Stress Characterization in Visco-Elastic Asphalt Mixes under Different Dynamic Loadings

Pabitra Rajbongshi\textsuperscript{1)}, Mitali Saharia\textsuperscript{2)} and Kh. Lakshman Singh \textsuperscript{3)\textsuperscript{a}}

\textsuperscript{1)} Doctor, National Institute of Technology Silchar, India. E-Mail: prajbongshi@yahoo.com
\textsuperscript{2)} National Institute of Technology Silchar, India. E-Mail: sahariamitalis@gmail.com
\textsuperscript{3)} National Institute of Technology Silchar, India. E-Mail: lakshman_kh@yahoo.com

ABSTRACT

Asphalt mix is a visco-elastic material. That is, it exhibits a combination of time-independent stress-strain (elastic) behaviour and time-dependent stress-strain (viscous) behavior under loading. It imparts stress relaxation with time under constant strain and strain enhancement with time under constant stress conditions. Both stress and strain vary with certain phase differences under dynamic loading. Different types of dynamic loading can be used for material evaluation, like sinusoidal, halfwave sinusoidal, haversine, square, halfwave square,… etc. loadings, either at a stress-controlled mode or a strain-controlled mode. This paper aims to characterize the stress ($\sigma(t)$) variation for different loading patterns under strain ($\varepsilon(t)$) control condition. Four different types of $\varepsilon(t)$ are considered with and without rest period. It is observed that the $\sigma(t)$ characteristic is significantly different for different $\varepsilon(t)$ inputs, for the same strain amplitude and loading frequency. Asphalt pavement is being subjected to vehicular dynamic loading with rest time and a stress-based shift factor due to the rest period is developed. This study also explores the sensitivity of stress fluctuation with loading frequency ($f$) and relaxation time ($\tau$) of asphalt concrete, for each loading pattern. It is seen that both $f$ and $\tau$ affect significantly the stress characteristics under different loading patterns.

KEYWORDS: Asphalt concrete, Visco-elastic behavior, Dynamic loading, Stress-strain behavior.

INTRODUCTION

The stress ($\sigma(t)$) and strain ($\varepsilon(t)$) behaviors in asphalt concrete (AC) change with time ($t$), due to the visco-elastic nature of asphalt material. $\sigma(t)$ tends to relax with time at constant strain or $\varepsilon(t)$ tends to increase with time at constant stress. At the same time, AC possesses a time-independent modulus due to which $\sigma(t)$ does not continue to relax with time at constant strain or $\varepsilon(t)$ does not continue to increase with time at constant stress, after a certain loading period (Lundström, 2002). Such material is known as an elasto-visco-elastic material. The modulus parameter for such AC material can be represented by relaxation modulus or creep modulus under static loading and by complex modulus or dynamic modulus under cyclic loading conditions, along with the time-independent modulus parameter. Using the modulus parameter, the $\sigma(t)$ function can be evaluated for any given $\varepsilon(t)$ as input and \textit{vice versa} (Wahengbam and Rajbongshi) in asphalt mixes under static or dynamic loading.

An asphalt pavement is subjected to dynamic vehicular loading. Under certain dynamic loading conditions, various researchers (Lundström, 2002; Chailleux et al., 2006; De Bendetto et al., 2001; Hsu and Tseng, 1996; Pellinen et al., 2007; Plati et al., 2013;...
Pramesti et al., 2013; Shu and Huang, 2008; Yan et al., 2015; Yang and You, 2015; You et al., 2012; Zhao et al., 2012; Zhu et al., 2011) studied asphalt mix behavior from different aspects. Usually, a 3-point or 4-point sinusoidal or haversine continuous cyclic loading either at constant strain amplitude or at constant stress amplitude is considered for material evaluation (Lundström, 2002; Pramesti et al., 2013; AASHTO, 2007; Baburamani, 1999; Huang, 2004; Lundström et al., 2007; Oliveira et al., 2009). Moreover, vehicular loading in the field is neither at control stress-strain situation nor at continuous cyclic loading (due to time headway) like sinusoidal or haversine loading patterns. Other loading patterns, like halfwave sinusoidal or halfwave square loading patterns (with 50% cyclic rest time) may be better representative of field loading situations (Hsu and Tseng, 1996; Auang, 2004; Oliveira et al., 2009; Chakraborty, 2003). Four different types of typical strain ($\varepsilon(t)$) loading patterns are shown in Figure 1, with the same strain fluctuation ($\varepsilon_0$) and the same cycle time ($T$). It is expected that stress ($\sigma(t)$) variation would be different for different $\varepsilon(t)$ patterns, even if the strain fluctuation within the same cycle time is the same. This forms the scope of the present study to characterize the $\sigma(t)$ parameter under such dynamic loadings.

Further, repetitive vehicular loading is placed over the road surface for a small interval of time of 0.5 to 0.1 sec depending upon vehicle speed for a speed range of 10 to 50 kmph, followed by a time gap due to time headway. During the initial time of loading, the modulus value of AC significantly reduces with time ($t$) due to viscous effect and this is even more significant at relatively higher temperatures. Such relaxation and loading frequency characteristics would affect the stress amplitude depending upon the loading pattern. The objectives of the present paper are to (i) characterize stress ($\sigma(t)$) variation under different loading patterns, with and without cyclic rest time and (ii) analyze stress fluctuation under different strain loading frequencies ($f$) and relaxation times ($\tau$) of asphalt mix for different loading types, using visco-elastic principles.

Theoretical Development

Under dynamic loading, stress ($\sigma(t)$) and strain ($\varepsilon(t)$) parameters in visco-elastic materials can be expressed as complex numbers, where a complex number contains a real (Re) part and an imaginary (Im) part with a certain phase angle ($\theta$) (Lundström, 2002; Wahengbam and Rajbongshi, 2015; Huang, 2004). The amplitude of $\sigma(t)$ and $\varepsilon(t)$ is given by the absolute value of the respective complex number. For given $\varepsilon(t)$ as input with $\theta = 90^\circ$ between Re($\varepsilon$) and Im($\varepsilon$), the rotational vector diagram of $\sigma(t)$ is presented as shown in Figure 2, for cases of simple visco-elastic and elasto-visco-elastic materials. In Figure 2, the parameter $\varphi$ indicates the phase difference between $\varepsilon(t)$ and $\sigma(t)$ and $\theta = \pi/2 - \varphi$ represents the angle between Re($\sigma$) and Im($\sigma$). $\omega t$ indicates the angular displacement at time $t$ (where, $\omega = 2\pi f = 2\pi/T$) and $\sigma_e$ is the elastic stress component which has no contribution to the imaginary part (Im($\sigma$)) and no phase difference with $\varepsilon(t)$ like in elastic materials.
Figure (1): Typical dynamic strain loading patterns

Figure (2): Rotational vector diagram of stress parameter containing an imaginary part
Under cyclic loading and after few cycles, it can be recognized that the imaginary part of modulus parameter is the loss while the real part of modulus parameter is the storage (Lundström, 2002; Wahengbam and Rajbongshi, 2015). That is, the imaginary part vanishes with time and in that case, the rotational vector diagram can be represented as given in Figure 3, for both visco-elastic and elasto-visco-elastic cases. In the present work also, the imaginary parts of \( \sigma(t) \) and \( \varepsilon(t) \) are ignored while analyzing AC as an elasto-visco-elastic material. The elasto-visco-elastic behavior of AC under relaxation condition can be presented by a Maxwell model along with one spring element connected in parallel (known as Weichert model) as given in Figure 4. Thus, the relaxation modulus \( (E_r(t)) \) parameter for an elasto-visco-elastic asphalt mix using Wiechert model can be represented as given in Equation (1).

\[
E_r(t) = E_e + E e^{-t/\tau}; \quad \varepsilon(t) = \text{const.} \quad (1)
\]

where, \( E_e \) represents the elastic component and \( E \) and \( \tau \) are the Maxwell parameters of visco-elastic material. \( \tau \) is known as the relaxation time. \( E_r(t = 0) = E_e + E \), \( E_r(t = \infty) = E_e \) and \( E_e = 0 \) or \( E_r(t = 0) = 0 \) for a simple Maxwell model represent a visco-elastic material. That is how simple visco-elastic and elasto-visco-elastic asphalt mixes can be mathematically differentiated.

Using convolution integral theorem, the stress \( (\sigma(t)) \) function for any strain \( \varepsilon(t) \) input can be given as in Equation (2).

\[
\sigma(t) = \int_0^t E_r(t - t') \varepsilon(t') dt' = E_e \varepsilon(t) - \frac{E}{\tau} e^{-t/\tau} \int_0^t e^{t'/\tau} \varepsilon(t') dt' \quad (2)
\]
where, \( t' \) is an integrating variable and \( \dot{E}_v(t) = dE_v(t)/dt \). For any \( \varepsilon(t) \), loading input function, the \( \sigma(t) \) function can be evaluated using Equation (2). Four different loading patterns are considered in the present study. These are: sinusoidal, haversine, halfwave sinusoidal (i.e., 50% cyclic rest) and halfwave square (i.e., 50% cyclic rest) as shown in Figure 1. These patterns are referred to as Case-1, Case-2, Case-3 and Case-4, respectively. Mathematically, the \( \varepsilon(t) \) functions for these cases are expressed in Equations (3)-(6). It is to mention that in all the cases, the strain fluctuation (\( \varepsilon_0 \)) and cycle time (\( T \)) remain the same.

\[
\varepsilon(t) = \frac{\varepsilon_0}{2} \sin(wt); \quad \text{Case-1} \\
\varepsilon(t) = \varepsilon_0 \sin^2(wt/2); \quad \text{Case-2} \\
\varepsilon(t) = \frac{\varepsilon_0}{2} + \frac{\varepsilon_0}{2} \sin(wt/2) - \frac{2\varepsilon_0}{\pi} \sum_{n=1,2,3,\ldots} \frac{\cos(nwt)}{(4n^2-1)}; \quad \text{Case-3} \\
\varepsilon(t) = \frac{\varepsilon_0}{2} + \frac{2\varepsilon_0}{\pi} \sum_{n=1,3,5,\ldots} \frac{1}{n} \sin(nwt/2); \quad \text{Case-4}
\]

where, \( \varepsilon_0 \) is the strain fluctuation, \( w \) is the angular velocity (rad/sec) and \( w = 2\pi f = 2\pi/T \), where \( f \) is the loading frequency (1/sec) and \( T \) is the cycle time (sec). Using Equation (2), the \( \sigma(t) \) functions for different cases of \( \varepsilon(t) \) input as given in Equations (3)-(6) can be derived as in Equations (7)-(10).

\[
\sigma(t) = \frac{E\varepsilon_0}{2} \sin(wt) - \frac{E\varepsilon_0}{2\left(\tau^2w^2+1\right)} \left\{ \sin(wt) - tw \cos(wt) + tw e^{-\tau/t} \right\} \\
\sigma(t) = \frac{E\varepsilon_0}{2} \sin^2(wt/2) + \frac{E\varepsilon_0}{2\left(\tau^2w^2+1\right)} \left\{ \cos(wt) + tw \sin(wt) + \tau^2w^2e^{-\tau/t} - \tau^2w^2 - 1 \right\} \\
\sigma(t) = \frac{E\varepsilon_0}{\pi} \left[ \frac{\pi}{2} \sin(wt/2) + 1 \right] + E \left( e^{-\tau} - 1 \right) - \frac{E\varepsilon_0}{\left(\tau^2w^2+1\right)} \left[ \frac{1}{4(n^2-1)} \left\{ -E' \cos(nwt) + \frac{E'}{\left(\tau^2w^2+1\right)} \right\} \left( 2 \sin(wt/2) - tw \left( \cos(wt/2) - e^{-\tau} \right) \right) \right] \\
\sigma(t) = \frac{E\varepsilon_0}{2} + \frac{E\varepsilon_0}{2} \left( e^{-\tau} - 1 \right) - \frac{2E\varepsilon_0}{\pi} \sum_{n=1,3,5,\ldots} \left[ \frac{1}{n} \left( E_e \sin(nwt/2) - E \left( 4 \sin(nwt/2) + 2\pi n \sin(e^{-\tau/2} - \cos(nwt/2)) \right) \right) \right]
\]

Equations (7)-(10) represent the expressions of stress parameter for different loading patterns as considered in the present work, under strain-controlled situation. This is further elaborated through numerical analysis in the next section.

**Numerical Analysis**

The elasto-visco-elastic property of asphalt mixtures is represented by three paramedics (\( E_v, E \) and \( \tau \)) as given in Equation (1) considering Weichert model as shown in Figure 4. Asphalt mix properties used in the present analysis are given in Table 1 (Lundström, 2002).

![Figure (4): Weichert model with single Maxwell arm](image-url)
Table 1. Properties of asphalt concrete used in the analysis

<table>
<thead>
<tr>
<th>Elastic property</th>
<th>Visco-elastic properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_g )</td>
<td>( E )</td>
</tr>
<tr>
<td>100 MPa</td>
<td>9900 MPa</td>
</tr>
<tr>
<td>10 sec</td>
<td>99000 MPa-sec</td>
</tr>
<tr>
<td>30 sec</td>
<td>297000 MPa-sec</td>
</tr>
</tbody>
</table>

The loading frequency \( (f) \) or cyclic time \( (T) \) of loading can be correlated with the vehicular speed. Under moving load, a pavement section is influenced by the wheel load for a distance of nearly \( \pm 6a \), where \( a \) is the radius of contact area (Huang, 2004). That is, the loading time \( (T) \) can be estimated as \( T = 12a/v \), where \( v \) is the speed. The equivalent standard wheel load \( (P) \) is considered as 32.5kN for a dual wheel configuration of 40kN (or standard axle load of 80kN) (IRC, 2012). The tyre pressure \( (p) \) is taken as 0.6MPa and thus, the \( a \) value can be calculated as \( a = \sqrt{P/(\pi p)} = 130 \)mm. Accordingly, the wheel load frequency \( (f) \) and cycle time \( (T) \) for different vehicle speeds can be estimated as \( T = 1/f = 12a/v \). The variations in \( f \) and \( T \) with speed are presented in Figure 5. \( f \) and \( T \) considered in the present study are also tabulated in Table 2. For numerical analysis of stress parameter, the strain fluctuation \( (\varepsilon_0) \) is taken as 200 micro-strain for all cases. Using this data, the subsequent sections present the computational results for different loading conditions.

Table 2. Loading time and load frequency for different vehicle speeds

<table>
<thead>
<tr>
<th>Speed in kmph (in m/sec)</th>
<th>( T ) (sec)</th>
<th>( f ) (Hz)</th>
<th>( w ) (rad/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 (2.78)</td>
<td>0.56</td>
<td>1.8</td>
<td>11.3</td>
</tr>
<tr>
<td>20 (5.56)</td>
<td>0.28</td>
<td>3.5</td>
<td>22.0</td>
</tr>
<tr>
<td>40 (11.11)</td>
<td>0.14</td>
<td>7.0</td>
<td>44.0</td>
</tr>
</tbody>
</table>

Figure (5): Loading time and load frequency for different vehicle speeds
Stress Characteristics Based on Loading Patterns

For different strain \( (\varepsilon(t)) \) functions (Equations (3)-(6)), the different stress \( (\sigma(t)) \) functions are evaluated using Equations (7)-(10). The \( \varepsilon(t) \) and \( \sigma(t) \) variations with time are presented in Figure 6. It is seen that the trend of \( \sigma(t) \) is significantly different for different loading patterns, though their strain fluctuation \( (\varepsilon_0) \) and cycle loading time \( (T) \) are the same for all the cases. The \( \sigma(t) \) variations for all the cases are shown in Figure 7 for comparison purposes.

Figure (6): Strain and stress variations with time for different loading patterns
The maximum positive and negative stress variations are presented in Figure 8 for each case. Stress fluctuation and phase angle (lagging) \( \phi \) between \( \sigma(t) \) and \( \varepsilon(t) \) are represented in Figure 9.

**Stress Characteristics Based on Loading Time**

Loading period or cycle time \( T \) is different for different vehicular speeds as depicted in Table 2. Reciprocal of \( T \) indicates loading frequency \( f \) under cyclic loading. Usually, 8-10 Hz loading frequency is used in dynamic tests for asphalt mixes (Lundström, 2002; Pramesti et al., 2013; Huang, 2004; Oliveira et al., 2009). For different \( f \) values, as given in Table 2, the \( \sigma(t) \) function has been evaluated for all loading patterns as presented in Figure 10 for one cyclic period in each case. A comparison of \( \sigma(t) \) fluctuations within the cycle time has been presented in Figure 11. From Figures 10 and 11, it is observed that the trend of stress variations does not vary with \( f \). Moreover, stress fluctuations are significantly different for different \( f \) values; i.e., loading
time (or vehicular speed values). It was also observed that for a given loading pattern, the phase angle ($\phi$) remains the same irrespective of $f$.

**Stress Characteristics Based on Relaxation Behavior**

Asphalt mix exhibits different relaxation behaviors at different temperatures. The relaxation time ($\tau$) in visco-elastic materials is defined as the ratio of viscosity ($\eta$) to elasticity ($E$) of the mix; i.e., $\eta = E \times \tau$. An attempt has been made to study the effect of $\tau$ on the stress ($\sigma(t)$) parameter for different strain ($\varepsilon(t)$) loading patterns. For the data given in Table 1, the $\sigma(t)$ functions are evaluated using Equations (7)-(10) for different $\tau$ values. This is shown in Figure 12 for one cyclic period with $f = 3.5$ Hz for all the cases. A comparison of $\sigma(t)$ fluctuations within the cycle time has been presented in Figure 13 for each case. From Figures 12 and 13, it may be concluded that the trends of stress variation and stress fluctuation considerably vary with the relaxation parameter of the asphalt mix.

![Figure (9): Comparison of stress fluctuations and phase angles (lagging) for different cases](image)

### Stress-based Shift Factor

As observed in Figures 9, 11 and 13, there is no difference in stress fluctuation between Case-1 and Case-2 (i.e., $\varepsilon(t)$ without rest time), irrespective of the loading frequency ($f$) and relaxation time ($\tau$) of AC. In Case-3 and Case-4; i.e., for $\varepsilon(t)$ with cyclic rest periods, stress fluctuations are considerably higher than in Case-1 and Case-2. This is due to the combined effect of visco-elasticity and unloading or rest time within the cycle period. Thus, to evaluate the impact of rest period on $\sigma(t)$ amplitude, a shift factor ($SF$) can be defined as:

$$SF = \frac{\sigma(t) \text{ fluctuation with cyclic rest time}}{\sigma(t) \text{ fluctuation without cyclic rest time}}$$

(11)
Figure (10): Stress variations for different frequencies

$SF$ values for Case-3 and Case-4 (with rest time) are presented in Figure 14 in respect of Case-1 and Case-2 (without rest time). It is seen that the $SF$ value decreases with increasing relaxation time ($\tau$); i.e., decreasing temperature. It was also observed that the loading frequency ($f$) has a negligible impact on the $SF$ parameter.
Figure (11): Comparison of stress fluctuations with different frequencies

Figure (12): Stress variations for different relaxation times
This paper presented the stress ($\sigma(t)$) characteristics considering AC as an elasto-visco-elastic material under different strain ($\varepsilon(t)$) loading patterns with different loading frequencies ($f$) and relaxation times ($\tau$) of AC. The following conclusions are drawn from the study results:

- The stress ($\sigma(t)$) parameter for four different types of $\varepsilon(t)$ loading has been analytically obtained, using the principles of visco-elasticity and considering Weichert model. Through numerical evaluation, it is seen that the trend of $\sigma(t)$ profile is significantly different for different $\varepsilon(t)$ loading patterns (Figure 7), for the same strain fluctuation and loading frequency. The $\sigma(t)$ fluctuation and the phase difference ($\varphi$) have been observed to be different for different $\varepsilon(t)$ loading types, though the strain fluctuation is the same for all cases. Of course, there is no difference in the stress fluctuation between Case-1 and Case-2 (with no rest time) and no difference in the phase angle between Case-3 and Case-4 (with rest time) as depicted in Figure 9.

Similar results were also observed for different
loading frequencies \( (f) \) and different relaxation times \( (\tau) \) of AC.

- The \( \sigma(t) \) parameter with different \( \varepsilon(t) \) loading types has been evaluated for various loading frequencies \( (f) \) and it is seen that the trend of \( \sigma(t) \) variations does not change with \( f \) (Figure 10). However, the stress amplitude is affected by \( f \) or vehicle speed (Figure 11). The higher the vehicular speed (or \( f \) value), the lower the stress fluctuation.

- The \( \sigma(t) \) function has been also evaluated for different relaxation times \( (\tau) \) of AC and it is seen that both the trends of stress variation and stress amplitude vary with relaxation parameter (Figures 12 and 13). The higher the \( \tau \) value (or the lower the temperature), the lower the stress fluctuation.

Notations

The following symbols are used in this paper:

- \( E_e \), \( E \) and \( \eta = \) Material properties representing elasto-visco-elastic characteristic of AC
- \( \tau = \) Relaxation time of asphalt mix = \( \eta/E \)
- \( E_r(t) = \) Relaxation modulus at time \( t \)
- \( \sigma(t) = \) Stress function
- \( \varepsilon(t) = \) Strain function
- \( \varepsilon_0 = \) Strain amplitude
- \( f = \) Load frequency
- \( \omega = \) Angular velocity = \( 2\pi f \)
- \( T = \) Cyclic loading time = \( 1/f \)
- \( \theta = \) Phase angle between imaginary and real parts of strain parameter or stress parameter
- \( \varphi = \) Phase difference between stress and strain parameters

REFERENCES


