving pressure $\Delta\sigma_{f,N}$ for different values of P_0 are obtained. As shown in Fig. 11, The frost heaving pressure $\Delta\sigma_{f,N}$ for P_0 values of 0MPa, 2MPa, 4MPa, 6MPa, 8MPa and 10MPa with the number of freeze-thaw cycles is obtained. The results indicate that *in-situ* stress has a significant impact on the frost heaving pressure. Additionally, with the increase of *in-situ* stress P_0, the frost heaving pressure $\Delta\sigma_{f,N}$ increases from 0.629MPa to 0.959MPa after 200 freeze-thaw cycles. This indicates that frost heaving pressure acting on the lining increases with *in-situ* stress. Therefore, it is necessary to apply insulation and antifreeze measures to cold-region tunnels for the safety of lining structure.

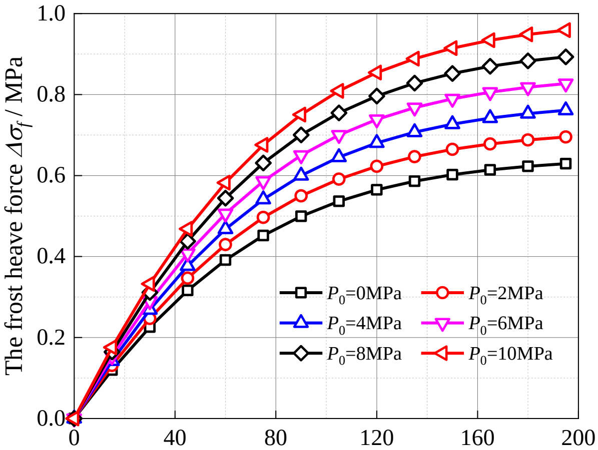

Figure (11): Variation of frost heaving pressure $\Delta\sigma_{f,N}$ with *in-situ* stress and number of freeze-thaw cycles

CONCLUSIONS

In this paper, the analytical solutions of frost heaving pressure for cold-region tunnels considering freeze-thaw cycles and *in-situ* stress are obtained using the elastic theory. The conclusions drawn are:

(1) The calculation model of frost heaving pressure for cold-region tunnels considering the influences of the frost heaving and *in-situ* stress is established and analytical solutions of frost heaving pressure for cold-region tunnels are analyzed by the elastic theory. Meanwhile, the single-factor analysis method was used to obtain the variation law of frost heave pressures $\Delta\sigma_f$ and $\Delta\sigma_h$ with factors such as linear frost heaving ratio ε_0, radius r_0 of zero-frost heaving displacement and *in-situ* stress P_0.

(2) The elastic modulus $E_{f,N}$ after N freeze-thaw cycles was fitted by the compression test data, which shows that elastic modulus $E_{f,N}$ decreases with the freeze-thaw cycles. Also, the frost heaving ratio $\varepsilon_{V,N}$ gradually increases and eventually is subjected to a steady value. The formula for calculating the frost heaving ratio is proposed, which considers the change of the longitudinal wave velocity and the rock porosity ratio. Additionally, the rationality for this calculation formula is verified by contrastive analysis of frost heaving feature.

(3) The radius r_0 of zero-frost heaving displacement is stable at 4.110m with the freeze-thaw cycles. After 200 freeze-thaw cycles, the frost heaving pressures $\Delta\sigma_{f,N}$ and $\Delta\sigma_{h,N}$ gradually tend to be stable, increasing from 0 to 0.659MPa and 0.925MPa with the condition of P_0=2MPa. The frost heaving pressure $\Delta\sigma_{f,N}$ increases with *in-situ* stress, which means that insulation and antifreeze measures should be applied in the deep positions of cold-region tunnels.

Data Availability

All calculated results are included in this paper.

Conflict of Interest

The authors declare that there is no conflict of interest.

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