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Passive Earth Pressure of Normally and Over-consolidated Cohesionless Soil Retaining Horizontal and Inclined Backfills Overlaying Natural Deposit

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ABSTRACT

Passive earth pressures play an important role in soil-structure interaction problems. They provide stabilizing forces for anchor blocks, laterally loaded pile foundations and retaining walls. Quite often, passive earth pressures are used to resist lateral movement of structures.

The passive earth pressure behind retaining walls depends on the strength of the backfill material, the wall-soil frictional angle and the stress history of the soil. The stress history, which is represented by the so-called over-consolidation ratio (OCR), is a major governing parameter in the determination of passive earth pressure on the wall.

The present paper presents a theoretical model to predict the coefficient of passive earth pressure for the case of normally consolidated or over-consolidated cohesionless backfill overlaying deep deposit. The theory utilizes the method of slices and the limit equilibrium method of analysis. Design theory, design charts and design examples are presented. Coefficients of passive earth pressure as deduced by the present study and Yong and Qian (2000) method were very close for example at low friction angles, such as φ of 10° and δ_p of 10°. The present study estimates K_p at 1.67, while it was estimated by Yong and Qian to be 1.66. At higher values of friction as φ of 30º and δ_p of 15°, the present study estimates K_p at 4.29, while it was estimated by Yong and Qian to be 4.61.

KEYWORDS: Passive earth pressure, Over-consolidated cohesionless soil, Inclined backfill, Deep deposit.

INTRODUCTION AND BACKGROUND

The most important component in the design of rigid retaining walls, which are widely used in engineering practice, is the determination of the coefficients of active and passive lateral earth pressures (Al-Zoubi, 2015). The determination of soil characteristics and accordingly an appropriate coefficient of passive earth pressure is one of the major obstacles in geotechnical engineering. In the literature, several reports can be found dealing with the subject matter. Coulomb (1776) first suggested a mathematical solution for the calculation of earth pressure behind a retaining wall, taking into consideration the wall-soil friction angle δ_p. Rankine (1875) introduced a mathematical solution assuming that the earth mass is in a plastic equilibrium state. Terzaghi (1941) adopted a method for predicting the passive earth pressure assuming that the failure surface consists of two parts, logarithmic spiral and straight plane. Terzaghi and Peck (1948) developed empirical charts for predicting lateral pressures. Shields and Tolunay (1972) adopted Terzaghi's failure mechanism and used the method of slices similar to that of Bishop for slope stability analysis to compute the values of passive earth pressure P_p. Khoury (1994)

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studied the passive earth pressure of over-consolidated homogeneous and layered cohesionless soil experimentally. Kumar and Subba (1997) developed comprehensive charts based on an assumed failure surface consisting of a logarithmic spiral part and a plane part to determine the magnitudes of passive earth pressure coefficients. Yong and Qian (2000) proposed a procedure for determining the passive earth pressure coefficients using triangular slices within the framework of the limit equilibrium method. The soil mass behind the wall is divided into a series of triangular slices. The interslice forces are expressed in terms of a force coefficient. They derived equations for solving the interslice coefficients and then the passive earth pressure coefficient is determined by using the principle of optimality (the critical inclinations of the slice bases, minimum interslice force coefficients).

Utilizing a kinematical method of the limit analysis theory, the passive earth pressure problem was studied by Soubra (2000). The static and seismic passive earth pressure coefficients were estimated in the base of the translational kinematically admissible failure mechanism composed of a sequence of rigid triangles.

In practice, Coulomb's (1776) and Rankine's (1857) theories are widely used for predicting the passive earth pressure on retaining walls. The logarithmic spiral theory is less popular due to its complexity; however, it provides predictions that are more accurate than those given by the empirical formulae. A number of investigators (Liu et al., 2018; Soubra and Regenass, 2000) have developed alternative procedures using the logarithmic spiral theory for evaluating the coefficient of passive earth pressure (K_P). The results confirmed the accuracy of the logarithmic spiral theory for a wide range of angle of shearing resistance, φ and the angle of wall-soil friction, δ_p. For soils with cohesion, Bell (1915) developed an analytical solution that used the square root of the pressure coefficient to predict the cohesion's contribution to the overall resulting pressure. Many other reports can be found in literature dealing with the coefficient of passive earth pressure; i.e., Benmebarek et al. (2006), Benmebarek et al. (2008), Yu-Feng et al. (2008) and Yang and Yin (2008).

Benmebarek et al. (2008) estimated the 3D passive earth pressure coefficients for retaining walls subjected to translation using a numerical evaluation and found 3D passive earth pressure coefficients ($K_{pc(3D)}$, $K_{pq(3D)}$ and $K_{pc(3D)}$) which are affected by soil weight, surcharge loading and soil cohesion, while Anato et al. (2011) employed the three-dimensional numerical limit analysis upper bound theorem to determine the horizontal passive earth pressure coefficients.

Some of studies are concerned with finding the passive earth pressure for cohesionless soil; specifically, Benmeddour et al. (2012) used a series of dimensional finite element analyses to find a passive earth pressure coefficient for various geometrical and mechanical configurations of backfill retained by a vertical rigid wall. Recent study was achieved to find the passive earth pressure coefficient for a vertical retaining wall with inclined cohesionless backfill; Patki et al. (2015) used a method of a complete log spiral failure mechanism in such study. They utilized a limit equilibrium method in their investigation. On the other side, passive earth pressure coefficients were found for an inclined rigid retaining wall with inclined cohesionless backfill using Kotter's equation (Patki et al., 2016).

Liu et al. (2018) proposed using a modified logarithmic spiral method to determine the passive earth pressure and failure surface of cohesionless inclined backfill with the existence of wall- soil interface friction. Deng and Yang (2019) analyzed the passive earth pressure for unsaturated retaining structure, while Cao et al. (2019) calculated passive earth pressure using a simplified principle stress trajectory method of rigid retaining walls.

Nevertheless, little was reported with respect to passive earth pressure of over-consolidated cohessionless soils and the effects of natural deposit below the backfill.

Table 1 presents a comparison between the coefficients of passive earth pressure as deduced from several theories. It can be noted that the theory of Soubra and Regenass (2000) and the theory of Yong (2000) have almost produced the same results. Also, the results of Rankine's theory which is valid for smooth retaining walls (δ_p= 0) were given. Caqout and Kerisel's (1948) theory which gives good estimates for the case of $\varphi = \delta_p$ and the theory of Shield and Tolunay (1973) for the case of δ_p/φ <1.0 are given. It can be noted that, when the ratio δ_p/φ becomes closer to 1.0, Shields and Tolunay's method underestimates the coefficient of the passive earth pressure. As the value of δ_p increases, Coulomb's theory predicts increasingly erroneous values of passive earth pressure coefficients. However, Yong and Qian's theory provides better predictions under all these conditions, as it takes the effects of all these variables into consideration.

Table 1. Coefficients of passive earth pressure, K_p for normally consolidated cohesionless soils

φ (º)	δ (º)	Caqout and Kerisel (1948)	Rankine (1857)	Coulomb (1776)	Shields and Tolunay (1973)	Soubra and Regenass (2000)	Yong and Qian (2000)
20	0	------	2.040	2.040	2.040	2.040	2.040
	10	------	-----	2.640	2.520	2.580	2.560
	20	3.000	-----	3.520	2.880	3.120	3.060
30	0	-----	3.000	3.000	3.000	3.000	3.000
	15	-----	-----	4.980	4.450	4.690	4.610
	30	6.450	-----	10.10	5.770	6.860	6.590
40	0	-----	4.599	4.600	4.600	4.600	4.590
	20	-----	-----	11.770	9.080	9.990	9.660
	40	17.500	-----	92.580	14.420	19.620	18.240

Yong and Qian (2000) developed the following equation to predict the coefficient of passive earth pressure for homogeneous normally consolidated sand by considering force equilibrium for the passive Rankine zone (Eq. 1):

$$K_{PR} = \frac{sin(\theta_o + \alpha_o)\,sin(\theta_o)\,sin(\alpha_o + \phi\,)}{sin(\alpha_o)\,sin(\theta_o + \alpha_o + 2\phi\,)} \qquad (1)$$

The boundary conditions of Rankine zone were evaluated and used to calculate the stresses on the first slice in the deformation zone. Thus, by considering force equilibrium for the first slice in the deformation zone, the coefficient of passive earth pressure was given by Yong and Qian (2000) as:

$$K_1 = \left(\frac{sin(\theta_1 + \alpha_1)}{sin(\theta_0 + \alpha_1)\,sin(\theta_1 + \alpha_1 + \delta_1 + \phi\,)} \right) \times \left(K_{PR} \frac{sin(\theta_1 + \alpha_1)\,sin(\theta_0 + \alpha_1 + \delta_1 + \phi)}{sin(\theta_0 + \alpha_1)} + sin(\theta_1 - \theta_0)\,sin(\alpha_1 + \phi) \right) \qquad (2)$$

where;

φ : Angle of shearing resistance of the soil.

α_1 : First-slice base inclination.

δ_1 : Interslice frictional angle for the first slice.

θ_1 : Angle of the first slice from the backfill surface.

For the remaining slices, the coefficient of passive earth pressure was given as:

$$K_i = \left(\frac{sin(\theta_i + \alpha_i)}{sin(\theta_{i-1} + \alpha_i)\,sin(\theta_i + \alpha_i + \delta_i + \phi\,)} \right) \times \left(K_{i-1} \frac{sin(\theta_i + \alpha_i)\,sin(\theta_{i-1} + \alpha_i + \delta_{i-1} + \phi\,)}{sin(\theta_{i-1} + \alpha_i)} + sin(\theta_i - \theta_{i-1})\,sin(\alpha_i + \phi) \right) \qquad (3)$$

For i = 2, 3, 4…n.

where;

φ : Angle of shearing resistance of the soil.

α_i : i^{th}- slice base inclination.

δ_i : Interslice frictional angle of slice (i).

δ_{i-1} : Interslice frictional angle of the previous slice $(i-1)$.
θ_i : Angle of slice (i) from the backfill surface.
θ_{i-1} : Angle of the previous slice $(i-1)$ from the backfill surface.
K_{i-1} : Passive earth pressure coefficient of the previous slice $(i-1)$.

In the present investigation, the theoretical model developed by Yong and Qian (2000) to predict the coefficient of passive earth pressure of normally consolidated cohesionless soils is modified for the case of over-consolidated cohesionless horizontal and inclined backfills behind walls. Moreover, the theoretical model is further extended to predict the coefficient of passive earth pressure for the case of horizontally or inclined normally consolidated or over-consolidated cohesionless backfill overlaying deep deposit. The procedure for determining the passive earth pressure coefficients using triangular slices within the framework of the limit equilibrium method was adopted.

ANALYTICAL MODEL

The case of a vertical wall retaining a horizontal backfill made of homogeneous over-consolidated cohesionless soil subjected to passive earth pressure, is considered, Figure 1. In this analysis, the failure plane consists of a curve below the wall tip, then it continues as a plane until the ground level, Figure 2. The failing area can be divided into two zones; the deformation zone made of a logarithmic spiral curve and the Rankine zone. The deformation zone will be sub-divided into (n) number of triangular slices, where each will be analyzed using the limit equilibrium method of analysis. The Rankine zone will be regarded as one solid mass, which behaves as a rigid body. The Rankine zone, θ_o and the inclination of the base, α_o are determined according to Rankine, as follows:

$$\theta_o = 45^o - \frac{\phi}{2} \tag{4}$$

$$\alpha_o = \frac{\pi}{2} - \theta_o - \phi \tag{5}$$

The coefficient of passive earth pressure in the Rankine zone K_{PR} will be replaced by the formula of Yong and Qian (2000), which takes the effect of over-consolidation into account in terms of the over-consolidation ratio (OCR) as follows:

$$K_{PR} = \frac{b\sqrt{OCR}\,sin(\theta_o + \alpha_o)\,sin(\theta_o)\,sin(\alpha_o + \phi)}{sin(\alpha_o)\,sin(\theta_o + \alpha_o + 2\phi)} \tag{6}$$

where;
OCR = Over-consolidation ratio.
φ = Angle of shearing resistance of the backfill material.
b = A factor which depends on the angle of shearing resistance of the backfill material and is given by Khoury (1994) for OCR>1 and $\varphi < 45^o$ as follows:

$$b = \frac{1.25}{cos(45 - \varphi)} \tag{7}$$

θ_o and α_o = Previously defined.

The angle of shearing resistance of the backfill is φ, the unit weight of the backfill is γ and the frictional angle of the soil-wall interface is δ_p.

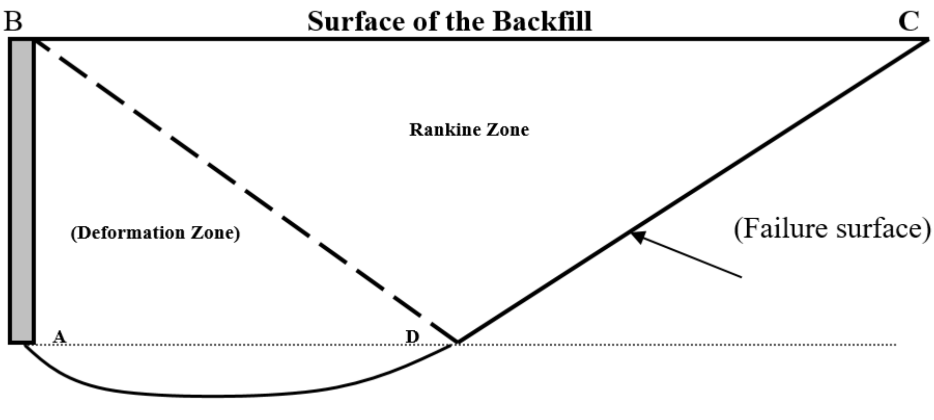


Figure (1): Failure mechanism behind a retaining wall subjected to passive conditions, showing Rankine and deformation zones

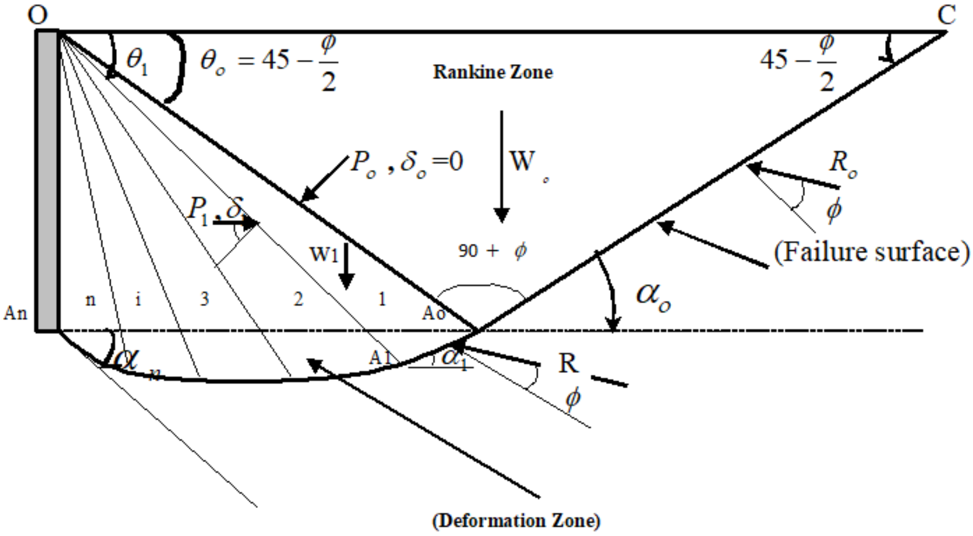


Figure (2): Failure mechanism behind a retaining wall subjected to passive conditions

Figure 3 presents the stresses acting on a typical slice (i). The resultant R_i is acting on the middle of the slice base, which makes an angle of φ with the normal. The forces P_i and P_{i-1} are acting at one-third of the radial lines $\overline{OA_i}$ and $\overline{OA_{i-1}}$ respectively. Then, the values of P_i, R_i and w_i as well as the lateral force P_p and its location can be determined by satisfying the moment equilibrium condition.

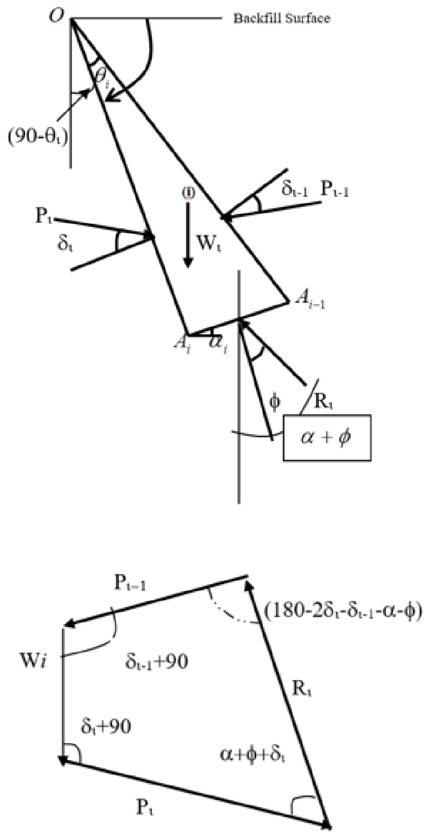


Figure (3): Slice forces' hodograph for a typical slice (*i*)

The passive earth pressure acting on the last slice (n), will be equivalent to the applied passive earth pressure on the wall, K_p. In this analysis, there are n unknowns (i.e., n slice base inclinations, α_1, $\alpha_2 \ldots \alpha_n$). The process of determining slice base inclination and the inter-slice force coefficients may be regarded as a multi-stage decision problem, in which K_i's are state variables and α_i's are decision variables. It could be noticed that K_i is related to the immediately previous coefficient, K_{i-1} and current slice base inclination, α_i.

Knowing the value of K_{PR}, the minimization of K_1 with respect to α_1 can be achieved by a procedure of simple optimization technique that will be used in this analysis, using the mathematical formulae that will provide the minimum passive earth pressure coefficient. The process is repeated for K_2, α_2…... K_n, α_n. With α_n, α_{n-1},……..,α_1,α_o known, the critical failure surface is immediately traced from the tip of the wall out to the backfill surface. Such process of K_p along with determination of the critical failure surface consists of n stages of minimization of K_i involving only one unknown, say α_i at each stage in order that K_n or K_p achieves a minimum. The effect of over-consolidation will transform to next slices in the deformation zone using Equations 2 and 3.

The above method of triangular slices is coded in a computer program of MATLAB (version 6.1) after minor changes: Due to the wall roughness, vertical shear stress was applied on the soil close to the wall; this shear stress will dissipate in between the first slice and Rankine zone along the radial line $\overline{OA_o}$. By definition, this zone ($\overline{OA_o}C$), as shown in Figure 1, is in Rankine passive state. The inter-slice angle of friction varied between values of zero at Rankine's zone to the maximum of wall-soil friction angle behind the wall, δ_p. The relationship was assumed to be linear as follows:

$$\delta_i = \delta_o + \frac{i}{n} \times \delta_p \qquad (8)$$

The coding procedure of simple optimization technique that will be used in this analysis was divided into the following steps:

1. The deformation zone $\overline{OA_o}A_n$ is sub-divided into (n) number of slices that have equal sub-angles; then, taking the horizontal backfill as a datum, the values of θ_i increase linearly according to the following proposed formula:

$$\theta_i = \left(\frac{\pi}{2} - \theta_o\right) \times \frac{i}{n} + \theta_o \qquad (9)$$

In order to determine the minimum coefficient of passive earth pressure, the angle θ_o (Eq. 4) was varied in a wide range (i.e., different values of φ were considered).

2. For the last slice, n (next to the wall), the angle at which the logarithmic spiral will depart from the bottom of the wall was determined by using Shields and Tolunay's formula as follows:

$$\alpha_n = \left(\frac{1}{2}\right)\left[arccos\{ \ cos(\varphi - \delta_p) - \frac{sin(\varphi - \delta_p)}{tan\,\phi} \ \} - \varphi - \delta_p \right] \qquad (10)$$

3. Referring to Figure 4, the tangential angle of the slice to the right side of the horizontal tangent of the curve ED, related to Rankine failure plane, which is inclined with the horizontal α_o, is given by the following formula:

$$\alpha_{(r,i)} = \left(\frac{i-10}{0.5 \times n}\right)^2 \times \alpha_o \qquad (11)$$

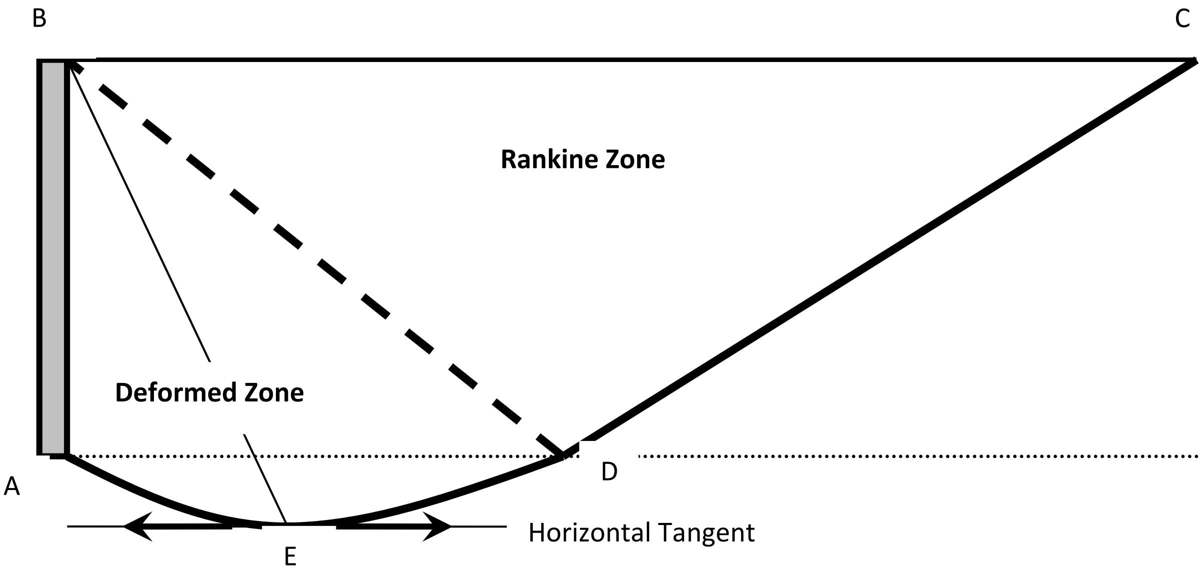
The tangential angle of the slice to the left side of the horizontal tangent of the curve EA, related to the last slice (n) and the base inclination α_n, is given by the following formula:

$$\alpha_{(l,i)} = \left(\frac{i-10}{0.5 \times n}\right)^2 \times \alpha_n \qquad (12)$$

These formulae are dependent on the number of slices that are used in the optimization process. After several trials, values of the passive earth pressure coefficients were determined and are given in Table 2. The proposed method was calibrated against Yong and Qian's method for normally consolidated backfill, where good agreement was noted.

Table 2. Coefficients of passive earth pressure as deduced by the present study and Yong and Qian (2000) method

φ^{o}	δ_p	K_p	
		Present study	**Yong and Qian (2000)**
10	0	1.48	1.42
	5	1.57	1.56
	10	1.67	1.66
20	0	2.18	2.04
	10	2.51	2.56
	20	2.99	3.06
30	0	3.27	3.00
	15	4.29	4.61
	30	6.24	6.59
40	0	5.16	4.59
	20	8.25	9.66
	40	18.39	18.24



Left Side

$$\boldsymbol{\alpha(l,i)} = \alpha(n) \times \left(\frac{i-10}{0.5 \times n}\right)^2$$

Right Side

$$\boldsymbol{\alpha(r,i)} = \alpha(o) \times \left(\frac{i-10}{0.5 \times n}\right)^2$$

Figure (4): The deformation zone divided into two parts by the horizontal tangent

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In this section, two layers are assumed; the top layer with φ_1 is overlaying a lower layer of sand with φ_2.

Rankine zone occurred within the upper layer, so that the upper layer shearing resistance angle φ_1 is considered for the calculation of the passive earth pressure coefficient for the Rankine zone, as follows:

$$K_{PR} = \frac{b\sqrt{OCR}\,sin(\theta_o+\alpha_o)sin(\theta_o)sin(\alpha_o+\phi_1)}{sin(\alpha_o)sin(\theta_o+\alpha_o+2\phi_1)} \quad (13)$$

where;

OCR: Over-consolidation ratio.

φ_1: Angle of shearing resistance of the soil of the backfill for the upper layer.

$\theta_o = \pi/4 - \varphi_1/2$.

$\alpha_o = \pi/2 - \theta_o - \varphi_1$.

b = Previously defined.

The deformed zone occurred in both layers; the upper layer and the lower layer. Then, a new shearing resistance angle will be composed of the two layers, considering the weight as a proportionate combination of the weights of the respective layers within the slice. This new proposed angle is given in the following formula:

$$\varphi_{new} = tan^{-1}\left(\frac{tan\varphi_1 + tan\varphi_2}{2}\right) \quad (14)$$

where;

φ_1: Upper layer soil friction angle.

φ_2: Lower layer soil friction angle.

This angle replaces the angle of shearing resistance of the homogeneous backfill for the deformation zone in Equations (2 & 3) and in Equation (10) for the last slice base inclination α_n. The computer program is adjusted to take these assumptions into consideration.

The results obtained (which will be presented in the form of charts hereafter) showed that all values of passive earth pressure of strong homogeneous sands are higher than that of strong sand overlaying weak sand. This can be explained by the fact that decreasing lower layer strength will result in a decrease in the passive pressure all over the soil mass, because the stress needed for the weak layer to fail is much less than that for the strong layer. Moreover, for weak sand overlaying strong sand, the passive earth pressure will remain the same as for weak homogeneous sand.

Passive Earth Pressure for Inclined Backfill

In Figure 5, a vertical rigid retaining wall with a cohesionless backfill of slope angle β is considered. The angle of shearing resistance is φ for the case of homogeneous backfill, φ_1 for the upper layer and φ_2 for the lower layer in the case of strong backfill overlaying deep deposit. The backfill inclination and the over-consolidated ratio, for the case of over-consolidated backfill, will be included in passive Rankine zone as follows:

$$k_{PR} = \frac{b\times\sqrt{OCR}\,sin(\theta_o+\alpha_o)sin(\theta_o+\beta)sin(\alpha_o+\phi_1)}{sin(\alpha_o-\beta)sin(\theta_o+\alpha_o+2\phi_1)} \quad (15)$$

where;

$$\theta_o = \frac{\pi}{4} - \frac{\phi}{2} - \frac{\beta}{2} - \frac{1}{2}arcsin\left(\frac{sin\beta}{sin\phi_1}\right) \quad (16)$$

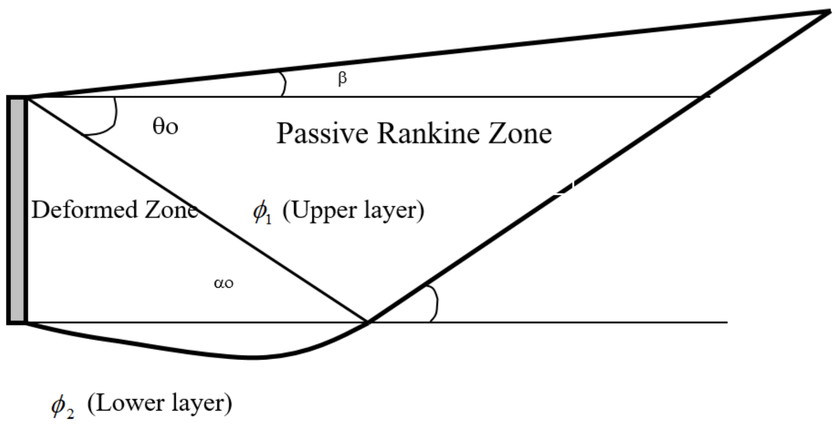

Figure (5): Failure plane for inclined backfill material of an angle behind a retaining wall

Equations (2) and (3) will be used to transfer stresses to the slices in the deformation zone and then to calculate the coefficient of passive earth pressure behind the retaining wall, by considering the case of homogeneous backfill ($\varphi_1 = \varphi_2$) and the case of two different layers ($\varphi_1 \neq \varphi_2$). In the last case, the composite angle of shearing resistance φ_{new} will be used for the calculations of the deformed zone. Consequently, the computer program is adjusted to take the previous assumptions into consideration.

In this study, a wide range of parameter values was used with the objective to develop design charts for practicing use. These charts will take into account the soil condition parameters that affect the coefficient of passive earth pressure, such as the over-consolidation ratio (*OCR*), shearing resistance angle of the backfill ϕ, the ratio of soil-wall friction $\frac{\delta_P}{\phi}$ and the backfill inclination β^o. Typical results are shown in Figures 6 to 9.

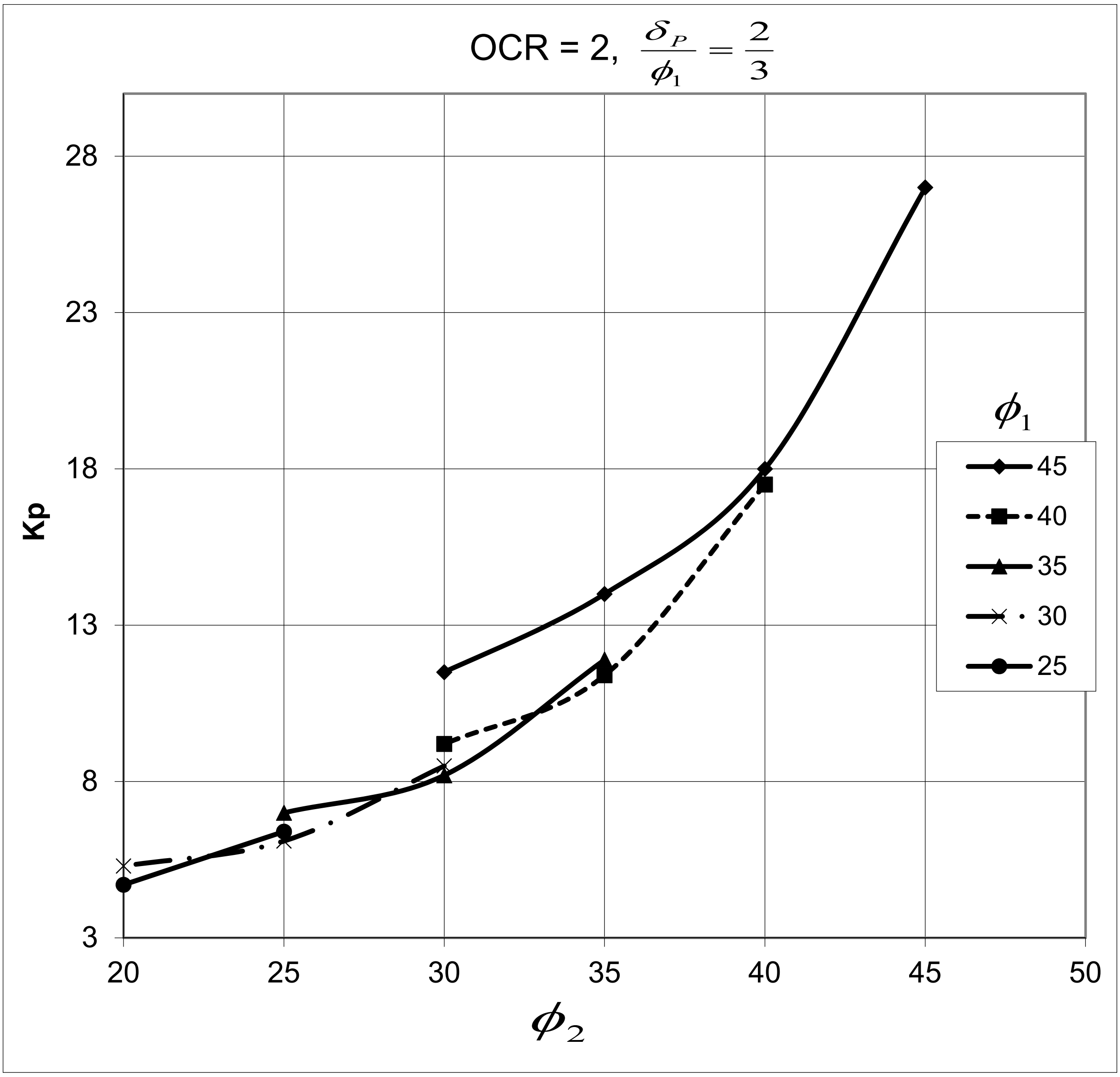

Figure (6): Coefficient of passive earth pressure for OCR =2, $\frac{\delta_P}{\varphi_1} = \frac{2}{3}$

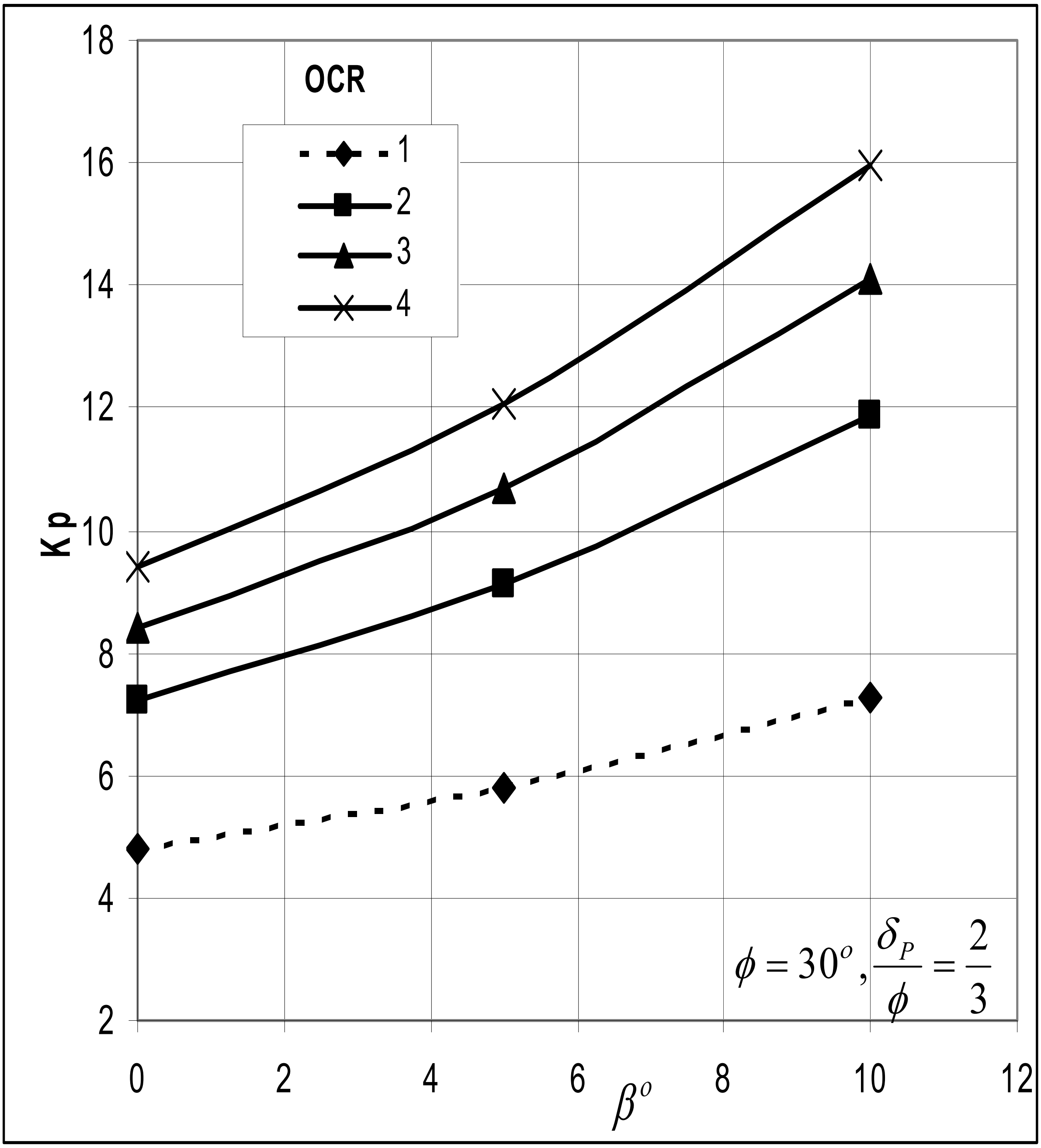

Figure (7): Coefficient of passive earth pressure for inclined homogeneous normally and over-consolidated backfill sand, $\varphi = 30^o, \frac{\delta_P}{\varphi} = \frac{2}{3}$

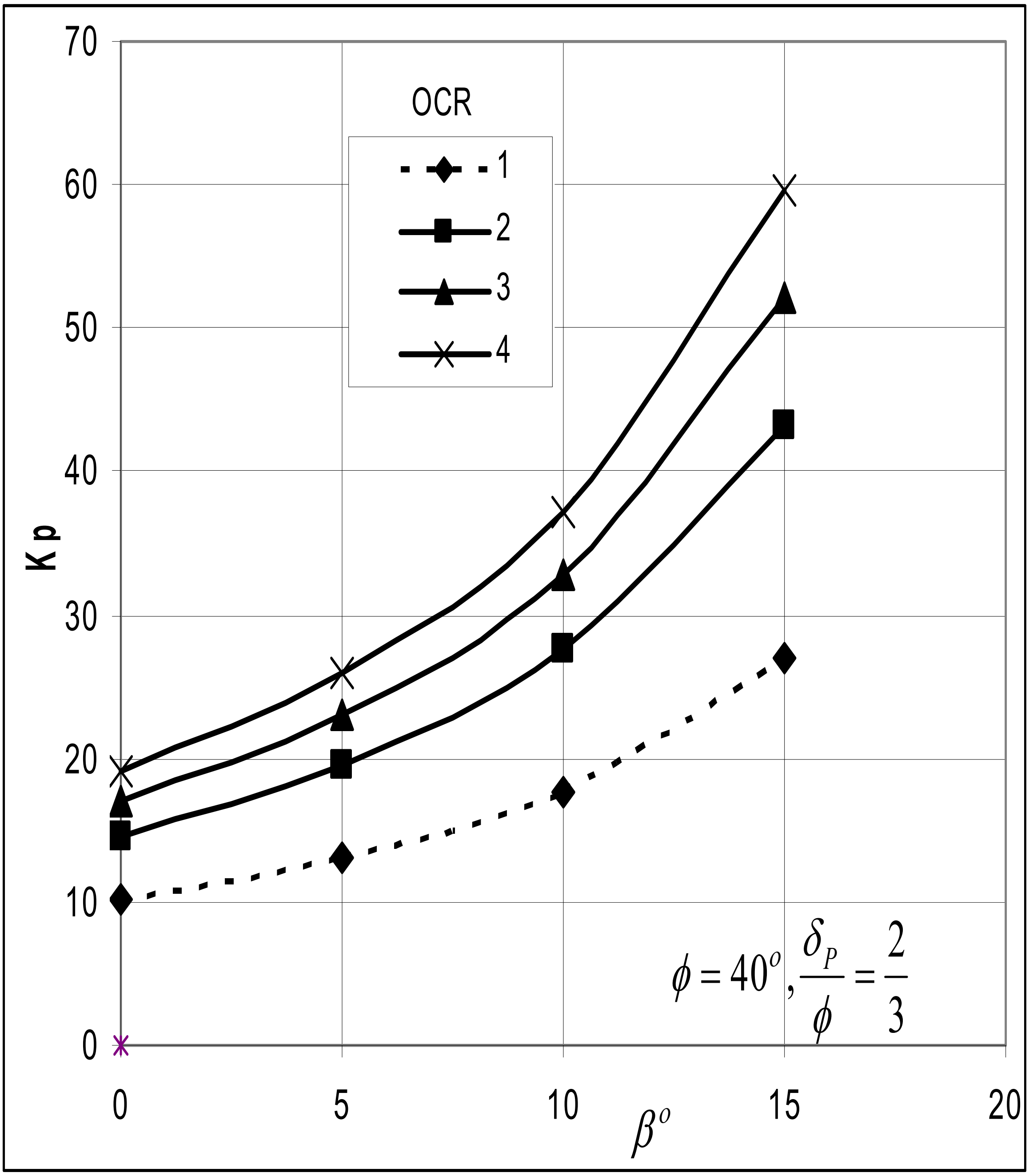

Figure (8): Coefficient of passive earth pressure for inclined homogeneous normally and over-consolidated backfill sand, $\varphi = 40^o, \frac{\delta_P}{\varphi} = \frac{2}{3}$

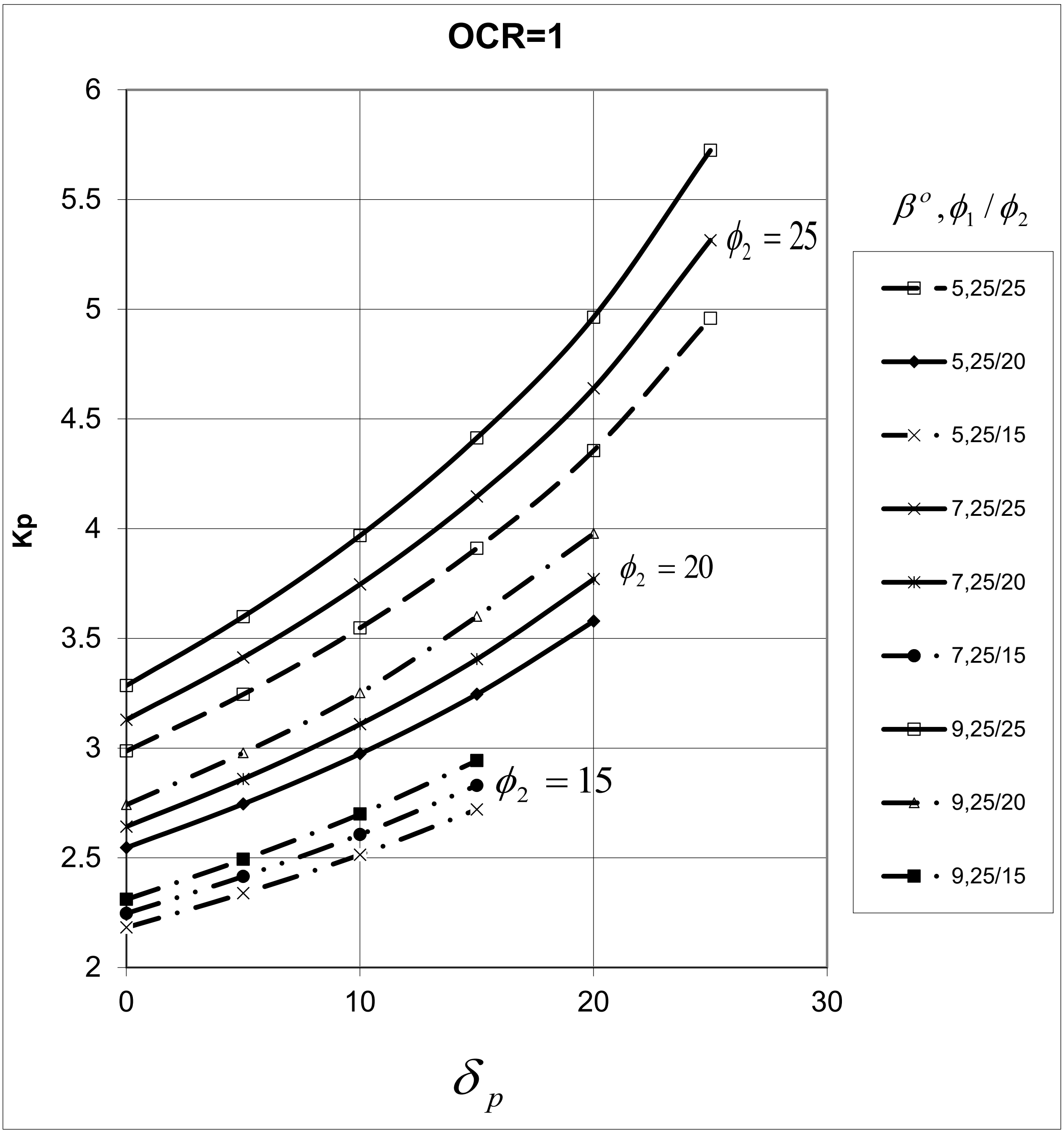

Figure (9): Coefficient of passive earth pressure for inclined backfill for normally consolidated sand

CONCLUSIONS

Based on this study, the following conclusions can be drawn:

1. The passive earth pressure behind a retaining wall is affected directly by several parameters, including the stress history. Passive earth pressure coefficient in the case of homogeneous over-consolidated sands increases while increasing the shearing resistance angle of sand and it is a factor of over-consolidation. The coefficient of passive earth pressure increases when increasing the value of over-consolidation ratio (*OCR*).
2. For the case of two layers of different types of normally consolidated sand, the passive earth pressure coefficients increased proportionally when increasing the soil-wall frictional angle (δ_p), the value of the composed shearing resistance angle (ϕ_{new}) and the value of *OCR*. Likewise, for the case of two layers of over-consolidated sands, the passive earth pressure coefficient increases

proportionally as increasing the soil-wall frictional angle (δ_p), the value of the composed shearing resistance angle (ϕ_{new}) and the value of *OCR*. In the case of weak soils overlaying very strong layers of sand, the coefficient of passive earth pressure remains approximately as that of the weak layer.

3. It would be logical to infer that the passive earth pressure is related to the shearing resistance of soil along the rupture surface. Backfill inclination upward the top of the wall increases the coefficient of passive earth pressure.
4. Design charts were developed for engineering practice to predict the coefficient of passive earth pressure for homogeneous over-consolidated sand or over-consolidated sand backfill overlaying natural deposit.

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