A Model for Fixed-cost Batch Transportation Problem under Uncertainty

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ABSTRACT

Shipping costs make a significant portion of total cost of goods. The fixed-cost transportation problem is an extension of the general transportation problem. This problem is one of the fundamental and important problems in the field of transportation and has recently received much attention by researchers considering real-world assumptions. The purpose of this research is to develop a mathematical model and an appropriate method to solve the fixed-cost batch transportation problem to reduce variable and fixed costs in a way that supply and demand constraints are met and the optimal transportation decision is made. In this research, it is assumed that products are transported in batches. It is also assumed that the decision variables have fuzzy numbers and the aim is to find the best shipping method with minimum cost. As the extension of transportation problem, which has been proven as an NP-hard problem, four frequently-used metaheuristic algorithms, including Simulated Annealing (SA), Imperialist Competitive Algorithm (ICA), Variable Neighborhood Search (VNS) and hybrid VNS, were used to solve this problem. The Taguchi method was applied to parameter setting. The results of these four algorithms were compared according to the three criteria of optimal value, computational time and dispersion rate. The comparison of the efficiency of the algorithms showed that the hybrid VNS algorithm outperforms the other three algorithms. Finally, the uncertain parameters of the problem were defuzzified by the fuzzy robust method and the dispersion rates of the solutions obtained by the algorithms were compared.

KEYWORDS: Fixed-cost transportation, Batch transport, Fuzzy environment, Meta-heuristic Algorithm, Uncertainty.

INTRODUCTION

In addition to dealing with organization and internal resources in the current competitive marketplace, firms and enterprises have found themselves in need of managing and overseeing related resources and components outside the organization. Therefore, today, supply chain management is a fundamental base of e-business implementation in the world. Supply chain management is a phenomenon that does this in a way that customers can get reliable and fast services with quality products at minimum cost. fixed-cost Transportation Problem (FCTP) is an extension of the general transportation problem, in which a certain number of products are shipped to demand locations while a fixed-cost is applied in addition to the previous variable cost. Various types of fixed-cost transportation problems have been addressed. They can be linear or nonlinear depending on the type of variable cost. The problem can also be symmetric or asymmetric in terms of the amount of demand and supply and the asymmetric form can be easily converted into a symmetric one. In this paper, a fixed-cost transportation problem is considered under

Received on 9/1/2020.
Accepted for Publication on 10/3/2020.

uncertainty. Therefore, the transport problem has been examined at one level. This level involves shipping of products from distribution centers to demand points (customers, retailers, … etc.).

The innovation of this research relates to how goods are transported. Previous research has focused on the number of products shipped from one place to another, whereas in this study, products are transported in batches. In this case, we try to minimize the sum of costs by considering the variable cost of batch shipping. Goods can be shipped from any supplier to any customer and a variable cost per shipping a batch as well as a fixed cost for opening the respective route are taken into account. Assuming a certain number of goods per batch, if the number of goods shipped is less than that certain number, we consider the cost of one batch. Each resource has a specific capacity and each destination has a specific demand. The objective function is to minimize the total sum of transportation costs, including both variable and fixed costs, so that supply and demand constraints are met at both source and destination nodes.

Problem variables are considered fuzzy. Uncertainty in transportation systems is one of the most important issues for decision makers in planning, especially long-term planning.

One of the positive effects of lowering freight costs, which results in increasing customer satisfaction, is the greater profitability for each component of the supply chain. This was the rationale behind this study. The results of this research may lead to increasing the scope of profitability, which includes the end customer and all components of the supply chain.

The purpose of this research is to develop a mathematical model and an appropriate method to solve the fixed-cost transportation problem by taking batch transportation of goods into account in order to reduce variable and fixed costs in a way that supply and demand constraints are met in each node and to facilitate the process of adopting optimal transport planning policies for linking of chain levels and demand management.

After presenting the problem and stating the importance of the topic under discussion in this section, we review the previous studies, proposed models and solutions in the following section. Solving tools will also be discussed briefly, followed by the presentation of the designed model along with explanations for the objective function and constraints. After determining the input parameters, the four proposed algorithms are compared using a number of indices. Then, to determine the efficiency of the proposed model and algorithms, the results of solving the problem will be shown in different sizes through several graphs. Finally, the results and guidelines obtained in this study will be presented and suggestions will be made for expanding this research and for future studies.

LITERATURE REVIEW

Nowadays, it is reiterated in the literature to emphasize Transportation Demand Management (TDM) strategies more than the traditional transportation strategies which are based on “supply-side” tactic, specifically in rapidly growing developing countries (Rahman and Al-Ahmadi, 2010).

For the first time, Balinski (1961) introduced the fixed-cost transportation problem presented its specific characteristics and proposed an imprecise method to solve it. Since then, there have been many studies on Fixed-cost Transportation Problem (FCTP). FCTP was introduced by Hirsch et al. in 1968. The purpose of such problem is to find the best routes to minimize the total sum of variable and dependent costs, while also estimating supply and demand requirements for each source and destination.

Michalewicz et al. (1991) used Genetic Algorithm (GA) for solving linear and nonlinear transportation problems. Adlakha and Kowalski (2003) proposed a simple heuristic algorithm to solve the fixed-cost transportation problem at a small scale. Jiménez and Verdegay (1998) investigated a fixed-cost transportation problem with Genetic Algorithm. Samanta and Roy (2005) proposed an algorithm to solve the multi-objective transportation problem in a fuzzy environment. Sheng and Yao (2012) studied FCTP with uncertain variables. Papageorgiou et al. (2012) studied a linear mode of the problem and examined FCTP with product composition. Yang and Feng (2007) investigated a two-criteria solid transportation problem with stochastic parameters. Molla-Alizadeh et al. (2011) studied the fixed-cost transportation problem with capacity constraint in a two-stage supply chain network. Adlakha et al. (2010) and Kowalski et al. (2014) developed a simple algorithm to obtain a general solution to a small-scale fixed-cost transportation problem. Hajiaghaei-Keshteli et al. (2010) considered the nonlinear fixed-cost transportation problem and proposed a new method for designing chromosomes using a Genetic Algorithm based on the Spanning Tree technique and Prüfer numbers. Altassan et al. (2014) proposed an algorithm to solve the fixed-cost transportation problem.

Sadeghi-Moghaddam et al. (2019) solved the fixed-cost transportation problem considering discount constraints.

Cheng et al. (2015) proposed an efficient hybrid algorithm to solve batch ordering and routing problem, to determine batch size, to allocate order in a batch and to determine transportation distance. Molla-Alizadeh et al. (2013) considered variable and fixed costs as fuzzy numbers and used three meta-heuristic algorithms to solve the problem. Mahmoodirad et al. (2014) proposed a new method for finding an approximate solution close to the optimal solution in FCTP in which all parameters (shipping cost, fixed cost, decision variables, supply and demand) are triangular fuzzy numbers. Ebrahimnejad (2016) proposed a new method for solving fuzzy transportation problem (FTP).

Rostami et al. (2015) tried to minimize the maximum delay and cost of batch delivery and completion time.

Halder et al. (2017) considered a fixed-cost transportation problem under uncertainty with replaceable and breakable items. Jana et al. (2017) solved a Fixed-Cost Multi-Objective Profit Transportation Problem (MOPTP) in a fuzzy environment.

Table 1. Summary of studies on FCTP issues

Author	Year	Deterministic Environment	Uncertainty		Solving method		Multi-objective	Cost		Problem type	
			Fuzzy	Probabilistic	Exact	Heuristic		Variable	Fixed	Linear	Non-linear
Balinski	1961	✓				✓		✓	✓	✓	
Hirsch	1968	✓				✓		✓	✓	✓	
Michalewicz *et al.*	1991	✓				✓		✓		✓	✓
Yang and Gen	1994	✓				✓		✓		✓	
Chanas and Kuchta	1996		✓				✓	✓	✓		
Jimenez	1996	✓				✓			✓	✓	
Sun *et al.*	1998	✓				✓			✓	✓	
Jimenez and Verdegay	1998		✓					✓		✓	
Adlakha and Kowalski	2003	✓				✓		✓	✓		
Liu *et al.*	2004		✓			✓		✓			
Samanta and Roy	2005		✓			✓	✓	✓			
Yang and Feng	2007			✓		✓		✓			✓
Kowalski and Lev	2008	✓						✓	✓		✓
Ojha *et al.*	2009		✓					✓		✓	
Hajiaghaei-Keshteli *et al.*	2010	✓				✓		✓	✓		✓
Molla-Alizadeh-Zavardehi *et al.*	2011	✓				✓		✓	✓		

Sheng and Yao	2012		✓					✓	✓		
Papageorgiou *et al.*	2012	✓						✓	✓		✓
Lotfi and Tavakkoli-Moghaddam	2013					✓		✓	✓	✓	✓
Molla-Alizadeh-Zavardehi *et al.*	2013		✓			✓		✓	✓		
Mahmoodirad *et al.*	2014		✓			✓		✓	✓	✓	
Kowalski *et al*.	2014	✓				✓		✓	✓		
Altassan *et al.*	2014	✓				✓		✓	✓		
Rostami *et al.*	2015	✓			✓	✓	✓	✓		✓	
Cheng *et al.*	2015	✓				✓				✓	
Pramanik *et al*.	2015		✓					✓	✓		
Ebrahimnejad	2016		✓			✓		✓			
Upmanyu and Saxena	2016		✓				✓	✓	✓	✓	✓
Baykasoğlu and Subulan	2017		✓		✓			✓		✓	
Halder *et al.*	2017		✓			✓		✓	✓		
Jana *et al.*	2017		✓		✓		✓	✓	✓		
Sadeghi-Moghadam *et al.*	2017		✓		✓	✓			✓	✓	

As it is clear from the review, until now, no study has addressed fixed-cost transportation problem so far by considering batch shipping. Also, given the complexity of transportation processes and interactions between the processes mentioned, the need to consider the cost of transportation for each batch - rather than for each commodity - becomes clear. One of the positive effects of reducing freight costs and, in turn, increasing customer satisfaction, is to generate more profit for each component of the supply chain. Because of such a situation in the real world, we have studied it. Therefore, by conducting this research, the scope of profitability, which includes the end customer and all components of the supply chain, is further enhanced.

METHODOLOGY

Simulated Annealing (SA) Algorithm

Simulated Annealing (SA) algorithm is a meta-heuristic algorithm which was introduced in the 1980s by Kirkpatrick et al. (1983) and since then has been widely used in combinatorial optimization problems due to its simplicity and efficiency. As an example, Millán-Páramo (2018) applied SA to minimize the weight of truss structures with discrete sizing variables. Given the above explanation of the basic SA algorithm, the advantages of this algorithm can be stated as follows: it can be matched with any alternative system and cost function, it can guarantee the likelihood of finding the alternative solution and it can be easily coded for any complex problem. In addition to that, it usually generates a good solution.

Figure (1) shows the pseudo-code of SA algorithm.

1. Defining the objective function and setting the algorithm's parameters
Defining the initial temperature T_0 and initial value x^0
Defining temperature reduction rate of a
2. until ($n<N$, $T>T_f$)
Neighborhood production and creating new solutions
Calculating $\Delta f = f_{n+1}(x_{n+1}) - f_n(x_n)$
3. If the new solution is better, it will be accepted; otherwise, a random r will be generated.
If $p = \exp\left(\frac{-\Delta f}{T}\right) > r$ works, it will be accepted.
End of the cycle
Updating x^* and f^*
End of the cycle

Figure (1): Pseudo-code of simulated annealing algorithm

Imperialist Competitive Algorithm (ICA)

The ICA algorithm is a new meta-heuristic method developed by Atashpaz and Lucas (2007) which has been used in many fields due to its efficient performance, convergence speed and high optimization capability compared to the other algorithms. In these problems, given that the algorithm has little information, such as search space and definable solution to the main problem, it can move toward better solutions by generating random solutions and progress in the search space as much as possible. The pseudo-code of the ICA algorithm is shown in Figure (2).

1. Selecting random points on the function and forming of early empires
2. Assimilate colonies: Moving the colonies toward the colonial country
3. If there is a colony in an empire which has lower cost than that of the imperialist, exchange the positions of that imperialist and the colony
4. Computing the total cost of an empire (by considering the respective cost of the imperialist and each colony)
5. Selecting a colony from the weakest empire and allocating it to the empire with the highest probability of domination
6. Eliminating of the weak empires
7. If there is only one empire left, stop; otherwise go to Step 2

Figure (2): Imperialist competitive algorithm pseudo-code

Variable Neighborhood Search (VNS)Algorithm

The variable neighborhood search algorithm was designed by Brimberg and Mladenowic (1996). This algorithm systematically searches the solution space by changing the neighbors around a solution. This algorithm starts with an initial solution, then randomly selects a neighbor from the nearest neighbor (N_1) on which a local search is done to find its local optimum. If the obtained solution is better than the previous one, another random neighbor will be selected from the same neighbor and the steps will be repeated. But, if the solution is not better than the previous one, then the neighbor with a higher rank will be used. The above-mentioned steps are repeated until we reach the neighbor with the highest rank (N_{max}). VNS is a stochastic algorithm in which a set of neighborhood structures *Nk (k = 1, ..., n)* is first defined. The features of this algorithm include the following:

- Simple and very practical;
- Coherent: Steps naturally follow principles, not hybrids;
- Efficient and effective: Very good solution quality at average CPU time;
- Strong and robust: Performs well on a variety of problems;
- User-friendly: Easy to understand and execute;
- Innovative: Capable of expansion and development.

Select an initial solution such as x
Repeat $\{1 \rightarrow i\}$
Select a solution from the i^{th} neighborhood of the current solution, such as x'
*Value
Apply a local search method to x' and name the result x''
*Local search
Calculate the difference between the objective function value of the current solution and the new solution (ΔF)
If $\Delta F \geq 0$, accept the solution
*Movement
$1 \rightarrow i$
*Return to the first neighborhood
Otherwise, $1+i \rightarrow i$
*Reject the obtained solution and go to the next neighborhood until $i \leq 1$
(until the stop condition is satisfied)

Figure (3): Overview of variable neighborhood search algorithm

Hybrid VNS and Simulated Annealing Algorithm (Hybrid VNS-SA)

In recent years, considerable research has been done on hybrid meta-heuristic algorithms in the field of optimization. The performance of the VNS algorithm is mainly related to the local search design, as enhanced work in the local search phase produces a better local optimization. According to the findings of our research, the combination of SA and VNS yields better results and increases the effectiveness of VNS. SA acts as a local search method, because it seeks good neighborhood. The main idea for integration of SA with VNS is to get rid of local optima. The steps of the proposed hybrid VNS algorithm are shown in Figure 4.

Select a set of neighborhood structures N_k^s, *for* $k = 1,\ldots,k_{max}$ which will be used at the shaking stage and select a set of neighborhood structures N_l^{ls}, *for* $l = 1,\ldots,l_{max}$ which will be used in local search. Set an initial temperature T_0. (n_{max} is the number of neighborhood search per each temperature and a is the temperature reduction rate).
Find the initial solution x
Repeat (internal cycle)
Shake
Generate the stochastic point $x' \in N_k^s(x)$
Local search (SA)
Take the solution x'
Set $1 \rightarrow n$
$x' \rightarrow x''$
While $n_{max} \geq n$ move
$x_{new} \in N_l^{ls}(x'')$
If $\Delta_{s,s'} \leq 0$
$1+n \rightarrow n$, $x \rightarrow x^n$
Once more, randomly generate R within (0,1):
if $R \leq \exp(-\frac{\Delta_{s,s'}}{T_n})$
$x'' \leftarrow x_{new}, n \leftarrow n+1$
End
End if $f(x'') \leq f(x')$
$x' \leftarrow x'', k \leftarrow 1 - l \leftarrow 1; k \leftarrow k+1$ و $l \leftarrow l+1$
And
$T_{n+1} = aT_n$
Until $k > k_{max}$ $(k_{max} = 3)$
Until the stop criterion is met.

Figure (4): Overview of hybrid VNS-SA algorithm

Problem Modeling

As mentioned in the previous sections, this study presents a model for transportation under uncertainty. We consider a symmetric transportation network, because the asymmetric network can be transformed into a symmetric one by introducing a conceptual framework. The geographical location of suppliers and the location of sales outside the scope of the research model are also determined in the operation. The possibility of vehicles' break-down has not been considered in the research, whereas the amount of supply and demand and number of goods in each batch are certain and specified. The innovation of this research relates to how to carry goods. Previous research has taken into account the number of products that are shipped from source to destination, but here, products are transferred in batches. Each batch has a particular capacity for which a specific cost has been allocated. Each vehicle also has a certain capacity to carry goods. Table (2) defines the indices used in the proposed model. The parameters of the proposed model are defined in Table (3). Table (4) shows the decision variables of the proposed model. These variables include integer variables and binary variables.

Table 2. Indices of the proposed model

I	Total points of source	$i \in \{1,2,...,I\}$
J	Total points of destination	$j \in \{1,2,...,J\}$
K	Total number of vehicles	$k \in \{1,2,...,K\}$

Table 3. Parameters of the proposed model

$\tilde{c}_{ijk}$: variable cost of transporting a batch from supplier *i* to destination *j* by vehicle *k*
$\tilde{f}_{ijk}$: fixed-cost of transporting a batch from supplier *i* to destination *j* by vehicle *k*
a_i: amount of goods provided by supplier *i*
b_j: amount of demand in destination *j*
e_k: capacity of vehicle *k*
d: capacity of the batch

Table 4. Decision variables

x_{ijk}	Number of batches shipped from supplier *i* to destination *j* by vehicle *k*
x'_{ijk}	Number of products shipped from supplier i to destination j by vehicle k
y_{ijk}	Zero and one binary variable

Objective Function and Constraints

The proposed mathematical model of the fixed-cost transportation problem is defined by considering batch transportation and constraints according to Equation (1). Values of fuzzy variable and fixed costs are addressed using triangular membership functions.

Indices

I: The set of source points $i \in \{1,2,\dots,I\}$

J: The set of destination points $j \in \{1,2,\dots,J\}$

K: The set of vehicles $k \in \{1,2,\dots,K\}$

Parameters

$\tilde{c}_{ijk}$: The variable cost of batch transportation from supplier *i* to destination *j* with vehicle *k*

$\tilde{f}_{ijk}$: The fixed-cost of batch transportation from supplier *i* to destination *j* with vehicle *k*

a_i: The amount of goods supplied by supplier *i*

b_i: The amount of demands of destination *j*

e_k: The capacity of vehicle *k*

d: The batch capacity

$$Min \quad \tilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} (\tilde{c}_{ijk} x_{ijk} + \tilde{f}_{ijk} y \qquad (1)$$

$$s.t.$$

$$\sum_{j=1}^{n} \sum_{k=1}^{K} x'_{ijk} \le a_i$$

$$\sum_{i=1}^{m} \sum_{k=1}^{K} x'_{ijk} \ge b_j$$

$$\sum_{i=1}^{m} \sum_{j=1}^{m} x'_{ijk} \le e_k$$

$$x_{ijk} = \left[\frac{x'_{ijk}}{d} \right] \qquad if\ (\frac{x'_{ijk}}{d}) \in Z$$

$$x_{ijk} = \left[\frac{x'_{ijk}}{d} \right] + 1 \qquad if\ (\frac{x'_{ijk}}{d}) \notin Z$$

$$x'_{ijk} \ge 0 \quad , \quad d \ge 0$$

$$y_{ijk} = 0 \qquad if\ x'_{ijk} = 0$$

$$y_{ijk} = 1 \qquad if\ x'_{ijk} > 0$$

$$i = 1, \dots, m$$

$$j = 1, \dots, n$$

$$k = 1, \dots, K$$

In the above model, the objective is to minimize the sum of fixed and variable costs for carrying a number of batches.

The first constraint indicates that the number of goods shipped must be less than or equal to the number supplied by source *i*.

The second constraint states that the number of goods shipped must be greater than or equal to the number demanded by destination *j*.

The third constraint shows that the number of goods shipped should not exceed the capacity of the vehicle.

The fourth and fifth constraints represent the process of calculating the number of batches shipped based on the batch capacity and the number of goods shipped. If the number of goods is a multiple of the capacity of the batch, it is calculated according to the fourth constraint; otherwise, one batch is added to the number of batches depending on the number of the remaining goods.

The sixth constraint does not allow the problem variables to become negative.

The seventh and eighth constraints indicate that y_{ijk} is a variable of zero or one.

Proposed Solution

In this section, we solve and prove the mathematical model by resorting to the logical relationships between the parameters estimated by the mathematical relations and a solution is eventually provided. Fixed-cost transportation problem is more complicated than the transportation problem due to the fixed costs involved in solving it. Fixed costs create discontinuities in the objective function and hence, this problem is known as an Np-hard problem. After that, it is time to solve and optimize the statements of the problems by techniques of operations research. Since the computational time for solving the problem increases exponentially by increasing the problem size, it is therefore fruitful to develop meta-heuristic algorithms. Here, the meta-heuristic algorithms of simulated annealing, imperialist competitive algorithm, variable neighborhood search and hybrid VNS were used and their respective pseudo-codes are presented below. After solving the system under optimization in different sizes, the obtained results were collected, classified and then analyzed. Finally, the results and strategies obtained through this research were presented and suggestions were made for expanding this research and for future studies.

Parameter Analysis

In this research, the parameters that affect the algorithm were first identified and evaluated on the basis of input indices. This study determines the combinations of different levels of parameters according to the orthogonal array and solves the problem in the appropriate size. The appropriate values are suggested for setting the parameters.

The problem input parameters for the three proposed algorithms were considered as shown in Table (5).

Table 5. Values of parameters for the three proposed algorithms

	MaxIt	Initial T	Reduction T	Sub-it	nPop	nEmp	Beta	Probability of Revolution	Zeta	Number of neighborhood
SA levels	100	1000	0.9	10						
	200	12000	0.99	20						
	300	15000	0.999	50						
ICA levels	100				100	20	0.3	0.1	0.1	
	200				150	30	0.5	0.2	0.2	
	300				200	40	0.7	0.3	0.3	
VNS levels	100				100					10
	200				150					20
	300				200					50
VNS levels	100	10000	0.9		100					
	200	12000	0.99		150					
	300	15000	0.999		200					

In this research, the parameters were considered at three levels. In the simulated annealing algorithm, there are 9 experiments with four 3-level factors; therefore, the total number of experiments ($9 * 3^4$) equals 729. The imperialist competitive algorithm has 27 experiments with six 3-level factors and the total number of experiments (27 * 3^6) equals 19683. Also, for variable neighborhood search, the total number of experiments was (9 * 33) and the hybrid VNS algorithm had 27 experiments with five 3-level factors which accounted to (27 * 3^5) experiments. The Taguchi method reduces the number of experiments by selecting the best orthogonal array. The L9 scheme was used for simulated annealing and variable neighborhood search and the L27 scheme was used for imperialist competitive algorithm and hybrid VNS.

Since the optimal values of the parameters may vary for different states of the problem, the model is solved for each of the four algorithms in two sizes of 30*10 and 50*200 using the Taguchi method. The results of the alternative parameters in these two sizes are given in Table (6). Parameters with larger signal-to-noise ratios are selected as superior values. The algorithms are programmed in MATLAB software and run on a computer system with a Core i5 2.53 GHz with 4GB of memory.

Table 6. Optimal values of parameters in different sizes obtained by Taguchi method

	Size	MaxIt	Initial T	Reduction T	Sub-it	nPop	nEmp	Beta	Probability of Revolution	Zeta	Number of neighborhood
SA	4	100	12000	0.999	20						
	7	200	12000	0.999	20						
ICA	4	300				200	20	0.5	0.1	0.2	
	7	300				100	30	0.5	0.3	0.1	
VNS	4	200				100					20
	7	200				200					10
Hybrid VNS	4	100	12000	0.999		100					50
	7	200	12000	0.9		100					50

Figures (5) through (12) show the signal-to-noise ratios of the values of the parameters of the four algorithms. Parameters with larger signal-to-noise ratios are selected as the superior values. For example, as shown in Figure (5), for the first parameter (i.e., MaxIt) in the SA algorithm in size 4, the first value is selected as the best value (i.e., 100) because of its higher signal-to-noise ratio. Similarly, for the parameter Initial T, the second value (12000), for the parameter Reduction T, the third value (0.999) and for the parameter Sub-it, the second value (20) are considered as the optimal values of the parameters.

Figures 5 through 12 show the signal-to-noise ratio of the parameters of the four presented algorithms. According to these figures, the parameters with higher signal-to-noise ratios are selected as superior values.

Data Generation

To evaluate the efficiency of the algorithms used, the problem is solved in seven different sizes. The various sizes of the problem are shown in Table (7). Each problem was run five times for higher efficiency. The size of each problem was determined by the number of sources, destinations and the amount of fixed and variable costs. Also, each batch was considered to contain 10 products in all examples.

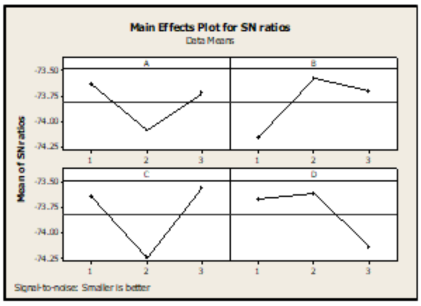

Figure (5): S / N ratio for the SA parameter in size 4

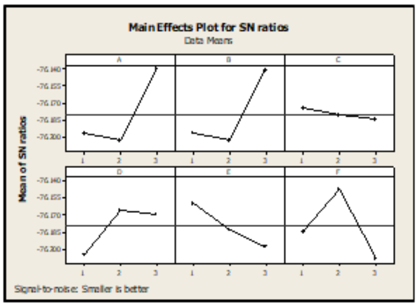

Figure (6): S / N ratio for the ICA parameters in size 4

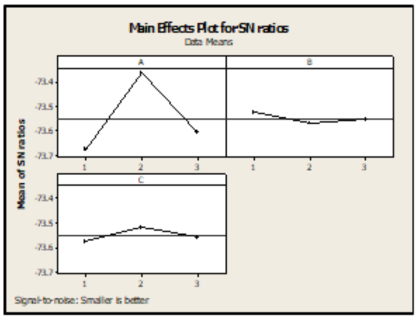

Figure (7): S/N ratio for VNS parameters in size 4

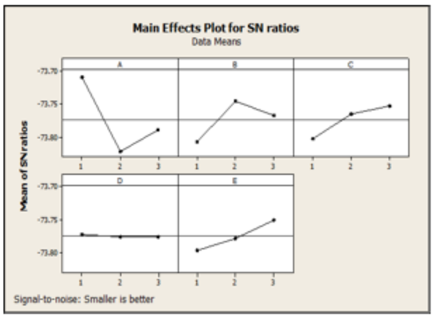

Figure (8): S/N ratio for hybrid VNS parameters in size 4

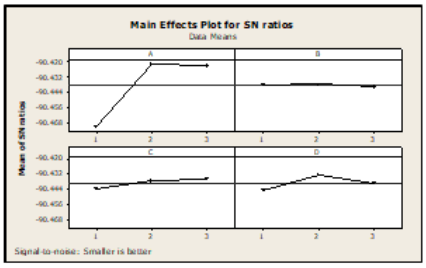

Figure (9): S/N ratio for SA parameters in size 4

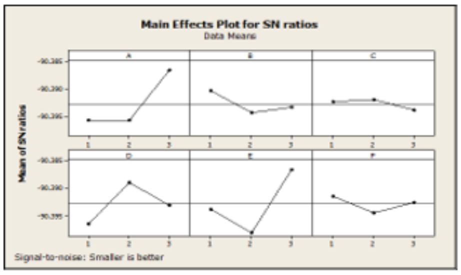

Figure (10): S/N ratio for ICA parameters in size 7

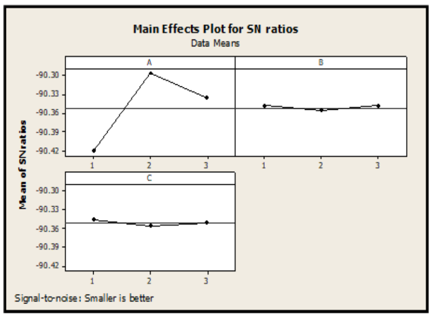

Figure (11): S/N ratio for VNS parameters in size 7

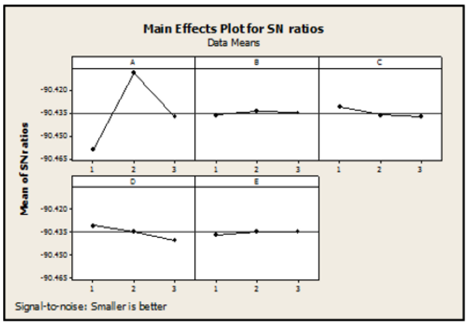

Figure (12): S/N ratio for hybrid VNS parameters in size 7

Table 7. Various sizes of the problem

Problem size	Total demand	Range of variable costs in triangular fuzzy		Range of fixed costs in triangular fuzzy	
		Lower limit	Upper limit	Lower limit	Upper limit
10*10*4	10,000	U(3,6)	U(6,10)	U(50,100)	U(150,200)
10*20*4	15,000				
15*15*6	15,000				
10*30*6	15,000				
50*50*8	50,000				
30*100*8	30,000				
50*200*10	50,000				

Classification of fuzzy subsets is very important in transportation problems. There are several ways to rank fuzzy sets in the definitions' section. Since the objective function is to minimize the integral value of fuzzy transportation costs, we use a simple and flexible method of fuzzy integral ranking proposed by Liu and Wang (2004). According to this method, the value of the whole integral of the convex combination of right and left integrals in the optimistic index is α∈ [0, 1]. The left integral is used to represent the optimistic view and the right integral is used to represent the pessimistic view.

$$\tilde{\mu}_A(x) = \begin{cases} (x-a)/(b-a) & a \le x < b \\ (c-x)/(c-b) & b \le x < c \\ 0 \quad x \ge c \quad or \quad x < a \end{cases} \tag{2}$$

The inverse functions can be displayed as follows:

$$\begin{aligned} \tilde{g}_A(y)^L &= a + (b-a)y \\ \tilde{g}_A(y)^R &= c - (c-b)y = c + (b-c)y \end{aligned} \tag{3}$$

In the above equations, y∈ [0,1]; therefore, the value of the right and left integrals can be formulated as follows:

$$\begin{aligned} I(\tilde{A})^L &= \int_0^1 \tilde{g}_A(y)^L dy = \frac{1}{2}(a+b) \\ I(\tilde{A})^R &= \int_0^1 \tilde{g}_A(y)^R dy = \frac{1}{2}(b+c) \end{aligned} \tag{4}$$

The total value of the integral is $\tilde{A} = (a, b, c)$

$$\begin{aligned} I_T^\alpha(\tilde{A}) &= \alpha L(\tilde{A})^R + (1-\alpha) I(\tilde{A})^L \\ &= \frac{1}{2}(\alpha c + b + (1-\alpha)a) \end{aligned} \tag{5}$$

An optimistic degree of [0,1] is given and in this model, we consider the alpha value to be 0.5.

RESULTS AND DISCUSSION

MATLAB software (version 16) was used to run each algorithm in different sizes. The behaviors of the algorithms are compared in terms of the obtained values of the objective function and the dispersion of the generated data in Table 7. In order to evaluate and compare the algorithms accurately, each algorithm was run in the 7 sizes stated in Table (6) and for each size, it was run 5 times and the resulting values were recorded. Finally, the mean of the obtained numbers was recorded in each run of each algorithm. Also, in order to compare the algorithms with respect to the size of the problem, an unscaling procedure was performed. RPD is a measure of data unscaling, meaning the percentage of relative deviation. Its formula is as follows:

$$RPD = \frac{Alg_{sol} - min_{sol}}{min_{sol}} \times 100 \tag{6}$$

Also, in order to measure the efficiency of the algorithms in terms of computational time, the problem was compared in different sizes with respect to the time taken to execute the algorithms. Here, each algorithm was run 6 times per problem size and every time, the duration was recorded and finally the mean of all 6 runs was calculated for each run of each algorithm.

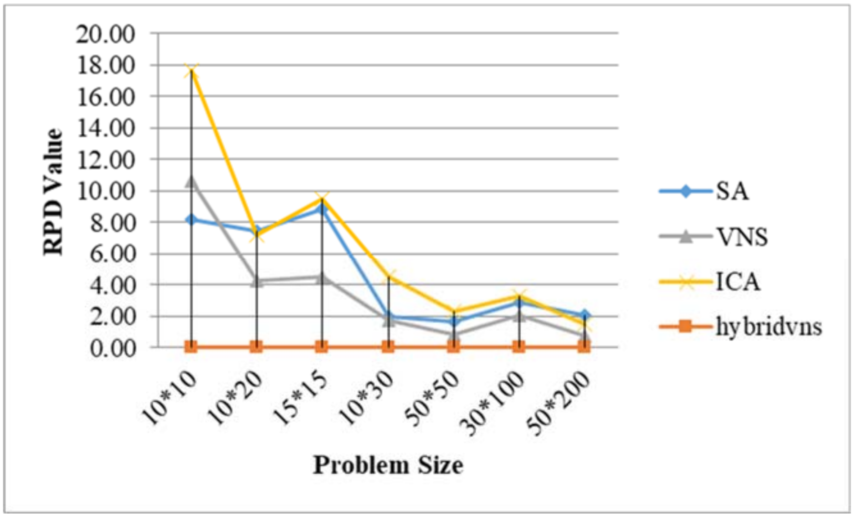

Figure (13): RPD chart for each algorithm in different sizes

Figure (13) shows the overall comparison of the algorithms. As can be seen, in all sizes, the hybrid VNS algorithm yields a smaller objective function than other algorithms. After that, at the smallest size, the SA performance is better than for the other algorithms; it has a lot of fluctuations in middle sizes. However, with the increase in the size of the problem, the performance of the VNS algorithm is still better than those of the other algorithms after the hybrid VNS. As can be seen in Figure (14), a comparison of the efficiency of the algorithms in terms of execution time reveals that the duration of SA algorithm is shorter than those of the other algorithms and the hybrid VNS algorithm is ranked last. The algorithms are also compared in Figure (15) in terms of the dispersion of the generated data.

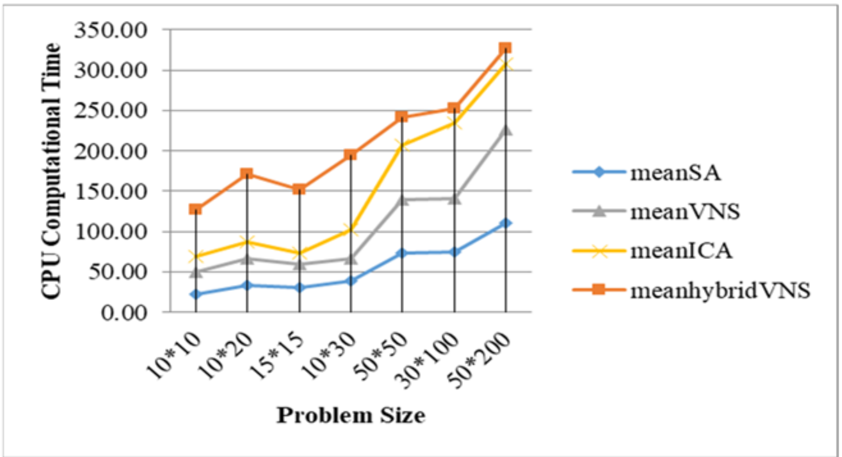

Figure (14): Computational time of each algorithm in different problem sizes

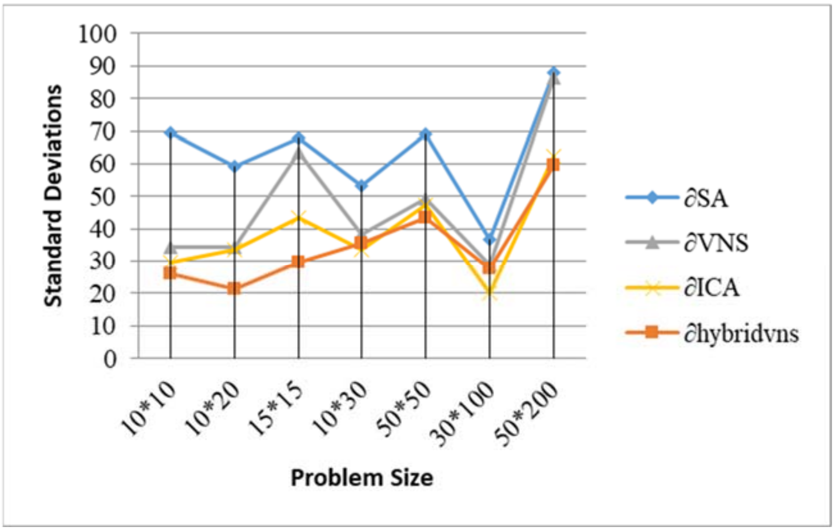

Figure (15): Standard deviation of each algorithm in different sizes

Then, in order to validate the results, the problem was solved in small sizes by GAMS software and exact solution method and the obtained values of the optimal objective function and the time it took to reach a solution through each algorithm were compared as indicated in Table (8).

Table 8. Comparison of algorithms' performance with the exact method

Size	GAMS		SA		VNS		ICA		HybridVNS	
	Answer	Time	Answer	Time	Answer	Time	Answer	Time	Answer	Time
10*10	1644	2	1796.00	22.85	1837.40	49.62	1954.50	69.39	1660.6	281.91
10*20	2970	179	3581.50	32.98	3473.40	65.73	3573.00	86.88	3375	171.81
15*15	2254	1138	2867.80	29.98	2754.17	59.03	2885.40	73.90	2634.6	151.73
10*30	3928	2106	4659.60	38.52	4816.60	66.88	4848.60	101.91	4638.4	195.00

According to Table (8), it can be concluded that as the size of the problem increases, the time to reach the optimal solution in GAMS software increases sharply. However, the proposed algorithms are able to achieve good results in a shorter time. The above comparisons and the obtained results show the validity of the mathematical model.

As stated above, there are several ways to rank fuzzy sets. Since the objective function is to minimize the fuzzy transportation costs, we used a simple and flexible method of fuzzy ranking to defuzzify the fuzzy values in the problem. This method does not take deviation into account and it may involve high risk. A small change in the values of nonlinear parameters can cause a large change in the system reliability. Therefore, using the fuzzy robust method is suggested.

$$I_T^{\alpha}(\tilde{A}) = \frac{1}{2}\left(\alpha c + b + \left(1-\alpha\right)a\right) + \left(c-a\right) \qquad (7)$$

In the second part of the formula, the maximum deviation is shown which should be minimized.

The results obtained from the dispersion of the solutions generated by the four algorithms using simple deffuzification method and the fuzzy robust method are shown in Figures (16) and (17). As can be seen, the dispersion rate of the solutions obtained from the fuzzy robust method is significantly reduced compared to the previous method.

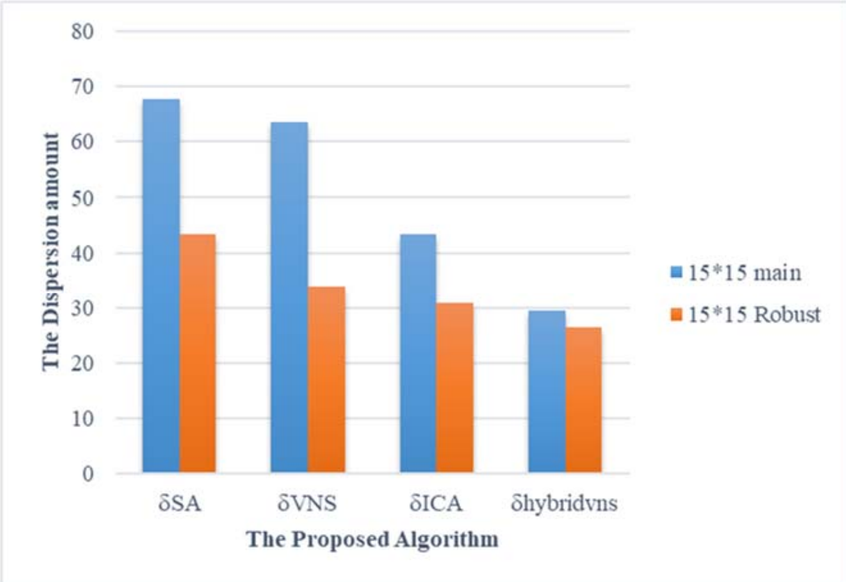

Figure (16): Comparison of dispersions of the algorithms used by different defuzzification methods in the small-sized problem

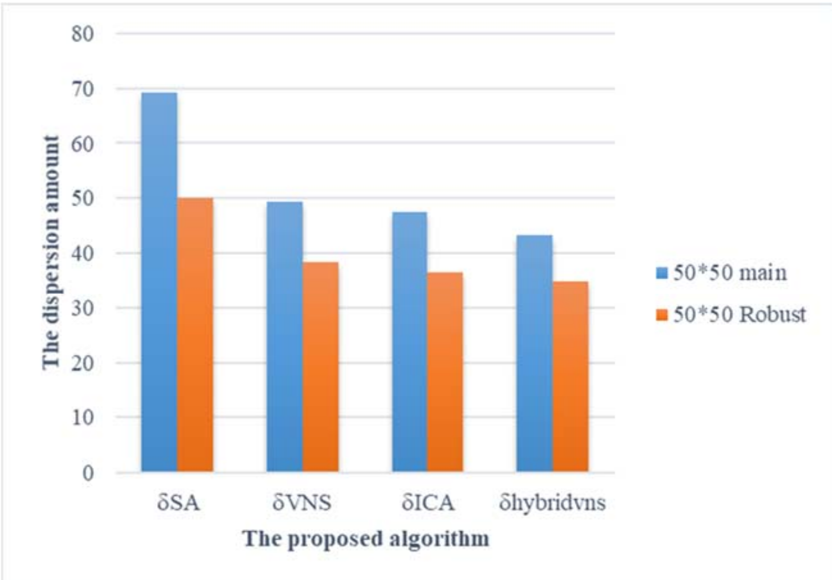

Figure (17): Comparison of dispersions of the algorithms used by different defuzzification methods in the large-sized problem

In this section, first, the process of establishing the parameters was considered. Then, by the Taguchi method, the orthogonal array of each algorithm and the optimal values of the parameters were determined. Then, the value of the objective function was obtained for each run of the algorithms. Unscaling was performed to compare the amount of algorithms in different sizes. In order to validate the algorithms, the problem was solved by the exact solution method and by GAMS software. Then, to evaluate the efficiency of the algorithms, the results of solving these four algorithms were compared in terms of three criteria of optimal value, time and dispersion rate by multi-criteria decision making method. The results of comparing the efficiency of the algorithms showed superiority of the hybrid VNS over the other algorithms. At the end, the uncertain parameters of the problem were defuzzified by the robust fuzzy method and the results of the dispersion rate of the solutions generated by the algorithms were compared with each other, which showed that the dispersion rate of the solutions obtained from the fuzzy robust method had significantly decreased.

CONCLUSION

Given the importance of transportation in today's world, this study examines the problem of fuzzy fixed-cost transportation in terms of cost savings and batch transportation. A considerable number of studies were conducted on fixed shipping cost; however, the present paper differs from the previous research in considering batch transportation to reduce costs. This research was conducted to achieve optimal decisions in terms of minimizing batch transportation costs under fuzzy environment.

Then, four meta-heuristic algorithms, including SA, ICA, VNS and hybrid VNS-SA, were implemented to solve the proposed model. The Taguchi method was used for parameter setting. Finally, to evaluate the efficiency of the algorithms, the results of these four algorithms were compared based on the three criteria of optimal value, computational time and dispersion rate. The comparison of the performance of the algorithms showed that the hybrid VNS-SA algorithm performs better than the other algorithms. The uncertain parameters of the problem were defuzzified using the fuzzy robust method and the results of the dispersion rate of the solutions generated by the algorithms were compared. The results showed that the fuzzy robust method has reduced the dispersion of the solutions.

Multiple batches with varying capacities for transportation of goods or several routes with different capacities between supplier and customer can be considered for future research. Also, a discount rate can be added to batch shipping or other objective functions, such as maximizing profits, reducing shipping duration, … etc., may be considered in the mathematical model. Moreover, other meta-heuristic algorithms can be applied to solve the proposed model and the results compared with the findings of this paper.

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