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Extension of the Enriched Discontinuous Galerkin Approach for Frictional Contact Involving Non-conforming Meshes with Plasticity

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ABSTRACT

The Enriched Discontinuous Galerkin Approach (EDGA) is typically implemented to solve the continuity problem at the interface of contact problems involving non-conforming meshes (NCMs). Through local enrichment, the EDGA enables a two-pass strategy for the enforcement of geometric compatibility at all nodes of the interface without the need of a master-slave definition. The local enrichment transforms the geometric compatibility condition to a set of node-to-node constraints by inserting a new node where a node meets a surface. Updating the set of Lagrangian shape functions to account for the additional node ensures the completeness of the finite element interpolation in the enriched element. In this study, the EDGA is extended to model large-deformation contact problems between bodies with plasticity. However, the enrichment introduces higher-order element shape functions associated with the interface nodes. To solve this problem without losing the material history at the existing integration points, Gauss-Kronrod method for numerical integration is utilized to add an additional set of integration points interlaced between the original Gaussian quadrature. Material history at the additional integration points is then interpolated from the existing points. After proper formulations that pass the contact patch test, this study, through various cases with and without plasticity, confirmed the necessity for proper treatment of contact problems to accurately capture the tractions transferred through the interface between two bodies. Without proper treatment, the results are not accurate, especially for critical contact problems, such as football helmet-skull interaction and oil pipe-soil interaction.

KEYWORDS: Galerkin approach, Contact formulation, Non-conforming meshes, Plasticity.

INTRODUCTION

Computational models for contact problems, in general, and frictional contact problems, in particular, are of high demand in structural engineering and many other fields, where accurate modelling of the interaction between different components across interfaces is required to simulate the behavior of systems, such as steel connections, bridge bearings, soil-structure interaction in piles or other foundations, among others. In modelling contact problems, if the bodies coming into contact are discretized using different finite element meshes, or in the presence of large sliding, the nodes of the first body will no longer coincide with those of the second body across the interface, therefore resulting in a non-conforming mesh (NCM). An NCM is a finite element discretization of a given domain, where point-wise displacement continuity does not hold along a given interface separating two domains discretized with

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conforming meshes. NCMs are created by large sliding or when different finite element mesh sizes are used to increase accuracy in capturing the behaviour in each component and/or along the interface. Interface behaviour can be unilateral, as is typical in contact problems where the two bodies are allowed to separate from each other, or bilateral ensuring full coupling regardless of loading/deformation conditions. The main challenge in both cases, however, is to ensure deformation compatibility and continuity of interface tractions in the absence of full displacement conformity along the interface. The difference between unilateral and bilateral coupling is that these conditions apply to normal components of the kinematic and traction fields, only, in unilateral contact. As such, methods for unilateral and bilateral coupling have traditionally been interchangeably used.

The earliest and simplest contact formulation is the node-to-surface method that enforces displacement continuity between a set of slave nodes at one side of the interface and their projections along the opposing master surface using a set of discrete Lagrange multipliers. This method is generally not capable of representing a state of constant pressure, therefore failing the well-known patch test (Papadopoulos and Taylor, 1992). The primal interface element method was widely used in literature. Zaman et al. developed a simple thin-layer element and used it in a finite element procedure for simulation of various modes of deformation in dynamic response (Zaman et al., 1984). Hird and Russell presented an analytical solution for the compression of a long elastic block, bonded along one side to a rigid material (Hird and Russell, 1990). Karadeniz introduced an interface 3-D beam element for the analysis of framed structures which interact with an elastic medium (Karadeniz, 1999). The formulations of the element were based on the assumption that the elastic medium can be represented by a two-parameter model of the Winkler model and Pasternak model (Pasternak, 1954). Luan and Wu proposed a non-linear elasto-perfect plastic model for the interface element to simulate the behavior of Soil-Structure-Interaction (SSI) contact problems (Luan and Wu, 2004). Swamy et al. analyzed SSI problems adopting the finite element method and the usage of link/interface elements between two elements of different materials (Swamy et al., 2011). Their study concluded that the presence or absence of interface elements affects the settlement, differential settlements and stresses in soil. Mahmood and Ahmed adopted a finite element approach to model an SSI system consisting of reinforced concrete plane frame, soil deposit and interface, which represents the frictional surface between foundation of the structure and sub-soil. The authors concluded that the thin-layer interface element method could successfully simulate the effect of slip and separation in the dynamic analysis of soil-reinforced concrete frame interaction problems (Mahmood and Ahmed, 2008). The mortar method (otherwise known as the segment-to-segment formulation) is a widely used dual approach, where the gap function is averaged along the contacting segments and the pressure at the slave contact points is interpolated in terms of the nodal pressures on the master surface (Puso and Laursen, 2004). The drawback in this method is the Ladyzhenskaya Babuška-Brezzi (LBB) condition that governs the stability of dual finite element as well as the bias of choosing the master and slave surfaces. One of the most widely primal coupling approaches is the Discontinuous Galerkin (DG). Approach. This approach is used for the coupling problem, since it readily assumes discontinuous discretization on all inter-element interfaces. The DG formulation is based on identifying a set of target continuous fields for the displacement and traction fields on each interface and mapping the discretized displacement and traction fields on each surface to these target fields in a weak weighted residual form (Brezzi et al., 1999). The clear advantages of the primal DG method over dual ones are the unbiased treatment of the interface and the absence of the LBB restrictions. These methods, however, require a mesh-dependent stabilization parameter.

The Enriched Discontinuous Galerkin Approach (EDGA) developed by Haikal and Hjelmstad for the coupling of NCMs is a primal interface formulation that ensures geometric compatibility and a complete transfer of surface tractions between the connecting elements at the non-conforming interfaces (Haikal and Hjelmstad, 2010). The approach is based on a local enrichment of the non-conforming interface that enables a simple enforcement of the continuity of the displacement field using a set of discrete node-to-node constraints, thereby eliminating the need for master/slave designations. The authors treated the interface using a form of the DG method that guarantees the complete transfer of forces along non-conforming inter-element boundaries. The proposed interface formulation was shown to be consistent, stable and including the continuous Galerkin as a subset. The key advantages of this method are that it uses finite element estimates of the stress fields on the interface and that it is able to accommodate sliding and non-linearity (material and geometric).

INTERFACE FORMULATION

The simplest and earliest method for enforcing contact conditions is the node-to-surface approach illustrated in Figure 1. The node-to-surface contact constraint measures the gap or oriented distance between a "slave" node and its projection on the opposing "master" surface. The bodies on either side of the interface are free to move apart or come in contact with each other and the sign of the gap function is used to distinguish between these two scenarios.

i) Contact Patch Test Using the Node-to-Surface Approach

The contact patch test is used to check the ability of node-to-surface contact algorithm to transfer the stresses uniformly through the interface. Figure 2 shows the typical contact patch test, which consists of a punch in contact with a rectangular foundation, with a distributed load applied at the top free surfaces of the structure.

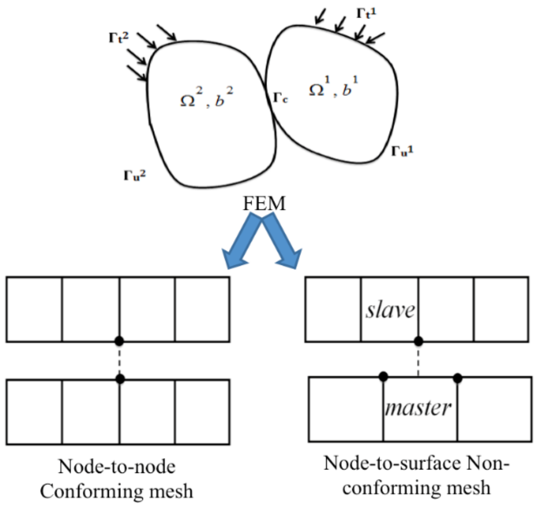

Figure (1): FEM interface discretization

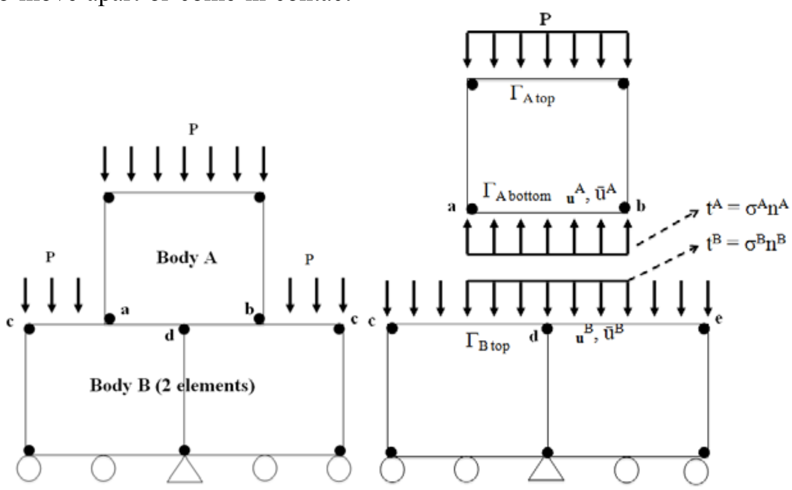

Figure (2): Contact patch test

From the equilibrium of the free body *A*, given that *u* is the displacement, the applied pressure *P* over the domain *Γ* must be equal to the internal stress t^A as follows:

$$\int_{\Gamma_{Atop}} P d\Gamma = \int_{\Gamma_{Abottom}} t^A d\Gamma \tag{1}$$

Assuming that the finite element discretization in element *A* is complete and can reflect a constant state of pressure, then:

$$\int_{\Gamma_{Atop}} P d\Gamma = \int_{\Gamma_{Abottom}} P d\Gamma \tag{2}$$

This implies that:

$$\int_{\Gamma_{Abottom}} t^A d\Gamma = \int_{\Gamma_{Abottom}} P d\Gamma \tag{3}$$

The work done by body *A* on body *B* is:

$$\int_{\Gamma_{Abottom}} t^A . \overline{u}^A d\Gamma = \int_{\Gamma_{Abottom}} P . \overline{u}^A d\Gamma \tag{4}$$

Similarly, the work done by body *B* on body *A* is:

$$\int_{\Gamma_{Btop}} t^B . \overline{u}^B d\Gamma \tag{5}$$

For the equilibrium to hold at the interface, the following must be true:

$$\int_{\Gamma_{Abottom}} t^A . \overline{u}^A d\Gamma = \int_{\Gamma_{Btop}} t^B . \overline{u}^B d\Gamma \tag{6}$$

$$\int_{\Gamma_{Abottom}} P . \overline{u}^A d\Gamma = \int_{\Gamma_{Bbottom}} t^B . \overline{u}^B d\Gamma \tag{7}$$

For the transfer of pressure to be complete, the traction field on $\Gamma_{Btop} = P$ is:

$$\int_{\Gamma_{Abottom}} P . \overline{u}^A d\Gamma = \int_{\Gamma_{Bbottom}} P . \overline{u}^B d\Gamma \tag{8}$$

Since $\Gamma_{Abottom} = \Gamma_{Btop} = \Gamma_C$, this leads to:

$$\int_{\Gamma_c} P . u^{-A} \, d\Gamma - \int_{\Gamma_c} P . u^{-B} \, d\Gamma = 0 \tag{9}$$

For the pressure transfer to be complete, the following condition has to hold:

$$\int_{\Gamma_c} u^{-A} \, d\Gamma - \int_{\Gamma_c} u^{-B} \, d\Gamma = 0 \tag{10}$$

Therefore, for the contact formulation to pass the patch test, the variational field needs to be continuous, at least in a weak sense, across the interface. The node-to-surface contact algorithm does not pass the patch test, since the gap function g^n guarantees continuity at the slave nodes only. The NCM node-to-surface algorithm does not pass the contact patch test due to the inaccurate transfer of forces from one side to the other. The difference between conforming meshes and NCMs is shown in Figure 3, which is a demonstration of the patch test using Abaqus. Figure 3 (left) shows a complete transfer of stresses along the interface in conforming meshes, while Figure 3 (right) shows incomplete transfer of stresses along the interface in non-conforming meshes.

ii) Interface Model: EDGA

The Enriched Discontinuous Galerkin Approach (EDGA) is implemented to solve the continuity problem at the interface of NCMs. The EDGA is a primal approach that enables a two-pass strategy for the enforcement of geometric compatibility along the interface through local enrichment. The method is based on a local enrichment designed to guarantee geometric compatibility at all nodes of the interface, without the need of a master-slave definition. The local enrichment transforms the geometric compatibility condition to a set of node-to-node constraints by inserting a new node where a node meets a surface. This local enrichment can be enforced at all nodes along the interface.

Completeness of the finite element interpolation in the enriched element can be preserved by updating the set of Lagrangian shape functions to account for the additional node. Since the displacement between the nodes remains discontinuous, the DG stabilization is applied to ensure the continuity of the tractions in a weak form along the interface. The EDGA is explained below. We start by discussing the fully coupled (bilateral) case and then move to release tangential displacement constraints to enable sliding along the interface.

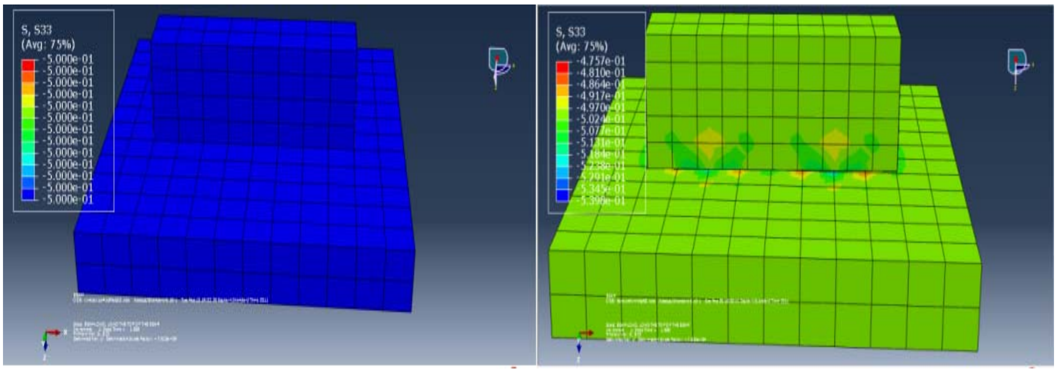

Figure (3): Complete *versus* incomplete transfer of stresses through the interface in conforming (left) and non-conforming (right) meshes

iii) Extension of the EDGA for Plasticity

In this study, the EDGA, developed for NCM coupling, is extended to model large-deformation contact problems between bodies with inelastic constitutive behaviour. The EDGA is a primal approach that enables a two-pass strategy for the enforcement of geometric compatibility along the interface by inserting a new node at contact locations. This local enrichment can be enforced at all nodes along the interface. The enrichment in the element introduces a higher order in the element shape function associated with the nodes located on that interface. Thus, the order of interpolation has to be increased in the direction where the node is inserted. Solving this problem in hyperelasticity is usually accomplished by increasing the order of the Gauss integration scheme, thereby introducing new integration points. This process, however, could be problematic for the case of inelasticity. For history-dependent materials in which plastic strains are stored and accumulated at the Gauss points after each converged load step, the computational history at the integration points before enrichment must be preserved. Therefore, a progressive integration rule such as the Gauss-Kronrod quadrature can be used alternatively. The Gauss-Kronrod quadrature inherits Gauss point locations and provides an additional set of integration points interlaced between the original Gaussian quadrature. To compute the number and locations of additional Kronrod points required to accurately evaluate a given integral, we begin by computing an estimate of the integral with the original Gauss quadrature. Then, we re-compute it using two sets of points combined; the original Gauss points set and the Gauss-Kronrod set. The difference between the values of the two sets gives an estimate of the error in the results.

Consider the enriched element shown in Figure 4. We assume an enrichment of the top surface $\zeta_2 = 1$,which introduces a quadratic term in ζ_1 in the element shape functions associated with the nodes located on this interface, while the order of interpolation with respect to ζ_2 remains the same. Therefore, for the element to be integrated properly, the integration rule order has to be increased in the direction of ζ_1. For the use of this element in contact simulations, two different sets of Gauss-Kronrod integration points are needed. The first set of points is used inside the element as illustrated in Figure 4, where the stresses and plastic strains are computed to find the internal forces and the stiffness of the element. In addition, Gauss-Kronrod integration points are needed on the interface, as illustrated in Figure 4, to be used for the stabilization terms, where the stresses and plastic strains are computed.

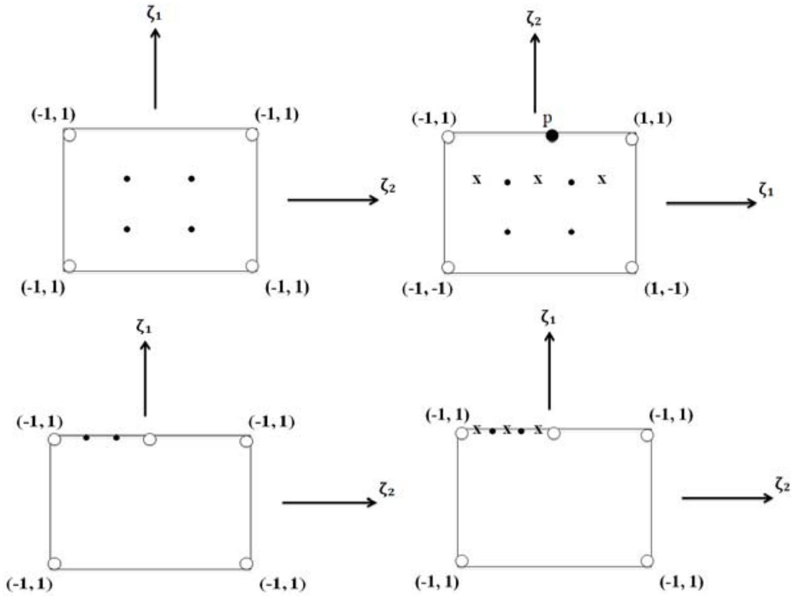

Figure (4): Q4 element with Gauss quadrature integration points inside and at the surface (left) and the enriched element with Gauss-Kronrod integration points (right)

RESULTS

i) Verification of the Gauss-Kronrod Integration Scheme

We use a single Q4 element under uniaxial tension for the verification of the Gauss-Kronrod integration scheme. The Q4 element shown in Figure 5 has an elastic modulus (E) =30,000 ksi, Poisson's ratio (υ) = 0.3 and yield stress (f_y) = 60 ksi. We assume plain-strain conditions. The applied distributed load P is equal to 65 ksi. This set-up is identical to the well-known patch test using finite elements and the expected solution is a constant pressure profile in the element. This element is enriched at the bottom surface, which introduces a higher order term in the element shape functions associated with the enriched node. The Gauss-Kronrod quadrature integration points are used along the enriched side as shown in Figure 5. The results in Figure 6 show that the Q4 element with Gauss-Kronrod quadrature integration points passes the patch test and exactly reflects a constant stress distribution. Figure 6 shows the Q4 element deformed shape, which is also consistent with the exact solution.

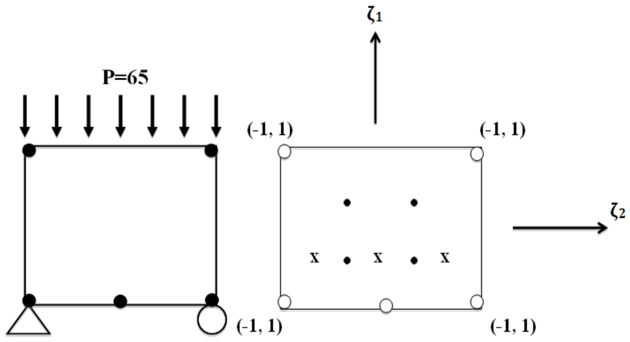

Figure (5): Patch test for Q4 element with enrichment (left) and Gauss-Kronrod quadrature integration points (right)

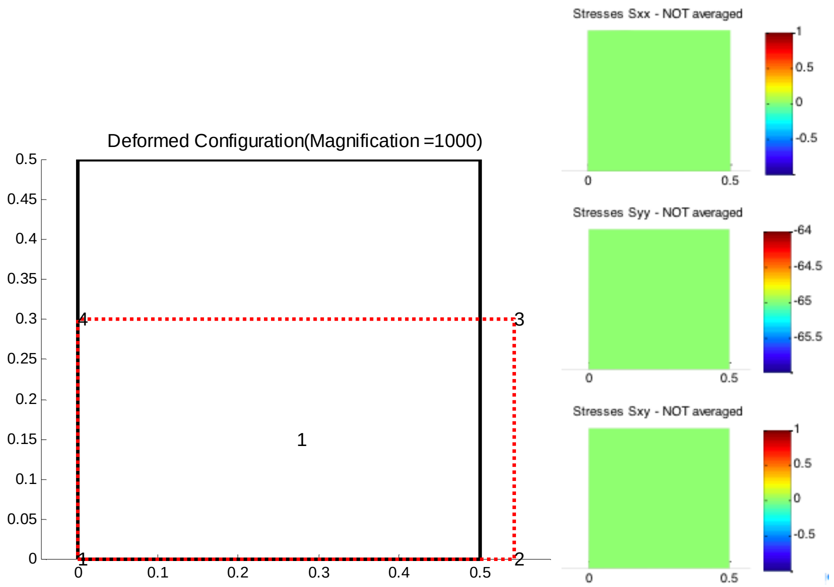

Figure (6): Deformed shape and stress distributions for Q4 element with Gauss-Kronrod integration points

ii) Small Deformation Linear Elastic Case

We consider the contact patch test configuration of Figure 7 and assume small deformations and a linear elastic material with E = 30,000 ksi and υ = 0.3. A distributed load of P = 200 ksi is applied on the top free surfaces of both bodies. The domains are discretized using Q4 elements under plane strain conditions. This case serves the purpose of verifying our implementation of the EDGA, showcasing its superiority in handling interface tractions when compared to the standard node-to-surface formulation. Figure 7 shows the deformed shape and stress distributions, respectively, obtained with a standard node-to-surface contact formulation without treating the non-conforming interface. The results show the deformed shape and stress distributions, respectively, obtained with a standard node-to-surface contact formulation without treating the non-conforming interface. The results show an incomplete transfer of the traction along the interface and inaccurate deformed shape. When the solution is obtained by applying the EDGA at the interface, the results show that the deformed configuration and the stress distributions reflect a state of constant pressure up to machine precision.

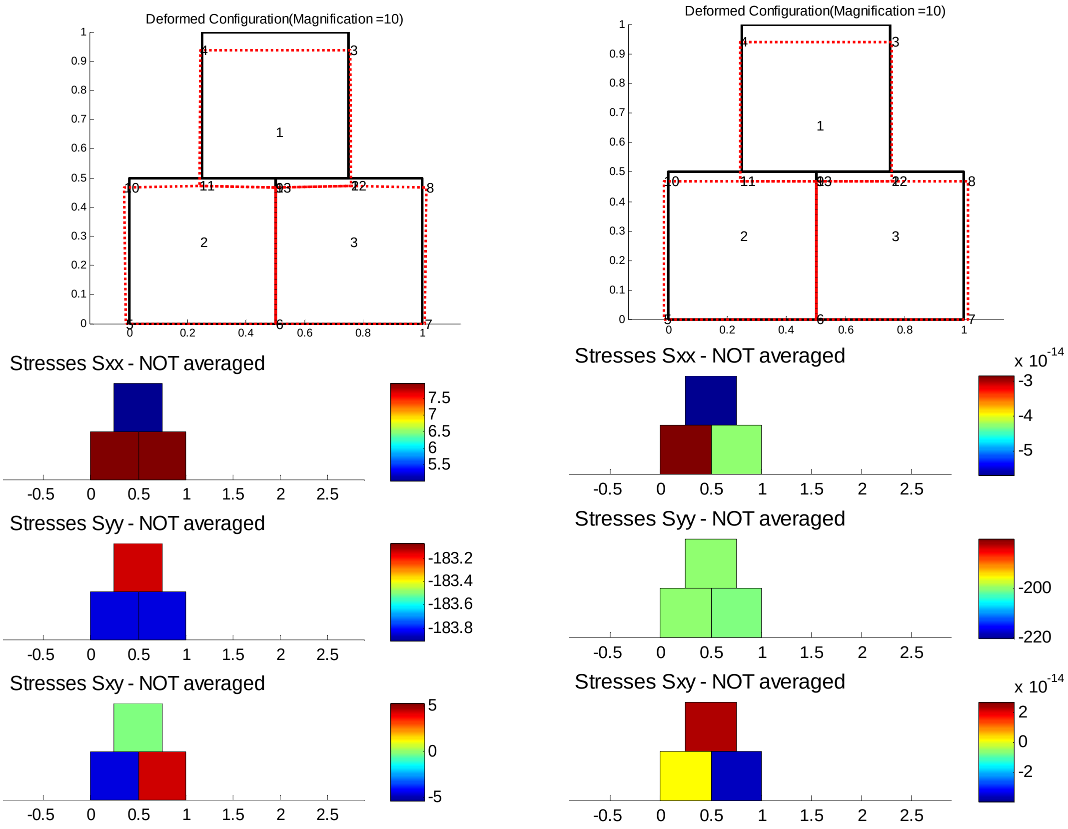

Figure (7): Contact patch test for the small deformation linear elastic case: deformed shape and stress field without EDGA (left) and with EDGA (right)

iii) Large Deformations with Hyperelasticity

In this case, we assume a hyperelastic material with the properties E = 30,000 ksi and υ = 0.3. A distributed load of P = 1000 ksi is applied to the top free surfaces of both bodies. The domains are discretized using Q4 elements under plane strain conditions. Figure 8 shows the solution obtained using the standard node-to-surface interface formulation without treating the non-conforming interface. Similar to the previous case, the results show an incomplete transfer of tractions along the interface and the deformed shape does not make sense, since the applied pressure is uniform and accordingly nodes 8, 9 and 10 are expected to deform similarly. When the solution is obtained by applying the EDGA at the interface, the results show that the deformed configuration and the stress distributions pass the patch test up to machine precision, which is also similar to the previous case.

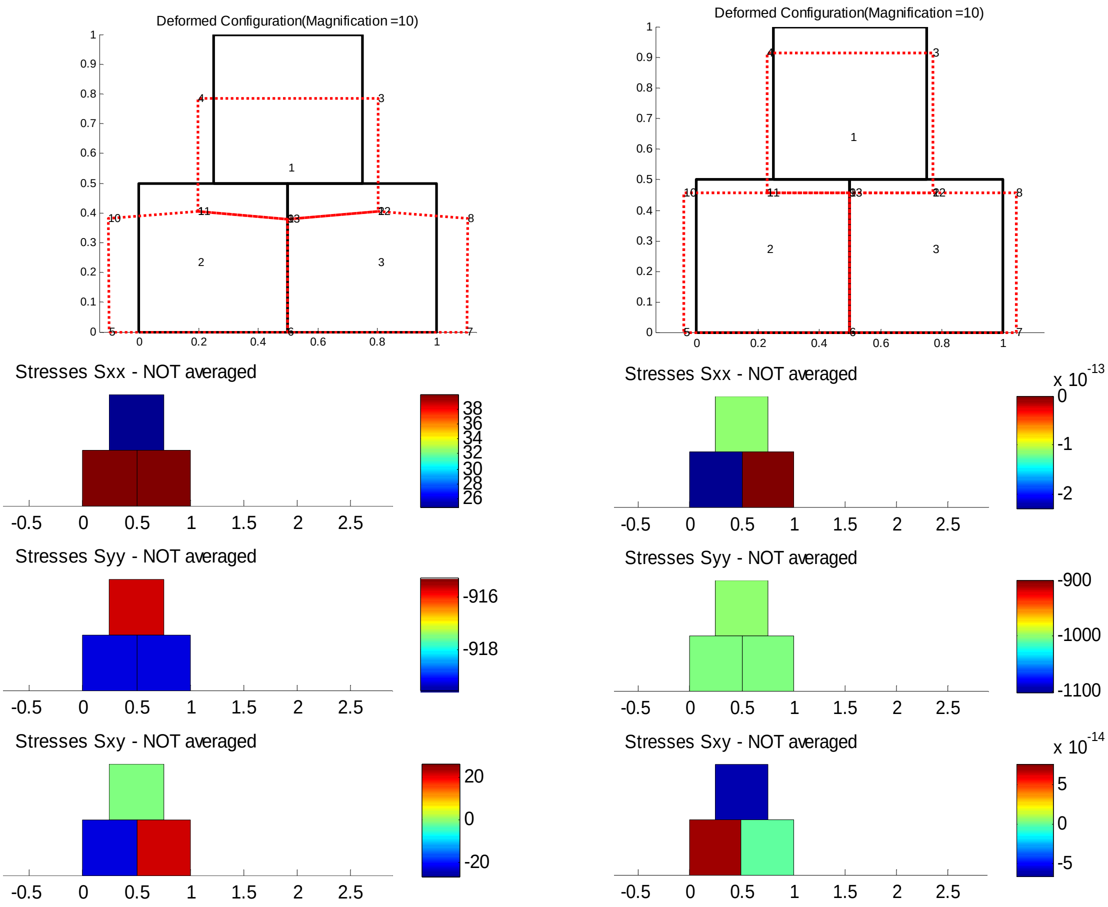

Figure (8): Contact patch test for the small deformation non-linear elastic case: deformed shape and stress field without EDGA (left) and with EDGA (right)

iv) Large Deformation with Von Mises Plasticity

In this case, an elasto-plastic material with the Von Mises yield criterion is used with E = 30,000 ksi, $\upsilon = 0.3$ and $f_y = 60$ ksi. A distributed load of P = 70 ksi is applied to the top free surfaces of both bodies. The domains are discretized using Q4 elements under plane strain conditions. Figure 9 shows the solution obtained without treating the non-conforming interface. The results show inaccurate pressure distribution and the deformed shape displays similar inaccuracies at nodes 8, 9 and 10. Plasticity in this problem is activated, since the applied load is greater than f_y. The plastic strain at the integration points is around 0.00965. Figure 9 shows the solution obtained by applying the EDGA for plasticity using Gauss-Kronrod quadrature integration points inside the element to find the tangent and the internal forces and at the interface for the stabilization terms. The results of this case also show that the deformed configuration and the stress distributions pass the patch test up to machine precision.

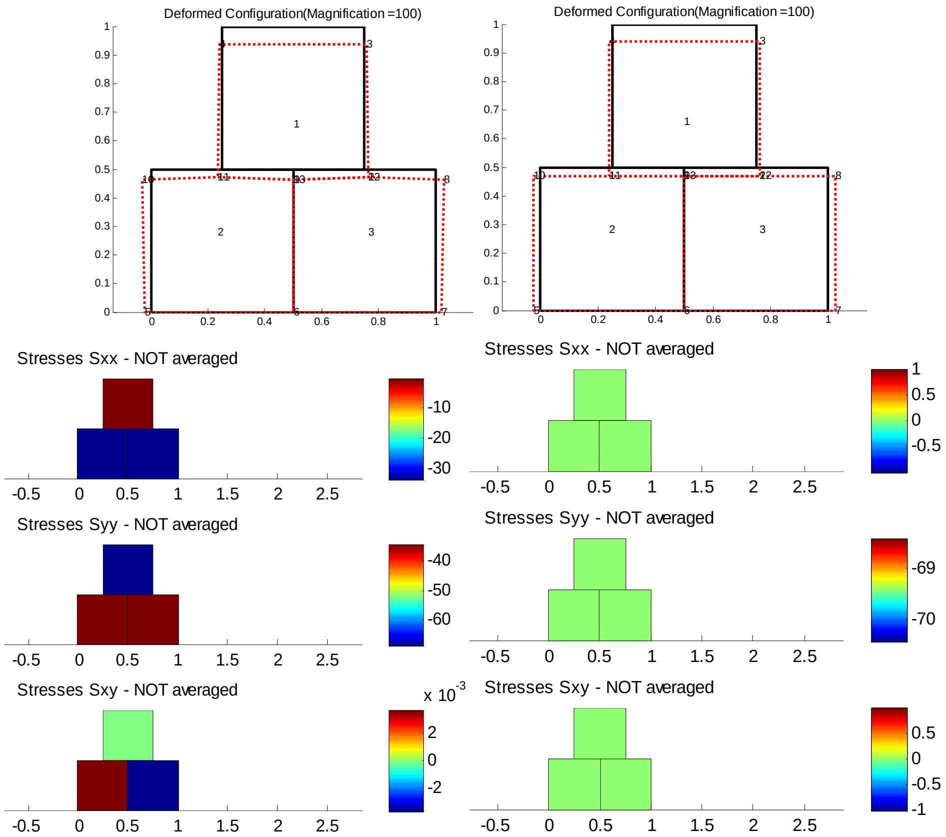

Figure (9): Contact patch test for the elasto-plastic case (Von Mises): deformed shape and stress field without EDGA (left) and with EDGA (right)

SUMMARY AND CONCLUSIONS

Numerical simulations of contact problems require proper modelling of all of its components: the domains and the interface. Large deformations, material non-linearities including plasticity, as well as interface friction, are important parameters that ought to be considered for accurate results. Contact simulations typically involve NCMs, either caused by sliding or due to adaptive refinement to increase accuracy in capturing localized behaviour as well as interface effects. The main challenge in using NCMs, however, is how to ensure geometric compatibility and complete transfer of tractions through the interface. Enforcing geometric compatibility at a set of nodes or discrete points using the node-to-surface approach does not reflect complete transfer of tractions at the interface. On the other hand, dual approaches that employ Lagrange multiplier field to enforce geometric compatibility in a weak sense is biased in choosing the master and the salve surfaces.

The EDGA is a primal approach that ensures geometric compatibility and complete transfer of surface tractions by virtue of a local enrichment in the element at the contact locations and a stabilization procedure along the interface. In this study, the EDGA is extended to problems with plasticity, where material history plays a major role in determining response, particularly in the presence of large deformations. Applying the EDGA involves the insertion of a node on the contact surface, thereby raising the order of interpolation in the contact element. To increase the order of integration without loss of material history at existing integration points, Gauss-Kronrod method for numerical integration is used that computes an additional set of integration points interlaced between the original Gaussian quadrature. Material history at the additional integration points is then interpolated from data at the existing points. The examples provided in this study for linear elastic, non-linear elastic and non-linear plastic cases confirm the necessity for proper treatment of contact problems to accurately capture the tractions transferred through the interface between two bodies such as in SSI problems. Without proper treatment, the results are not accurate.

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