Numerical Modeling of Longitudinal Wave Propagation for Damage Identification Using RBF-Based Meshless Method and Imperialist Competitive Algorithm

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ABSTRACT

This paper presents a novel approach to solve the wave propagation problem based on meshless method and its implementation for wave-based damage identification. Radial point interpolation method (RPIM) is a meshless technique which employs radial basis function (RBF) for shape function construction in the local support domain of the problem. RPIM satisfies the Kronecker delta property, which makes possible the enforcement of essential boundary conditions as easy as in finite element method. This study investigates the suitable parameters of RBF and dimension size of the support domain in longitudinal wave propagation. It is conducted by using minimum error between wave signals of benchmark and RPIM model of a rod. By considering the best parameters in RPIM modeling, the minimization of discrepancies between the benchmark and RPIM signals leads to identify the damage. The optimization problem was formulated by a new error function and Imperialist Competitive Algorithm (ICA) in such a way that the position, extent and length of damage are obtained. Various signal-to-noise ratios were concerned in the damage identification process to consider the measurement effect of practical manner.

KEYWORDS: Damage identification, Wave propagation, Meshless method, Radial basis function, Radial point interpolation, Optimization, Imperialist competitive algorithm.

INTRODUCTION

The changes in the physical properties of a structure (such as mass, stiffness and damping) which affect the performance of structures are identified as structural damage. In order to assess the damage, vibration-based methods have been extended based on the dynamic behavior of systems, especially using modal information. These methods received considerable attention during the past years (Doebling et al., 1996; Sohn et al., 2004). Since any cracks or other small defects lead to low changes in the global behavior of structure and modal data, the employment of methods based on modal data may be difficult to detect small damages. The recent trend for small damage detection is to use elastic wave-based techniques which can overcome the restrictions of traditional methods. This approach requires high-frequency excitation. The base of this approach is the well-known fact that traveling wave with high frequency content scatters from defect and discontinuity in solids.

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A variety of numerical techniques have been applied in order to model the wave propagation and scattering in time, frequency or time-frequency domains. The most frequent method is the conventional finite element method (FEM). Moser et al. (1999), Chakraborty et al. (2002) and Ong and Chen (2005) illustrated the capability of this method in guided wave propagation problems. Since wave propagation deals with loading of very high frequency content, a shorter wavelength of the structure and consequently a large number of degrees of freedom will be inevitable. Also, such conditions to obtain an accurate solution of the governing equation and to capture the effect of wave scattering at boundaries and discontinuities are a computationally prohibitive job. This limitation has motivated researchers to seek novel techniques. Ham and Bathe (2012) and Keramat and Ahmadi (2012) presented the modified FEM for wave propagation. Methods such as boundary element method (BEM) (Cho and Rose, 1996; Zhao and Rose, 2003), mass-spring lattice model (MSLM) (Delsanto and Scalerandi, 1998; Yim and Sohn, 2000) and local interaction simulation approach (LISA) (Delsanto et al., 1992) have been studied as alternative methods.

Recently, spectral FE method (SFEM) based on fast Fourier transform (Doyle, 1997; Horr and Safi, 2003; Palacz et al., 2005; Gopalakrishnan et al., 2008; Pahlavan et al., 2013), Laplace transform (Igawaa, 2013), wavelets (Gopalakrishnan and Mitra, 2006), orthogonal polynomials (Kudela et al., 2007; Peng et al., 2009) and wave FE (Zhou and Ichchou, 2010) have been reported in the literature as new algorithms for wave propagation to model and identify the damage. Conventional mesh-based methods are not well suited to treat problems with strong inhomogeneity, large deformations, mesh distortion and discontinuities that do not align with element edges. The existence of meshes may cause strong mesh dependency of the calculation for dynamic problems such as crack propagation and consequently the prognosis of remaining lifetime (Nguyen et al., 2008). In the past decades, researchers have been encouraged to explore the alternative methods to solve partial differential equations which do not have mesh-based difficulties. One of these methods is the meshless approach (Liu and Gu, 2005; Chen et al., 2006; Liu, 2010). In the field of wave propagation analysis, Wen (2010) utilized a meshless local Petrov–Galerkin (MLPG) method to solve wave propagation problem in three-dimensional poroelastic solids. Gao et al. (2007) proposed a new MLPG method to analyze stress-wave propagation in anisotropic and cracked media. Tadeu et al. (2007) investigated the method of fundamental solutions (MFS) as a meshless method in frequency domain. They evaluated the 3D scattered wave field generated by 2D empty cracks embedded in an elastic slab and a half space. Das and Kundu (2009) simulated ultrasonic wave field models for layered half-spaces with and without an internal crack using the meshless semi-analytical distributed point source method. The computed fields clearly demonstrated how the layered materials influence the amount of energy propagating through the layer and penetrating into the solid half-space. Zhang and Batra (2004) applied a modification to the smoothed particle hydrodynamics method for improvement of approximation accuracy to study the propagation of elastic waves in functionally graded materials. Li et al. (2010) formulated a meshless method based on the radial basis function collocation technique and generalized trapezoidal method for the numerical simulation of wave propagation problems.

Among various meshless techniques, point interpolation method (PIM) (Liu et al., 2001a; Liu et al., 2002) - as a series representation method for the function approximation - employs Galerkin's weak form and shape functions. In this method, the interpolation functions which are polynomial basis functions or radial basis functions (RBFs) possess the Kronecker delta function property and enforce to pass through nodes within the sub-domain. RBFs are functions whose values depend only on the distance from the point called center. In order to avoid the singularity problem in the polynomial PIM, RBFs are used to develop the radial point interpolation method (RPIM) shape function construction. The RPIM has been successfully applied to 1-, 2- and 3D solid mechanics (Liu et al., 2001b; Liu et al., 2005), plate structures (Liu and Chen, 2001), geometrically non-linear problems (Dai et al., 2006), material non-linearity problems (Wang et al., 2002) and additional mass detection (Ghaffarzadeh and Mansouri, 2013).

The construction of a mathematical model of the real structure is one of the main tasks of damage identification techniques in the model-based approach. Generally, damage is identified by comparison between mathematical and experimental models. The method relies on the computation of parameters in the mathematical model and gives the best correspondence with measured data which can be formulated as a basic optimization problem. Recently, computational intelligence methods have been applied for structural damage identification. Chou and Ghaboussi (2001) presented damage identification as an optimization problem and solved it by using a genetic algorithm. Static measurement of displacements at few degrees of freedom was used to identify the changes in the characteristic properties of structural members. Damage was indicated by the difference of measured and computed responses. Liu and Chen (1996) adopted a spectral FE method to formulate the equilibrium equation of the truss in the frequency domain field. Damage identification problem was formulated, in which the unbalanced nodal forces in the frequency domain field are minimized. Ruotolo and Surace (1997) used the modal parameters of the lower modes for non-destructive detection of cracks in beams. They used an FE model of structure to calculate the dynamic characteristics. Genetic algorithm method was utilized to optimize the inverse problem of damage identification. Sahoo and Maity (2007) proposed a hybrid neuro-genetic algorithm in order to automate the process of damage identification. They considered frequencies and strains as input parameters and the location and amount of damage as output parameters. Utilizing heuristic methods, such as ant colony optimization algorithm (Majumdar et al., 2012), incremental particle swarm optimization technique (Nanda et al., 2012) and combination of an adaptive real-parameter genetic algorithm with simulated annealing (Hea and Hwang, 2006) has been proposed to damage detection in structures. Bagheri et al. (2011) and Nicknam and Hosseini (2012) presented the method of damage assessment on the basis of modal parameters including natural frequencies and mode shapes using imperialist competitive algorithm which is a new global search heuristic method.

Several researchers have focused on the combination of elastic wave-based detection methods and optimization strategies. Tenenbaum et al. (2012) presented the formulation of damage identification based on wave propagation results and a hybrid optimization using Levenberg-Marquardt and particle swarm techniques. Inverse problem of damage identification was addressed in time domain field by minimizing the difference between the experimental echo and the one predicted by the Sequential Algebraic Algorithm. Xu et al. (2002) developed a method of damage detection for composite plates using Lamb waves and a projection genetic algorithm. This method formulated damage detection as an optimization problem by linking the measured and calculated surface displacement responses. These responses derived from Lamb waves in which the analytical model was the strip element method with the trial crack parameters. Wang and Tunsel (2007) proposed the application of S-transformation to identify the frequency–time domain characteristics and to reduce the noise of signals at the receiver. Two time domain models, time delay–attenuation and time delay–attenuation–phase, were utilized to represent the propagation of Lamb waves in a compact and meaningful model. The parameters of models were estimated by using genetic algorithm techniques.

As noted, elastic wave-based damage identification using the FE method requires the small sizing of discretization. FE models are discretized as well near damage locations if these positions are known. Efforts are required to get rid of convergence and mesh-related difficulties. In this study, the structural damage identification procedure is developed by longitudinal wave propagation analysis on the meshless model of a rod that dispersion was not observed. Radial point interpolation method is used to develop shape functions of the meshless model. By considering the proper shape parameters of adopted RBF and meshless parameters, damage identification can be established. For this purpose, the optimization process is applied on a new error function by means of Imperialistic Competitive Algorithm. The error function includes the discrepancies of meshless and benchmark scattered signals of wave propagation. The position, extent and length of damage are achieved as the optimization results.

Problem Formulation

Radial Point Interpolation Method Formulation

Using polynomials to create shape functions is one of the earliest interpolation schemes in various numerical analyses such as finite element method (FEM). In FEM, the interpolation is based on elements that have no gap and overlapping. However, in meshless techniques such as polynomial point interpolation method (PIM), the interpolation is defined on a set of nodes in the vicinity of a desired point called support domain. The overlapping of support domains is possible for different points (Figure 1). In order to avoid the singularity problem in the polynomial (PIM), the radial basis function (RBF) is used to develop the radial point interpolation method (RPIM) shape functions for weak-form methods.

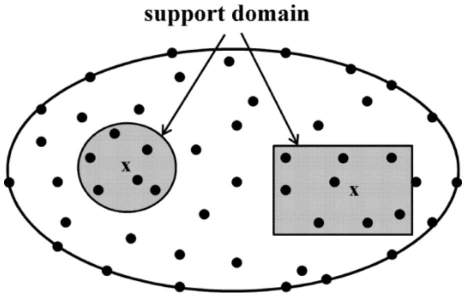

Figure (1): Representation of support domains which are used for interpolation

For the formulation of RPIM as a meshless method, a continuous function u(X) (i.e., displacement) can be approximated using radial basis function R_i (X) in the support domain that has a set of arbitrarily distributed nodes as:

$$u(\mathbf{X}) = \sum_{i=1}^{n} R_i(\mathbf{X})\, a_i = \mathbf{R}^{\mathrm{T}}\mathbf{a} \tag{1}$$

where n is the number of nodes in the support domain of $\mathbf{X} = [x, y, z]^{\mathrm{T}}$; a_i is the corresponding coefficient of the basis functions, which can be determined by enforcing $u(\mathbf{X})$ to be the nodal displacement at n nodes in the support domain:

$$\mathbf{d}_S = \mathbf{R}_Q\mathbf{a} \tag{2}$$

where $\mathbf{d}_s$ is the vector of nodal displacements and $\mathbf{a}$ is the vector of unknown coefficients as follows:

$$\mathbf{d}_S = \{u_1 \quad u_2 \quad \cdots \quad u_n\}^{\mathrm{T}} \tag{3}$$

$$\mathbf{a} = \{a_1 \quad a_2 \quad \cdots \quad a_n\}^{\mathrm{T}} \tag{4}$$

and $\mathbf{R}_Q$ is the moment matrix of RBF given by:

$$\mathbf{R}_Q = \begin{bmatrix} R_1(r_1) & R_2(r_1) & \cdots & R_n(r_1) \\ R_1(r_2) & R_2(r_2) & \cdots & R_n(r_2) \\ \vdots & \vdots & \ddots & \vdots \\ R_1(r_n) & R_2(r_n) & \cdots & R_n(r_n) \end{bmatrix} \tag{5}$$

In the radial basis function, the variable is only the distance between the point of interest *(x, y, z)* and a node at (x_k, y_k, z_k):

$$\mathbf{r} = \left[(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2\right]^{0.5} \tag{6}$$

Regarding the distance, it is directionless as:

$$R_i(r_j) = R_j(r_i) \tag{7}$$

Therefore, the moment matrix of $\mathbf{R}_Q$ is always a symmetric positive definite and invertible matrix. The existence of $\mathbf{R}_Q^{-1}$ is the major advantage of using RBF over the polynomial basis. The vector of unknown coefficients **a** may be obtained as a unique solution as:

$$\mathbf{a} = \mathbf{R}_Q^{-1}\mathbf{d}_s \tag{8}$$

Substituting Eq. (8) into Eq. (1) yields:

$$u(\mathbf{X}) = \mathbf{R}^T(\mathbf{X})\mathbf{R}_Q^{-1}\mathbf{d}_s = \Phi(\mathbf{X})\mathbf{d}_s \tag{9}$$

$$\Phi^T(\mathbf{X}) = \{\varphi_1(\mathbf{X}) \quad \varphi_2(\mathbf{X}) \quad \cdots \quad \varphi_n(\mathbf{X})\} \tag{10}$$

where $\Phi(\mathbf{X})$ is the vector of RPIM shape functions. Constructed RPIM shape functions are, in general, incompatible and inconsistent and have Kronecker delta function property which allows essential boundary conditions to be easily treated in the same way as in the standard FEM.

Among various RBFs, Gaussian (EXP) RBF is probably the best RBF to use and time can be saved in computing the best approximated value in the numerical analysis (Fasshauer, 2007). Gaussian form has been used as:

$$R_i(\mathbf{X}) = \exp\left[-(\kappa r)^2\right] = \exp\left[-\alpha_c\left(\frac{r_i}{d_c}\right)^2\right] \tag{11}$$

where κ is the shape parameter in original mathematical form; α_c is the shape parameter in dimensionless manner the value of which depends on the problem and d_c is the characteristic length that is usually the average nodal spacing for all *n* nodes in the support domain.

Wave Propagation Formulation

The equations of motion in an elasto-dynamic problem are given in matrix form as:

$$\mathbf{L}^T\sigma + \mathbf{b} = \rho\ddot{\mathbf{u}} + \eta_c\dot{\mathbf{u}}, \quad \text{in} \quad \Omega \tag{12}$$

$$\sigma = \mathbf{D}\varepsilon, \quad \varepsilon = \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right] \tag{13}$$

where **L** is differential operator, σ and ε are stress and strain, **b** is the body force; ρ is the mass density and η_c is the damping parameter. $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$ and **u** are acceleration, velocity and displacement, respectively, while $\nabla\mathbf{u}$ is the displacement gradient matrix. The natural and essential boundary conditions are:

$$\sigma\cdot\mathbf{n} = \bar{\mathbf{t}}, \quad \text{on} \quad \Gamma_t, \quad \mathbf{u} = \bar{\mathbf{u}} \quad \text{on} \quad \Gamma_u \tag{14}$$

where $\bar{\mathbf{u}}$ is the prescribed displacement on the essential boundary Γ_u and $\bar{\mathbf{t}}$ represents tractions on the natural boundary Γ_t. $\Gamma = \Gamma_t \cup \Gamma_u$ presents the complementary parts of the whole boundary in which **n** is the unit outward normal to this boundary. The Galerkin weak form of dynamic problem in Eqs. (12)-(14) over the global problem domain is expressed as:

$$\int_\Omega \delta\varepsilon^T\sigma\, d\Omega - \int_\Omega \delta\mathbf{u}^T(\mathbf{b} - \rho\ddot{\mathbf{u}} - \eta_c\dot{\mathbf{u}})\, d\Omega - \int_{\Gamma_t} \delta\mathbf{u}^T\bar{\mathbf{t}}\, d\Gamma = 0 \tag{15}$$

It has been found that the unconstrained Galerkin weak form works well with RPIM shape functions (Liu et al., 2005) which are used in this study. By using Eq. (13) and Eq. (9), the discretization of Eq. (15) yields:

$$\mathbf{M}\ddot{\mathbf{U}}(\mathbf{t}) + \mathbf{C}\dot{\mathbf{U}}(\mathbf{t}) + \mathbf{K}\mathbf{U}(\mathbf{t}) = \mathbf{F}(\mathbf{t}) \tag{16}$$

where **M**, **C, K** and **F** are the global matrices of mass, damping, stiffness and time dependent excitation signal, respectively. The components of matrices are derived in each support domain as:

$$\mathbf{m}_{ij} = \int_\Omega \Phi_i^T \rho \Phi_j \, d\Omega \tag{17}$$

$$\mathbf{c}_{ij} = \int_\Omega \Phi_i^T \eta_c \Phi_j \, d\Omega \tag{18}$$

$$\mathbf{k}_{ij} = \int_\Omega \mathbf{B}_i^T \mathbf{D} \mathbf{B}_j \, d\Omega \tag{19}$$

$$\mathbf{f}_i = \int_\Omega \Phi_i^T \mathbf{b}\, d\Omega + \int_{\Gamma_t} \Phi_i^{\mathbf{T}} \bar{\mathbf{t}}\, d\Gamma \tag{20}$$

In Eq. (19), **B** is the strain-displacement matrix defined as:

$$\mathbf{B}_i = \begin{bmatrix} \phi_{i,x} & 0 & 0 \\ 0 & \phi_{i,y} & 0 \\ 0 & 0 & \phi_{i,z} \\ \phi_{i,y} & \phi_{i,x} & 0 \\ 0 & \phi_{i,z} & \phi_{i,y} \\ \phi_{i,z} & 0 & \phi_{i,x} \end{bmatrix} \tag{21}$$

For the numerical integrations of Eqs. (17)-(20), a background mesh is necessary which is independent of nodes for interpolations that are not overlapping. Gauss-quadrature is used for numerical integration in this paper.

Time Integration Scheme

The central difference time integration scheme is used to solve Eq. (16) as an explicit method:

$$\left(\frac{1}{\Delta t^2}\mathbf{M}+\frac{1}{2\Delta t}\mathbf{C}\right)\mathbf{U}_{t+\Delta t}=\mathbf{F}_t-\left(\mathbf{K}-\frac{2}{\Delta t^2}\mathbf{M}\right)\mathbf{U}_t-\left(\frac{1}{\Delta t^2}\mathbf{M}-\frac{1}{2\Delta t}\mathbf{C}\right)\mathbf{U}_{t-\Delta t} \quad (22)$$

where t and Δt denote time and time step of integration, respectively. This scheme is stable if $\Delta t \leq \Delta t_{cr} = 2/\varpi_{max}$, where ϖ_{max} is the largest frequency of the system (Chen et al., 2006). Zero initial conditions, $\mathbf{U}= 0$ and $\dot{\mathbf{U}} = 0$ at t = 0 are assumed to be implemented as initial displacement and velocity.

General Description of ICA

Imperialistic competitive algorithm (ICA) is a novel global search heuristic method that uses imperialism and imperialistic competition process as a source of inspiration. Like other evolutionary methods, it does not need the gradient of the function during the optimization process.

ICA starts with an initial population which is called countries. These countries are divided into two groups: imperialists and colonies. Through evaluating the cost function of each country, some of the best of them (countries with the least cost) are selected to form the initial empires by controlling the other countries (colonies) based on their powers. The more powerful imperialists have the more colonies.

After creating the initial empires, the competition starts and the colonies begin moving toward their relevant imperialist state. This movement is a simple model of assimilation policy which was pursued by some of the imperialist states. While moving toward the imperialist, a colony might reach a position with higher cost than that of the imperialist state. In this case, the imperialist and the colony change their positions. During competition, the weak empires collapse and the powerful ones improve their colonies.

After a while, all the empires except the most powerful one will collapse and all the colonies will be under the control of this unique empire. This means that the algorithm converges to the best solution. In this real world, there is no difference; not only among colonies, but also between colonies and imperialists. More details about this algorithm can be referred to in (Gargari and Lucas, 2007; Kaveh and Talatahari, 2010).

Damage Identification Based on ICA

The fundamental of model-based damage identification is to establish an analytical model and to analyze changes in this model that are relevant to experimental data. This task can be formulated as an inverse problem by minimization of an error function. In accordance with this optimization approach, ICA can be employed as a tool to develop wave-based damage identification. In this regard, the analytical wave signals that are functions of damage parameters (location, length and extent) link to corresponding experimental ones and optimization process leading to damage identification.

Covariance matrix is one of the suitable quantities for comparison of two signals and the determinant of this measurement can be used as the base of error function in optimization technique. This determinant is a scalar measure and it can be considered as a residual term. If there is a high similarity between two signals, this quantity tends to zero; otherwise this measure will be raised by increment of discrepancy. Error function which calibrates the difference between the theoretically calculated and experimentally measured wave signals - relative to three damage parameters - is introduced as:

$$E\left(D_P, D_E, D_L\right)=\sum_{i=1}^{N_p}\det\left[cov\left(f_i\left(D_P, D_E, D_L\right), f_i^{\,m}\right)\right] \quad (23)$$

where cov is the covariance matrix, D_P, D_E and D_L are position, extent and length of damage, respectively. f_i and f_i^m are the analytical (using RPIM) and experimental signals at i^{th} point (sensor), respectively.

N_p is the number of sensors on the wave guide. ICA-based optimization of Eq. (23) will be executed by assuming the initial countries as preliminary damage parameters. By determination of initial imperialists, the competition will be performed until an empire remains. The results that correspond to this condition are the best values of damage parameters.

Numerical Experiment, Benchmark Model and Damage Scenario

In this study, benchmark structure is the FE model of a homogeneous rod which is analyzed using MATLAB. Its length is 500 mm which consists of 1000 FEs to perfectly monitor wave propagation. Damage status is introduced with 40% reduction in the modulus of elasticity, whereas its length equals 5 mm and it is located 200 mm from the left end. The rod has the cross-sectional dimensions: width = 20 mm, height = 20 mm and material properties are: Young's modulus = 70 GPa, mass density = 2700 kg/m^3. Damping matrix applies as $\mathbf{C} = \eta\mathbf{M}$ with η=0.02. Figure 2 represents damage position, length and points S_1 and S_2 that capture response signals. The excitation force impulse is a Hanning-windowed sinusoidal wave with the central frequency of 150 kHz applied at the right end (Figure 3).

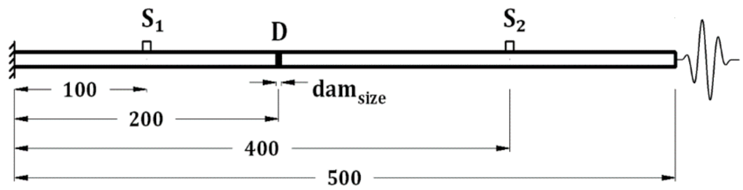

Figure (2): The configuration of the rod as a benchmark model

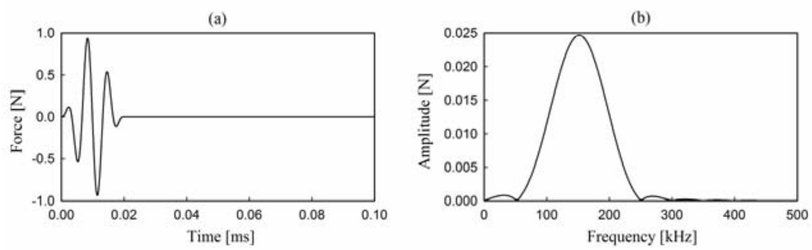

Figure (3): Form of an excitation signal in: (a) time domain, (b) frequency domain

In order to simulate the practical conditions, Gaussian noise was superimposed on normalized FE outputs with four levels of signal-to-noise ratio (SNR): SNR=30, 20, 10 dB and noiseless level. Figures 4 and 5 represent the normalized amplitude of displacement signals in healthy and damaged state, respectively, obtained from wave propagation analysis.

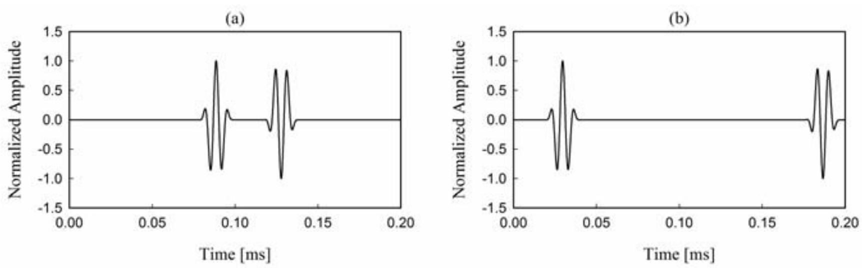

Figure (4): Normalized displacement response of healthy benchmark model captured in (a) S_1, (b) S_2

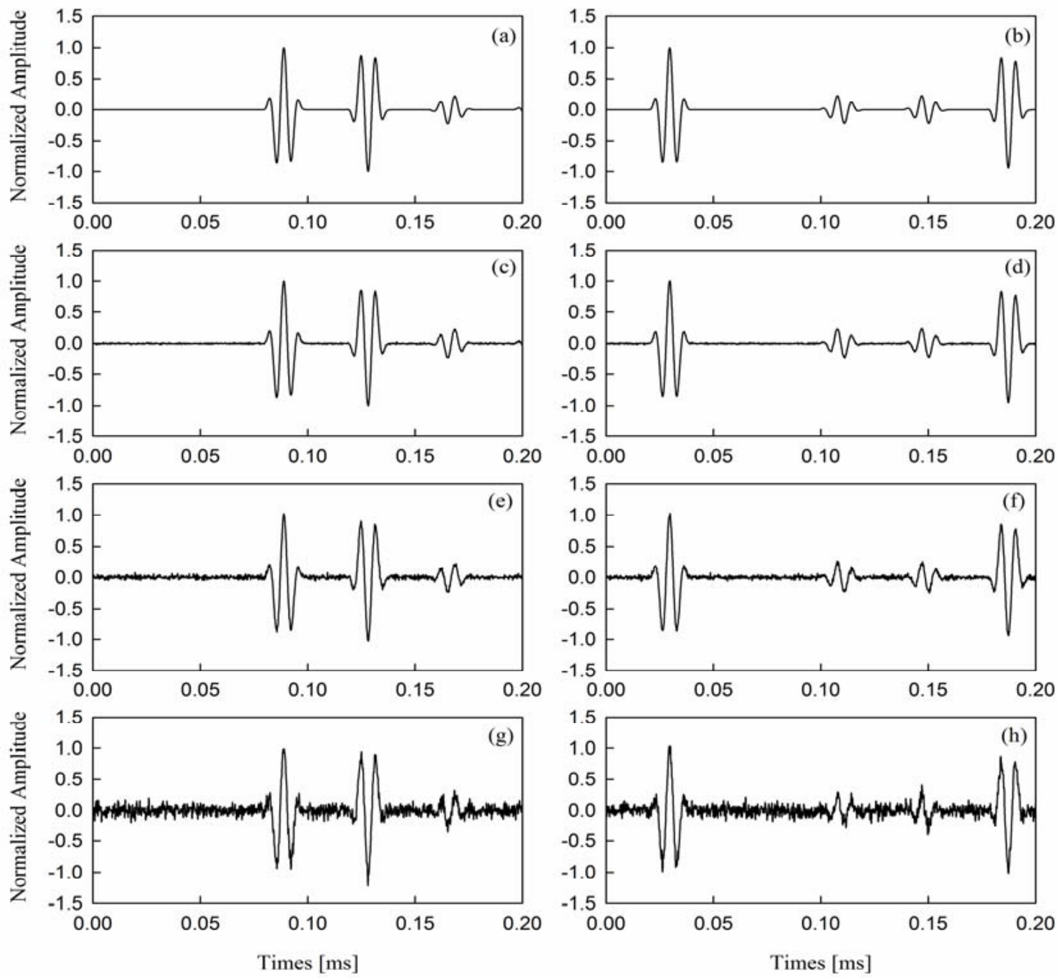

Figure (5): Normalized displacement response of damaged benchmark model captured in: (a) S_1, noiseless (b) S_2, noiseless (c) S_1, SNR=30 (d) S_2, SNR=30 (e) S_1, SNR=20 (f) S_2, SNR=20 (g) S_1, SNR=10 (h) S_2, SNR=10

Numerical Analysis

Optimal Parameters of RPIM for Wave Propagation

In this section, wave propagation analysis has been carried out for the selection of optimal parameters regardless of damage existence. The parameters of RPIM which affect the analysis include shape parameter of Gaussian RBF (α_C), number of nodes (N_{nodes}) implicitly considered as characteristic length (d_C) and the size of support domain (α_S). Smaller shape parameter (α_C) causes the RBF to become flatter, whereas its increment leads to more peaked RBF with local effects. Moreover, number of nodes and distance between them are significant due to two reasons:

Firstly, with respect to the discussion of the interpolation given in Eq. (1), the interpolation accuracy is affected by the number of field nodes. Secondly, with regard to the discussion of wave propagation, since the wavelength of applied load is very short, exact input wave energy propagation along the structure requires the reduction in the distance between nodes and consequently the increment of their number.

It is significant to mention that high accuracy of results is not achieved just by increasing the number of nodes, but extension of the size of support domain (α_S)

can increase the number of nodes involved in the interpolation related to the specific support domain. The number of nodes can remain constant, but the size of support domain should give rise to the participation of more nodes in the interpolation process (see the increasing of dark areas in Figure 1).

The benchmark model is simulated based on RPIM using the regularly distributed field nodes in MATLAB platform. The minimum number of Gauss points identical to the number of unconstrained field nodes is acceptable to avoid the singularity of system matrices. Therefore, background cells are defined between two field nodes and the integrating points are placed in the center of cells. The study on the number of integrating points is not the aim of this research.

Root Mean Square Error (RMSE) is explored to measure the difference between signals predicted by RPIM and the signal which is obtained from the benchmark model as:

$$\mathrm{RMSE}(\%)=\sqrt{\sum_{t=1}^{N}\left(f_t-f_t^{m}\right)^2 / N}\cdot 100 \quad (24)$$

where f_t and f_t^m are RPIM and benchmark signals, respectively and N is the number of sampling times.

Figure 6 represents the variation of RMSE with regard to α_C and α_S for outputs of S_1 and S_2 in a different number of nodes. It is concluded that RMSE is much more sensitive to α_S over the interval (0,1) for all node numbers. It is noticeable that if α_C increases further than this interval, more discrepancy occurs, tending to high error slowly independent from α_C and α_S.

Figure 7 shows the least average RMSE between S_1 and S_2 signals. Based on this result, among the various sets of (N_{nodes}, α_C, α_S), the triple values of (500, 0.09, 3) give the minimum error equal to 0.75%. It can be observed that the other sets yield an admissible error which can be used for static or other transient problems, but owing to the purpose of damage detection, the least response is adopted. Moreover, the highly dense node distribution is required because of the short wavelength of input signals. Application of these values for wave propagation in both healthy and damaged states is represented in Figure 8 for a range of times. Wave scattering from damage is recognizable in Figure 8(b). This phenomenon is initialized from a position equal to 300 mm from the excitation point.

Damage Identification and Discussion

Damage identification based on error function in Eq. (23) has been carried out in accordance with the flowchart of Figure 9. ICA starts by generating an initial country which is an array of variables whose values are to be optimized. According to the characteristics of benchmark model, country and constraints of its variables are expressed as follows:

$$\text{country}=[\,D_P, D_E, D_L] \quad (25)$$

$$D_P \in (0,500)\,\mathrm{mm},\quad D_E \in (0,100)\%,\quad D_L \in (0,10)\,\mathrm{mm} \quad (26)$$

The number of initial population is considered equal to 100 and the half of population is selected to form the empires. These values have been tuned through a trial and error procedure, since there is no suitable mathematical tool to achieve the parameters of the initial population and initial empires. Figure 10 represents the optimization process. The number of iterations considered is 50, 100, 150 and 200. Each iteration is performed independently to regard the effect of randomness nature of ICA initialization. It is noticeable that the increment of noise content in benchmark signals leads to approximately high error. However, it does not result in poor prediction of damage diagnosis. Moreover, it is obvious that the increment of iteration leads to apparent stalling state which can be accepted for the completion of the process.

The results of damage identification are listed in Table 1 and the identification percentage errors are illustrated in Figure 11. It can be seen that the error generally tends to be a small error by the increment of iteration; even high noise content (10 dB) leads to an acceptable solution.

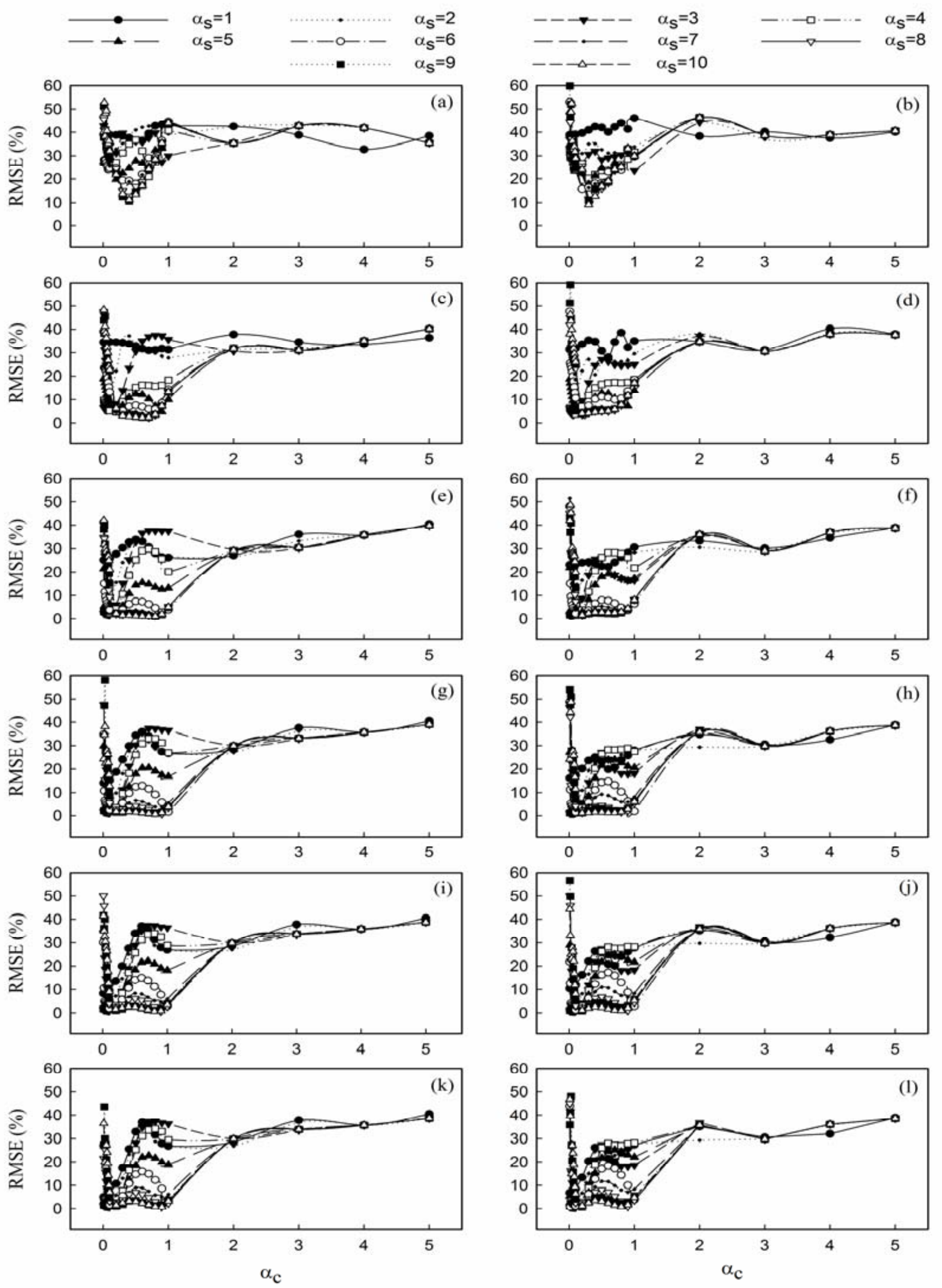

Figure (6): Variation of α_c and α_s relative to the number of nodes. (a) N_{nodes} = 50, S_1 (b) N_{nodes} = 50, S_2 (c) N_{nodes} = 100, S_1 (d) N_{nodes} = 100, S_2 (e) N_{nodes} = 200, S_1 (f) N_{nodes} = 200, S_2 (g) N_{nodes} = 300, S_1 (h) N_{nodes} = 300, S_2 (i) N_{nodes} = 400, S_1 (j) N_{nodes} = 400, S_2 (k) N_{nodes} = 500, S_1 (l) N_{nodes} = 500, S_2

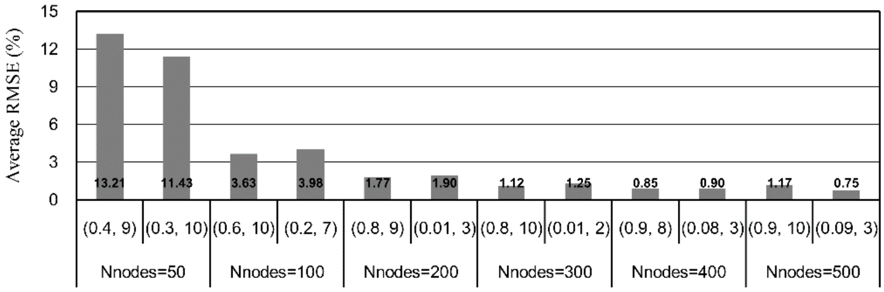

Figure (7): Variation of average RMSE between S_1 and S_2 relative to N_{nodes} and (α_c, α_s)

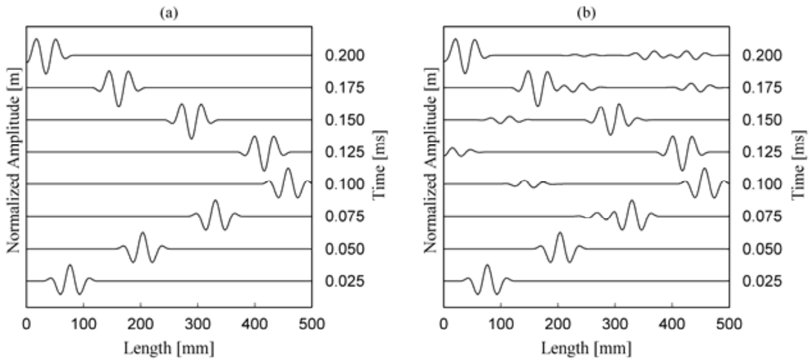

Figure (8): Wave propagation in the length of rod using RPIM (a) healthy state (b) damaged state

Results show that the evaluation of damage position has a negligible error percentage compared to the other two parameters. Even low iteration has a satisfactory precision regardless of the noise content. Substantial errors occur in low iteration for the prediction of extent and length of damage. For iterations higher than 100, the sensitivity moves toward reduction.

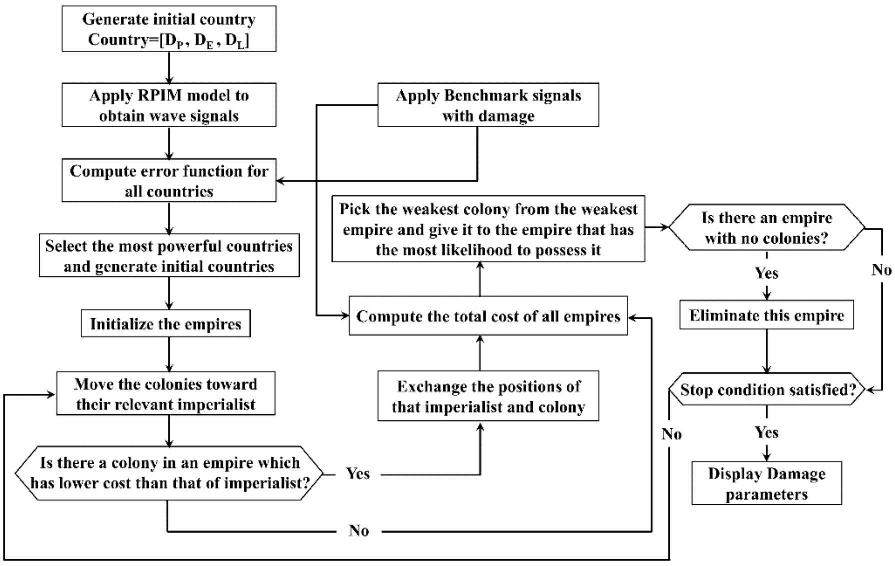

Figure (9): Flowchart of damage identification based on ICA

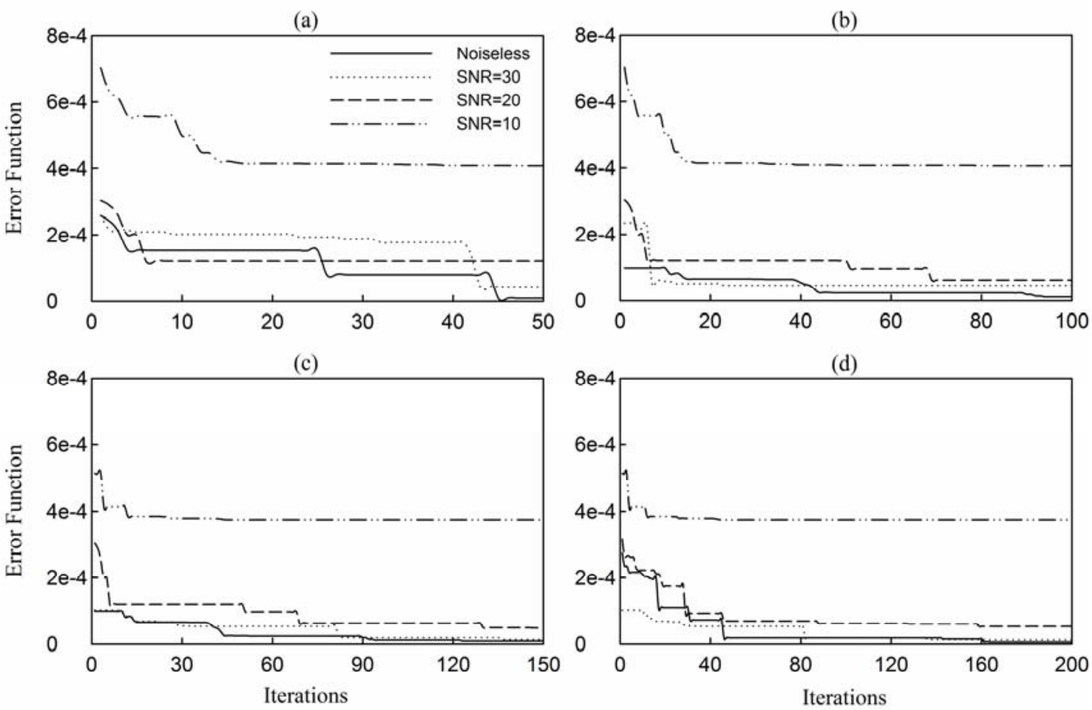

Figure (10): Results of optimization using ICA for number of iterations (a) 50 (b) 100 (c) 150 (d) 200

Table 1. Results of identification and percentage error [a]

Noise Content	# Iterations	Position Diagnosis (D_P)		Extent Diagnosis (D_E)		Length Diagnosis (D_L)	
		Calculated Value (mm)	Percent Error (%)	Calculated Value (%)	Percent Error (%)	Calculated Value (mm)	Percent Error (%)
Noiseless	50	200.93	0.47	34.04	14.90	6.53	30.60
	100	200.91	0.45	32.76	18.10	6.72	34.40
	150	201.17	0.58	34.14	14.65	6.72	34.40
	200	199.93	0.03	41.40	3.50	4.47	10.60
SNR=30	50	200.87	0.44	38.62	3.45	7.07	41.40
	100	201.27	0.64	27.94	30.15	6.98	39.60
	150	200.26	0.13	36.85	7.88	6.18	23.60
	200	200.25	0.13	37.69	5.78	6.04	20.80
SNR=20	50	202.94	1.47	22.87	42.83	9.03	80.60
	100	200.38	0.19	33.77	15.58	5.58	11.60
	150	200.40	0.20	37.86	5.35	5.62	12.40
	200	201.13	0.56	32.68	18.30	6.48	29.60
SNR=10	50	199.74	0.13	44.17	10.43	5.50	10.00
	100	199.73	0.14	43.84	9.60	5.31	6.20
	150	200.59	0.30	41.37	3.42	5.30	6.00
	200	200.59	0.30	41.37	3.42	5.30	6.00

[a] Actual values: D_P=200 mm, D_E=40%, D_L= 5 mm.

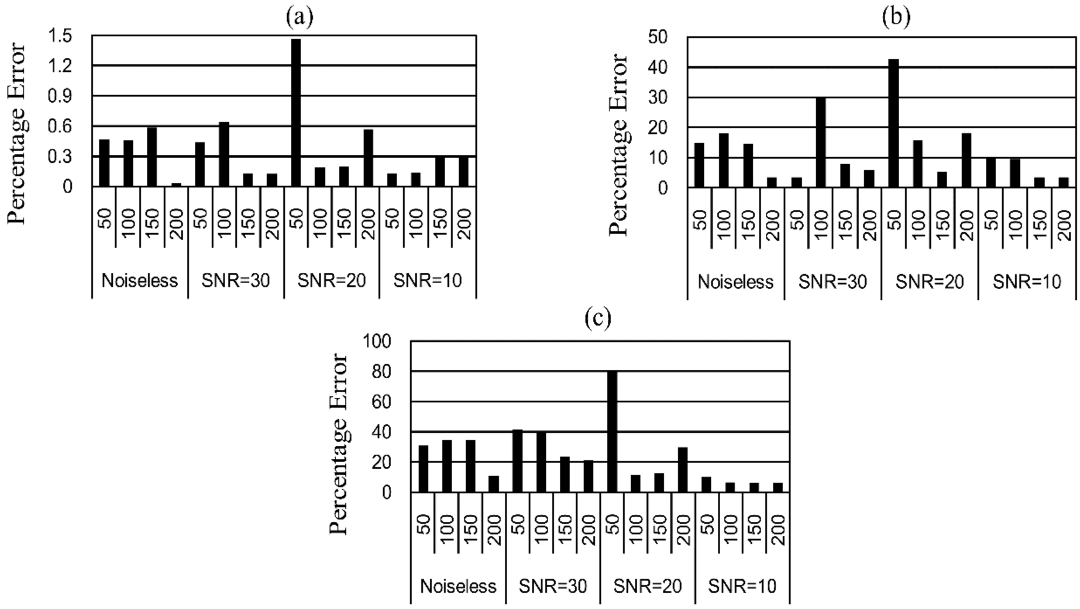

Figure (11): Results of relative errors (a) position (b) extent (c) length

CONCLUSIONS

Wave propagation phenomenon in damaged media can be a helpful mechanism to identify damage, particularly for small sizes. The implementation of this approach is applicable for both practical and numerical models and the comparison between the obtained signals gives damage parameters. This paper focuses on the numerical modelling of wave propagation for damage identification using optimization technique in 1-D wave guide. In the first section, wave propagation has been simulated using radial point interpolation method (RPIM) as a new meshless approach which uses RBF for shape function construction. According to the FE model of a rod, the validation of numerical RPIM model is executed and the appropriate shape parameters of RBF and support domain are determined. This work shows that the accuracy of simulation is acheived while shape parameter α_C is in the range of (0,1). In the last part, new damage identification techniques are proposed based on RPIM model and imperialist competitive algorithm (ICA). The optimization method is defined by minimizing the discrepancy of benchmark signals and damage-dependant RPIM signals using covariance matrix. Determinant of this matrix can be used as a residual of two signals which should be minimized. The results of optimization have led to approximatly exact diagnosis of damage position for high noise effect and low iteration number. However, acceptable values are achived in high iteration for extent and length of damage. In general, the results show that the noise content has a slight influence on the identification method.

The important matter which should be considered is the applicability of the proposed strategy for one-dimensional models such as a rod or beam. It becomes less effective for complex cases. The description of an appropriate error function in the optimization technique is difficult for wave propagation analysis in high dimensional structures.

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