Inelastic Dynamic Behaviour of Simply Modeled Explicit and Implicit Non-Ductile RC Beam-Column Joints

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ABSTRACT

This study focuses on evaluating the responses of five variants of simple implicit and explicit beam-column joint modeling schemes. A non-linear open-source finite element platform, OpenSees, was employed in performing the structural analysis of a beam-column joint sub-assemblage and a reinforced concrete frame building subjected to pseudo-static and dynamic analysis, respectively. The explicit rotational spring models with rigid links (SLM) and without rigid links (SM) were fairly able to estimate the hysteresis behaviour accurately, whilst the explicit ACI/ASCE joint model (ASCEM) was conservative in predicting the joint shear strength and post peak drift capacity, in addition to its inability to capture the stiffness and strength degradation accurately. The implicit centerline models (CLM and CLRBM) were unable to mimic the energy dissipation as well as the induced flexibility of the non-ductile joint imposed on the system. The explicit joint models (CM, CLM and ASCEM) generally observed an incremental shift in the modal periods of vibration, whilst for the implicit joints, there was no change in individual modal periods of vibration and the associated mass participations. The SLM, ASCEM and CLRBM joint models experienced fairly the same peak drift and rotational angle distribution along the story levels, whilst the other models were outliers.

KEYWORDS: Beam-column joint, Reinforced concrete frames, Bond slip, Shear deformation.

INTRODUCTION

Recent earthquakes have shown that older type non-ductile reinforced concrete buildings are very vulnerable and do sustain significant damage under seismic action. Beam-column joints of these buildings are deemed to have detailing deficiencies that can impose significant strength and stiffness loss and as such contribute to their global collapse (Moehle et al., 1991). The major deficiencies that are typical of such buildings include the absence of transverse hoops, insufficient anchorage of beam reinforcement, splicing longitudinal reinforcement and short embedment length of bottom beam reinforcement within the joints. Hence, in such frames, the development of shear resisting mechanisms (strut and truss mechanisms) to induce ductile failure under dynamic loading are not present (Hoffmann et al., 1992). Under seismic action, the primary inelastic mechanisms that govern joint response are bond slip and shear deformation (Celik and Ellingwood, 2008). These mechanisms are characterized by cracking of concrete, crushing of confined and unconfined concrete, closing of concrete cracks under load reversal, shearing across concrete crack surfaces, yielding of reinforcing steel and damage to bond zone concrete (Mitra 2007).

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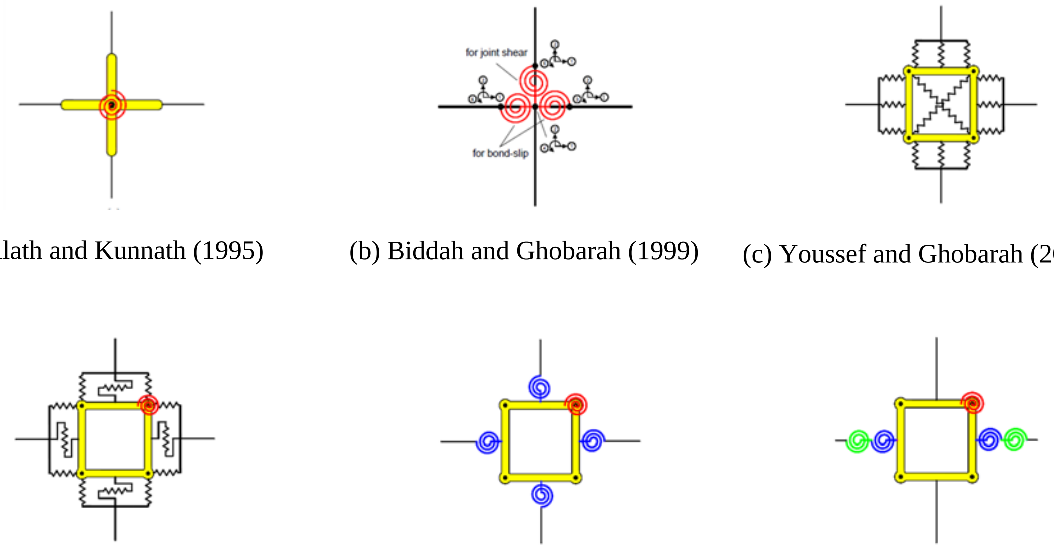
In providing a better understanding to the behaviour of beam-column joint sub-assemblages, several rotational spring idealizations (Fig.1) which can characterize the inelastic mechanism in the finite region of the beam-column joint have been proposed (Alath and Kunnath, 1995; Chao-Lie et al., 2016; Zhang et al., 2016).



(a) Alath and Kunnath (1995) (b) Biddah and Ghobarah (1999) (c) Youssef and Ghobarah (2001)

(d) Lowes and Altoontash (2003) (e) Altoontash (2004) (f) Shin and LaFave (2004)

Figure (1): Existing beam-column joint models

Simulating these inelastic mechanisms is imperative for seismic vulnerability assessment (Kien et al., 2012). The use of a simple rigid element or one-component rotational spring for joint modeling is advantageous in terms of computing resources and sophistication of analysis so far global inelastic dynamic behaviour of buildings is concerned. This study focuses on evaluating the performance and variations in the responses of what the authors deem to be the most widely used simple implicit and explicit beam-column joint modeling schemes for the seismic assessment of non-ductile RC frame structures. The principles underpinning simple basic implicit and explicit beam-column joint modeling are discussed, in order to model five variants of an interior prototype joint for evaluation. The five joint models comprised two rigid joint models and three flexible rotational spring joints of varying flexibility. The rigid joint models were modeled with non-linear elements but without a degradable hysteresis model, whilst the flexible joint models were modeled with a pinching and degradable hysteresis model. The five joint models are first validated using sub-assemblages subjected to pseudo-static loads before being implemented in a hypothetical non-ductile RC frame for an analytical inelastic dynamic study.

MODELING JOINT LOAD-DEFORMATION BACKBONE CURVES

The computational platform, OpenSees, is adopted for RC frame simulation. Traditionally, the numerical simulations of dynamic response of reinforced concrete frames assume that beams and column framing into a joint intersect at the centerline. These reinforced concrete moment resisting frames have been deemed to be rigidly connected and as such the orthogonality of their adjoining structural components (beams and columns) is assumed to be maintained under lateral loading. Researchers have found that this conventional approach to simulation of end zones of structural frame components as rigid can be valid for buildings that have been designed to modern seismic provisions. However, for buildings that do not conform to ductile detailing requirement, the inclusion of joint flexibility in RC frame simulation has proven to be significant in seismic vulnerability assessment. In this section, the fundamental principles of the derivation of joint load-deformation backbone curve for an explicit and implicit beam-column joint are presented.

Explicit Single Rotational Spring Joint Shear Backbone Curve

In accounting for the two major inelastic mechanisms that affect unreinforced and reinforced concrete joints under seismic action (i.e., shear and anchorage failure), a single rotational spring element that spans the joint region is typically used in RC frame simulation.

Alath and Kunnath (1995) recommended the inclusion of rigid links across and along the finite joint region to improve the analytical prediction of inelastic responses. The main input for the rotational spring consists of a joint shear monotonic backbone curve that defines the one-dimensional constitutive relationship and a hysteretic damage model that can account for the relative lower energy dissipation behaviour (pinching) that RC beam-column joint sub-assemblies experience under reverse cyclic loading (Favvata et al., 2014) (Fig. 2).

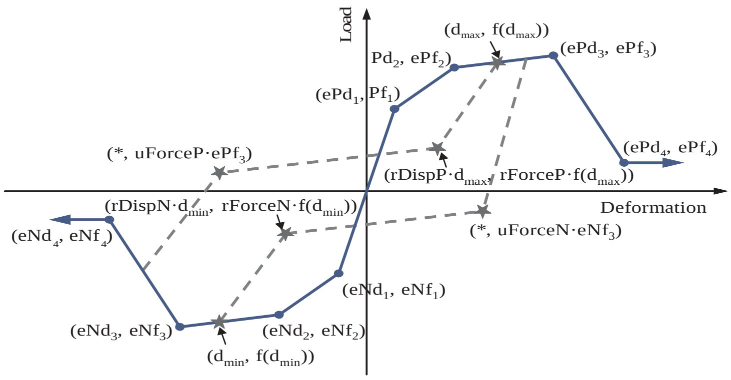


Figure (2): Pinching 4 damage material model

In order to provide a reliable estimate of this modeling approach, researchers have aimed at predicting the shear strength of non-conforming joints of varying in-plane and out-of-plane geometries, with an appreciable level of accuracy. In most of these shear strength models, the compressive strength of the in-place concrete has proven to be the most statistically significant parameter that characterizes RC joint behavior. The unified joint shear-stress Equation (1) of Kim and LaFave (2009) is adopted in the definition of the backbone curve (Fig. 3) for the explicit analytical models considered in this study.

$$V_j = 1.21 TB^{0.981} JI^{0.136} BI^{0.301} JP^{1.33} fc^{0.774} \left(\frac{e}{b} \right)^{0.67} \quad (1)$$

where *BI* is the beam longitudinal reinforcement index, *JI* is the joint transverse reinforcement index, *TB* is the confinement factor, *JP* is the parameter that describes the in-plane geometry, f_c is the compressive strength, *e* is the joint eccentricity and *b* is the column width.

Similarly, ACI 369-R11 and ASCE/SEI 41-06 have proposed a backbone curve that can be used in the seismic assessment of reinforced concrete frames and sub-assemblages. The code recommends that a generalized monotonic load-deformation curve (Fig. 3) be combined with modeling the joint panel zone as a rigid element, depending on the relative flexural moment capacities of the beams and columns framing into the joint.

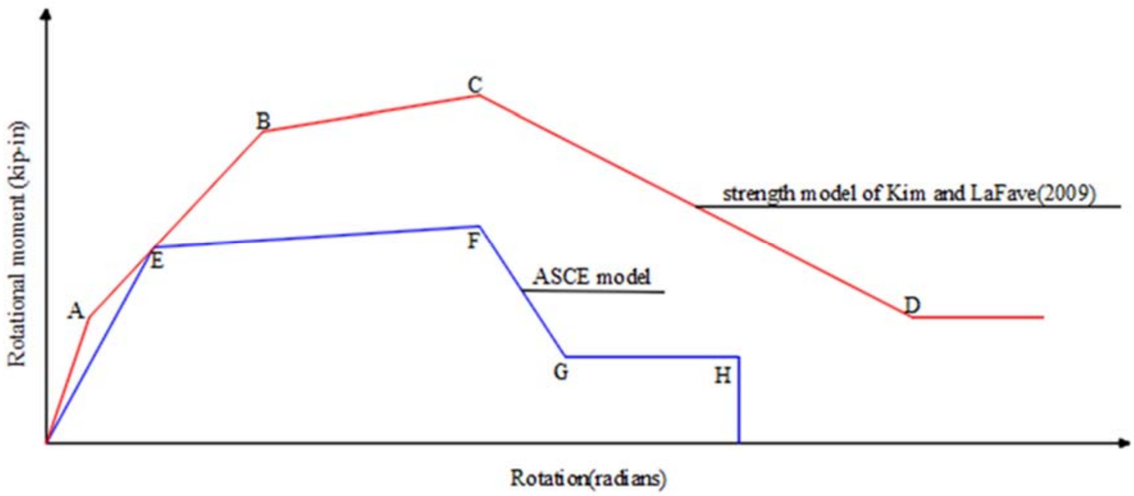


Figure (3): Generalized load deformation curve for explicit single rotational spring backbone curves

Implicit Equivalent Joint Moment-Rotation Backbone Curve

Depending on the mode of failure that RC frames experience, another approach to simulation is to reduce the flexural moment capacity of the beams and columns framing into the joint to levels that will induce joint shear failure. The limiting horizontal and vertical joint shear capacity can be estimated by considering the type of joint sub-assemblage under study. For instance, the maximum shear capacity for the interior sub-assemblage shown in Fig. 4 after a little algebra and satisfying equilibrium equations is defined as:

$$V_{jh} \geq 2M_b\left(\frac{1}{j_d} - \frac{1}{L_c\left(1-\frac{b_j}{L_b}\right)}\right) \tag{2}$$

$$V_{jh} \geq 2M_c\left(\frac{1}{j_d} - \frac{1}{L_c\left(1-\frac{b_j}{L_c}\right)}\right) \tag{3}$$

where V_{jh} is the horizontal joint shear capacity, V_{jv} is the vertical joint shear capacity, M_b is the flexural moment capacity of the adjoining beam, M_c is the flexural moment capacity of the adjoining column, j_d is the distance between the top and bottom reinforcing bars, L_c is the total length of column, L_b is the total length of beam and b_j is the width of the joint panel.

The lesser of these two orthogonal joint shear capacities can then be used to estimate the joint shear strength which can then be converted into the reduced moment capacity as:

$$\tau = \frac{V_j}{A_j} \tag{4}$$

$$M_j = \tau_j A_j \frac{1}{\left[\frac{1-b_j/L_b}{j_d} - \frac{1}{L_c}\right]} \quad (5)$$

where M_j is the joint rotational moment capacity, τ_j is the joint shear stress, A_j is the effective area of the joint panel. In order to incorporate the effect of bar-slip due to discontinuous bottom beam reinforcement, a reduction factor can be either applied to its area or yield strength in defining the pullout moment capacity of the section in the plastic hinge zone of the beam element (Hoffman et al., 1992; Jeon et al., 2012). The factor relates the ratio of the provided embedment length to the required development length (Elwood et al., 2007).

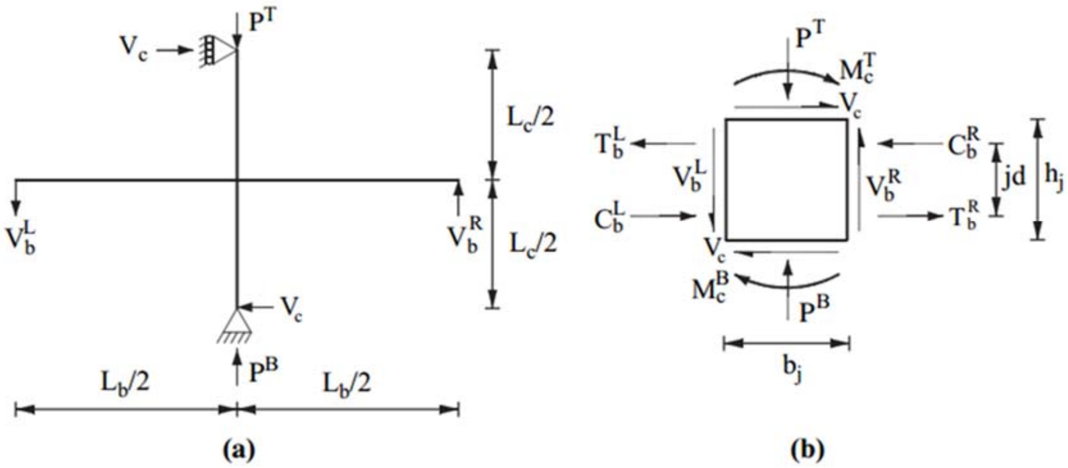
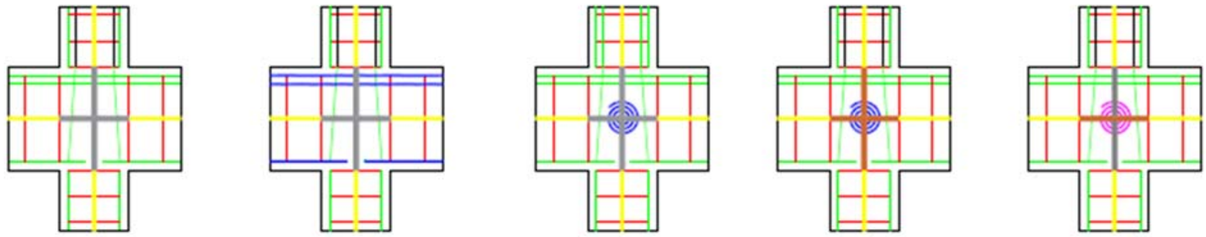


Figure (4): The free body diagrams of (a) a typical interior beam-column joint test setup and (b) its joint panel

Joint Modeling Techniques and Validation

The aforementioned analytical joint modeling techniques were investigated to determine the variations in their dynamic response quantities as well as their performance to experimental results of joint sub-assemblages. The developed RC joint models (Fig. 5) included the conventional centerline model (CLM), scissors joint model without rigid links (SM), scissors joint model with rigid links (SLM), the ASCE non-linear joint model (ASCEM) and the reduction in flexural capacity of framing beams to levels that induce joint failure (CLRBM). Three of the developed RC joint models (SM, SLM and ASCEM) employed the explicit single rotational joint shear rotation backbone curve with the OpenSees *Pinching 4* uniaxial damage model, whilst the other two models (CLM and CLRBM) used the implicit equivalent joint moment-rotation backbone curve with a non-degradable hysteresis model.

Component modeling of structural elements is required to be accurate enough to represent the full range of response; from diagonal cracking of concrete to significant strength and stiffness degradation, through to global collapse (Haselton et al., 2008). In addressing this issue and coupled with the computational demands from non-linear response history analysis, a fiber model and a plastic-hinge model capable of analytically reproducing the non-linear behaviour RC frames under seismic forces are adopted. The fiber-model (*forcebeam-column element*) that integrates the section response of the composite materials in the RC member as well as the spread of inelasticity along the member was used to define the column elements. A plastic-hinge model (*beam with hinges*) that localizes the inelastic response in the plastic hinges at the ends of the beams was used to characterize the beams' non-linear deformations. The chosen plastic hinge length spanned a distance of half the depth of the beam (Park et al., 1982). Following recommendations of ACI 369 R-11, a flexural strength reduction factor of 0.3 was applied to the elastic intermediate portion of the beam elements. The modified Kent-Park model (Park et al., 1982) was used to quantify the marginal increase in strength of concrete due to transverse steel confinement. Also, in assessing the experimental performance of the joint sub-assemblage under study, Aycardi et al. (1994) recommended a reduction factor of 0.5 for the yield strength of the bottom reinforcement in defining the pullout moment capacity of the framing beams.



(a) (b) (c) (d) (e)

Figure (5): (a) Centerline joint model (CLM), (b) scissors model without rigid links (SM), (c) scissors model with rigid links (SML), (d) centerline model with reduced beam flexural capacity (CLRBM) and (e) ASCEM model

The experimental work of Bracci et al. (1992), who evaluated the seismic performance of a one-third scaled three-story gravity loading design RC frame satisfying similitude requirements, was used in this study. The scaled model frame (Fig. 6a) possessed deficient reinforcing details, such as absence of transverse hoops, insufficient anchorage of beam reinforcement, splicing longitudinal reinforcement and short embedment length of bottom beam reinforcement within the joint. The force- drift response of the interior sub-assemblage was used to evaluate the developed analytical joint models as seen in the results and discussion section.

DESCRIPTION OF CASE STUDY RC FRAME

The five different analytical joint models were then implemented in a hypothetical three-story three-bay reinforced concrete frame model of weak column-strong beam design, with an expected soft-story collapse mechanism. The developed model is designed to represent the transverse interior frame of a hypothetical building (DL = 4.22kN/m^2 and LL = 4.0kN/m^2) designed to BS 8110 (1985) with 4m center to center in both longitudinal and transverse directions (Fig. 7).

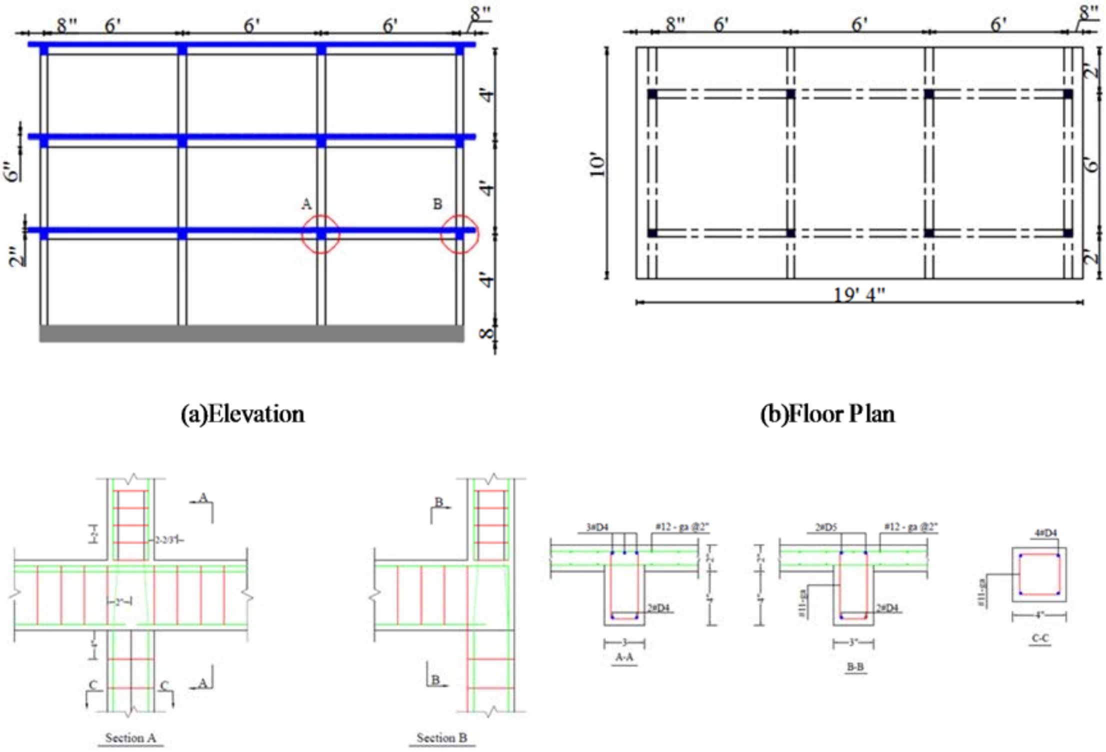
A factored uniform distributed load from gravity loads of contributing tributary areas was applied to the beam per load combination (1.4DL+1.6LL), with dynamic masses lumped at each connection per load combination (1.05DL + 0.25LL). The model frame was built in the open-source computational software OpenSees and a non-linear time history analysis was performed by utilizing ten historical records that were scaled to match a target spectrum at the first modal period of the model frame, T=0.59 (Fig. 8).

RESULTS AND DISCUSSION

Validation of Analytical RC Beam-Column Joint Models

Fig. 9 shows the plots for the base shear and lateral drift ratio for the interior sub-assemblage of the various joint models.

The tested sub-assemblage was able to sustain a maximum horizontal load of 1.65 kip at a drift amplitude of 2%. Comparing this load with analytical models, the scissors with rigid links (explicit model) gave the closest prediction of 1.67 kip at that particular drift level. The scissors models with or without rigid links (SLM & SM) were fairly able to account for the highly pinched hysteretic behavior as well as the bar-slip envelope response caused by anchorage failure of the bottom beam reinforcement.



(c) Details of beam-column joints

Figure (6): Details of prototype frame and sub-assemblages

This trend is also observed in the work of Celik and Ellingwood (2008), Jeon et al. (2012) and Hassan (2011), who assessed the performance of their proposed joint shear strength models by utilizing zero length rotational spring elements. The centerline models (CLM & CLRBM) were unable to mimic the energy dissipation as well as the induced flexibility of the non-ductile joint imposed on the system.

Lastly, the ACI 369-R11 and ASCE/SEI 41-06 explicit joint model (ASCEM) was conservative in predicting the joint shear strength and post peak drift capacity and could not capture the stiffness and strength degradation accurately. The sharp decrease in strength after attaining the maximum shear capacity was also observed in the work of Hassan (2011), who assessed the force–drift simulated response of ASCE 41 supplement with semi-rigid joint model. This was also in agreement with the work of Park et al. (2012), who adopted the backbone curve of ASCE 41 together with a bilinear bond-slip model.

In summary, the widely accepted notion that the joints of non-seismically designed reinforced concrete frames must be modeled as rigid is inappropriate. Therefore, the need for explicit modeling of joint behaviour of such frames in providing reliable estimates of performance levels under seismic vulnerability assessment cannot be overemphasized.

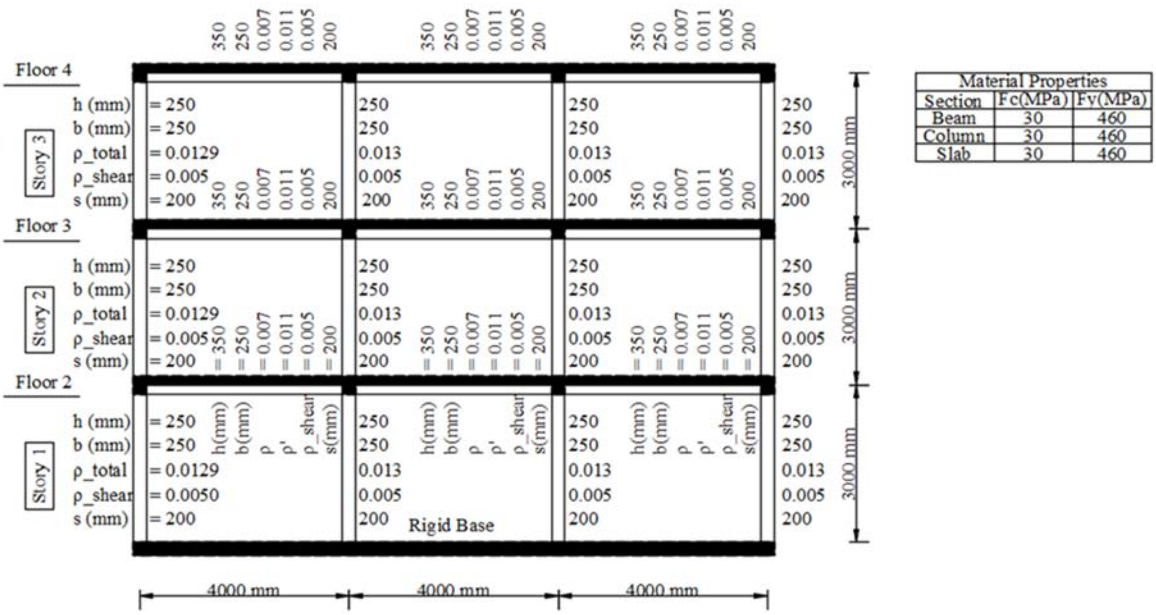


Figure (7): Geometry of model frame

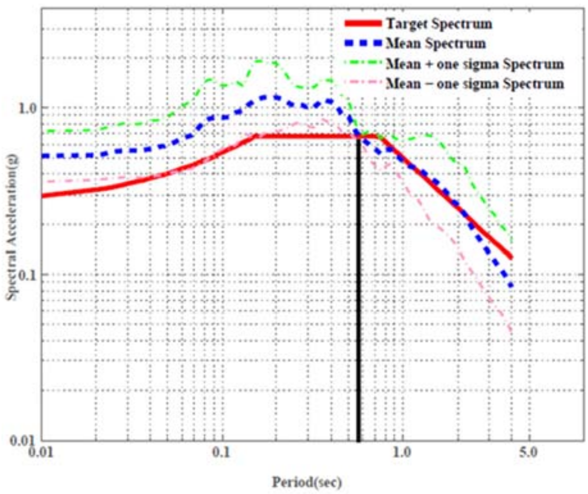


Figure (8): Response spectrum of the scaled ground motion record

Effect of Joint Model Type on Period of Building

At the building frame level, the natural modal periods of vibration for the various joint types were found to be sensitive to the implementation of a particular type. Table 1 shows the distribution of the first three modes of vibrations alongside their modal participation factors.

It is evident that the fundamental modal period is dominant in all cases, because it activates about 87% of the masses in non-linear analysis. For explicit joint models, there was an observed incremental shift in the modal periods of vibration. With the ASCE explicit model having the largest change in period, the behaviour of its force-drift response at the component test level imposed the highest joint flexibility. This affirms the fact that the inclusion of joint flexibility influences the intensity measures of the attracted seismic force from ground shaking. It is noteworthy that for implicit joint modeling achieved by adjusting the flexural properties of the adjoining framing members, there was no change in individual modal periods of vibration and the associated mass participations.

Table 1. Eigenvalue analysis results of various joint models

Joint Modeling Scheme	First Mode		Second Mode		Third Mode	
	T1	M1	T2	M2	T3	M3
CLM	0.5918	87.39	0.1911	10.22	0.1161	2.39
CLRBM	0.5918	87.39	0.1911	10.22	0.1161	2.39
SML	0.6012	86.90	0.1954	10.5	0.1203	2.59
SM	0.6036	86.89	0.1921	10.54	0.1146	2.57
ASCEM	0.6180	86.82	0.1968	10.58	0.1171	2.60

CLM - Centerline joint model; CLRBM - centerline model with reduced beam flexural capacity; SML - scissors model with rigid links; SM - scissors model without rigid links, ASCEM – ASCE/SEI 41-06 explicit joint model.

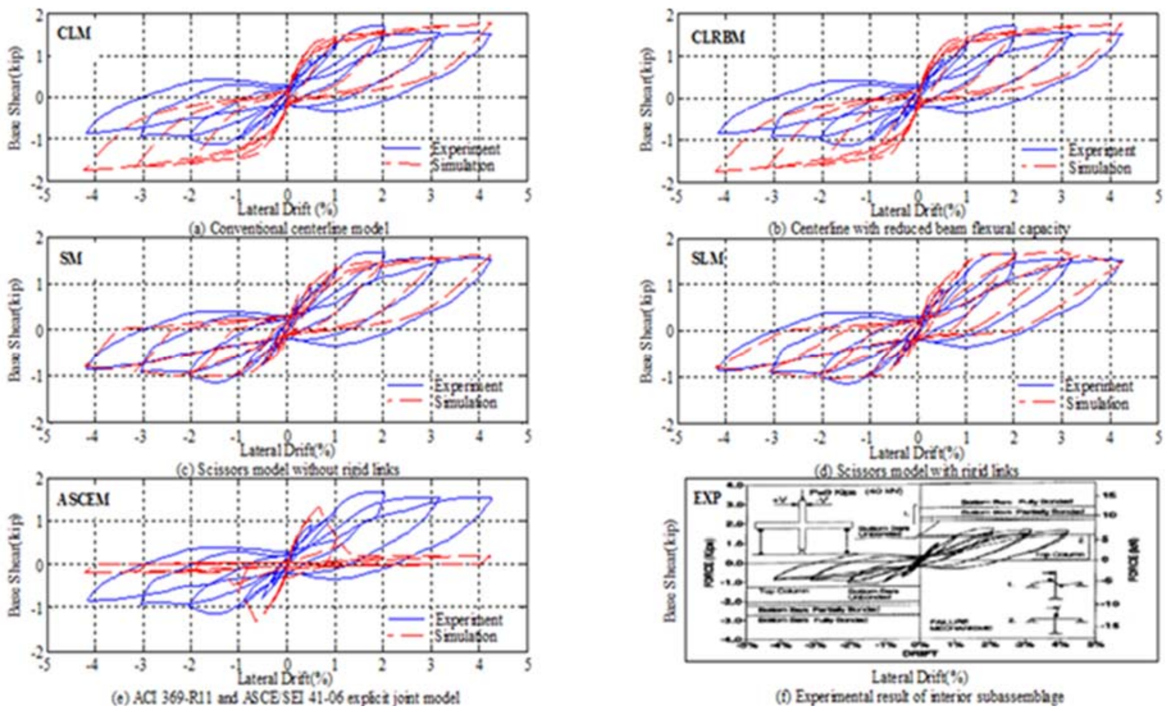


Figure (9): Comparison between hysteretic responses of experiment and analysis for interior joint sub-assemblage

Effect of Joint Type on Inter-Story Drift Ratios

The major engineering demand parameters that were used in assessing the seismic demand were the median responses of the peak inter-story drift ratio between floors and rotational demand at the joint. The model frame selected was a weak column-strong beam design, as such a soft story collapse mechanism is expected. From Fig. 10, the scissors model with rigid links produced the largest median value for the drift responses at the first floor level.

Comparing its first story drift responses with the conventional centerline model, there is an observed increase of about 36% and 7% in the maximum and average responses, respectively. The joint model of reducing the flexural properties of framing element to levels that reflect joint shear failure (CLRBM) also provided the smallest median response of the peak in-time drift ratios at the first floor. However, at the roof level, these two models, being at the extremes of estimated responses, showed an opposite trend; thus the scissors joint model tended to underestimate the roof drift response considerably. The maximum inter-story drift along the story levels, being the defining parameter used in relating the effect of ground shaking to the level of damage in seismic vulnerability assessment, the explicit scissors joint model gave the largest drift deformation. This was also observed in the work of Celik and Ellingwood (2008), who compared the variation of the rigid joint model (CLM) to their proposed joint model (SLM) in fragility assessment of gravity loaded design of reinforced concrete frames. A similar trend is seen in the observed plastic rotational deformations upon comparison between the two joint models. The ASCE/SEI 41-06 explicit joint model, centerline model with reduced beam moment capacities and the scissors model with rigid links experienced fairly the same peak drift and rotational angle distribution along the story levels. Since the scissors model with rigid links was able to simulate the experimental hysteresis response under cyclic loading for the interior slab-beam-column sub-assemblages with a better accuracy than the other models, its estimated engineering demand parameters (peak inter-story drift ratio and rotation angle) are deemed to be more realistic.

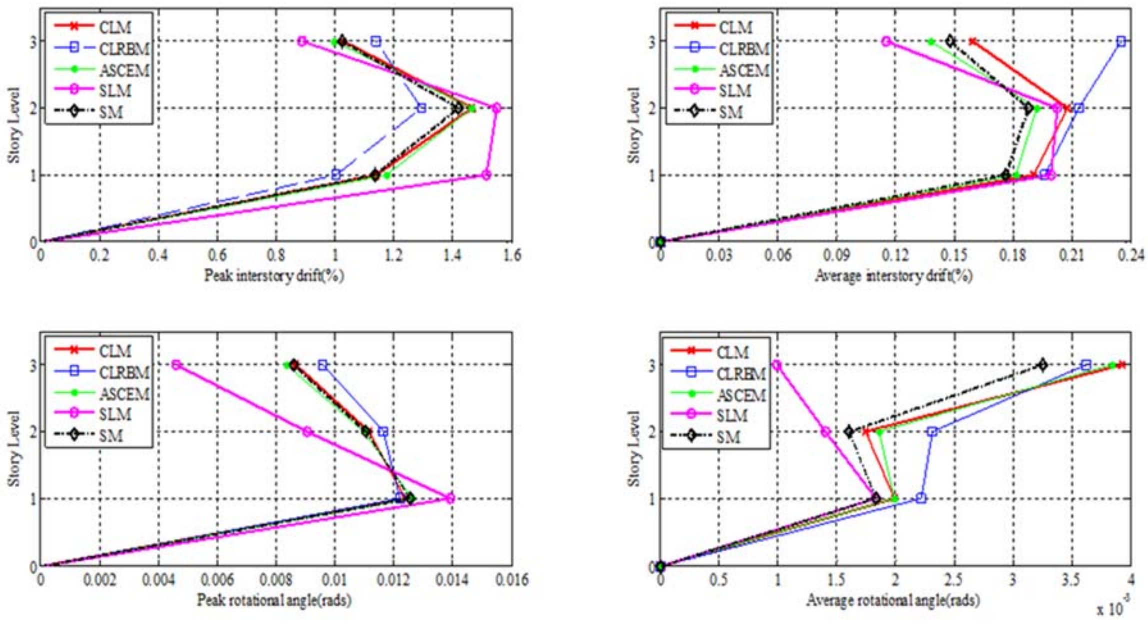


Figure (10): Drift ratio and rotational demand for the various joint models

CONCLUSIONS

The analytical modeling of the joint sub-assemblages showed that the centerline model could not accurately simulate the highly pinched responses of the experimental setup. The seismic performance of the scissor model with rigid links (SM) or without rigid links (SLM), in which the bond slip and shear behavior are modeled using rotational spring elements, was able to reduce the epistemic uncertainties, hence giving reliable estimates of the dynamic response quantities. However, by implicitly reducing the moment capacities of the framing elements to reflect joint capacity, the highly pinched hysteretic response of tested sub-assemblages could not be captured accurately.

The joint contribution to the median response of the peak inter-story drift ratio of non-seismically designed reinforced concrete frames can be significantly high and as such a critical appraisal has shown the importance of including explicit joint models in such frames. In a bid to simplify the assessment of the seismic performance of such frames for practical use, it is recommended that more research be conducted to generalize the dispersion of the joint flexibility to models that assume the joints to be rigidly connected.

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