Comparative Deterministic and Probabilistic Analysis of Two Unsaturated Soil Slope Models after Rainfall Infiltration

Manoj Kr. Sahis [1] ***and Partha Pratim Biswas*** [2]

[1] Jadavpur University, India. E-Mail: manojsahis@gmail.com
[2] Professor, Jadavpur University, India. E-Mail: ppb@const.jusl.ac.in

ABSTRACT

The prime aim of this paper is to develop a computational scheme for the reliability analysis of two unsaturated natural slope models after rainfall infiltration, considering the uncertain system parameters as random variables. Analysis has been conducted within the framework of the well known 'infinite slope' limit equilibrium model, suitably modified to take into account the shear strength for a partially saturated soil. The pore pressures in the shear strength equation may be positive or negative, depending on the position of the potential failure plane in relation to the water table or phreatic surface, and, more importantly, in relation to the wetting front at any given time. Analysis of the time dependent process of infiltration and the progression of the wetting front is based on the widely known Green-Ampt model.

The developed model is based on an approximate, yet simple method; namely Mean Value First Order Second Moment (MVFOSM) method of reliability analysis, treating the basic geotechnical parameters as random variables, where the random variables are assumed to be normally distributed and uncorrelated. The analysis clearly brings out the importance of carrying out a reliability analysis under the probabilistic framework.

KEYWORDS: Reliability analysis, Unsaturated slope models, Rainfall infiltration, Green-Ampt model.

INTRODUCTION

As we are well informed about the instant of rainfall, induced slope failure is nowadays becoming a vulnerable event. Innumerable research has been conducted on this topic, but it is still not properly understood and has remained unpredictable for many years. Several instants are there, where steep slopes remain stable for many years, while some gentle slopes fail during or immediately after rainfall. The basic mechanism of rainfall induced slope failure due to subsequent development of positive pore water pressure though initially negative pore water pressure (matric suction) plays a crucial role (Fredlund and Rahardjo, 1993). This positive pore water pressure reduces the shear strength, causing soil slopes to become unstable and then fail.

To find the infiltration depth, generally one-dimensional infiltration models, such as Green-Ampt model (Chow et al., 1988), are used. Green-Ampt model is the first physically based infiltration model to give wetting front depth in interaction with infiltration capacity of a particular soil slope.

It is well known that slope stability is a geotechnical problem which is surrounded by uncertainty. So, it is customary to consider all the parameters associated with slope stability as random

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variables, which necessitates requirements of a probabilistic analysis.

The fact that failure has been reported for computed safety factors larger than 1 indicates that no matter what number is found by analytical means, there is always some chance of failure. This gives rise to development of probabilistic approach. In probabilistic approach, the aim is to determine the probability of failure corresponding to the reliability index, taking different uncertainties into consideration.

FORMULATION

Infiltration Model

Green-Ampt (1911) model proposed the simplified process of infiltration theory. Water is ponded to a small depth on the soil surface; so, all the water which the soil can infiltrate is available at the surface. However, during a rainfall, water will pond on the surface only if the rainfall intensity is greater than the infiltration capacity of the soil. The incipient ponding time T_p is the elapsed time between the time rainfall begins and the time water begins to pond on the soil surface (Chow et al., 1988). If rainfall begins on dry soil, the vertical moisture profile in the soil may appear. Prior to ponding time ($T < T_p$), the rainfall intensity is less than the potential infiltration rate and the soil surface is unsaturated. Ponding begins when the rainfall intensity exceeds the potential infiltration rate. At this time ($T = T_p$), the soil surface is saturated. As rainfall continues ($T > T_p$), the saturated zone extends deeper into the soil and overflow occurs from the ponded water. The analysis of the time dependent process of infiltration and the progression of wetting front can be calculated based on the following four assumptions.

(1) The soil surface is maintained constantly wet by water ponding; (2) A sharp wetted front exists; (3) The hydraulic conductivity is constant throughout the soil; and (4) The soil matric suction at the wetting front remains constant.

The following Eqs. (1) through (3) are used to calculate the depth of the wetting front at incipient ponding (z_p), the cumulative infiltration at incipient ponding (F_p) and the incipient ponding time (T_p). These equations correspond to the conditions that the rainfall intensity i is greater than the minimum infiltration capacity of the soil and that the rainfall duration is greater than T_p (Cho and Lee, 2002; Xie et al., 2004; Muntohar and Liao, 2008).

$$z_P = \frac{K_S \psi_f}{i - K_s} \tag{1}$$

$$F_p = z_p \nabla\theta_i = \frac{K_s \nabla\theta_i \psi_f}{(i - K_s)} \tag{2}$$

$$T_p = \frac{F_p}{i} = \frac{K_s \nabla\theta_i \psi_f}{i(i - K_s)} \tag{3}$$

where, $\nabla\theta_i$ = the initial moisture deficit (m^3/m^3), ψ_f = the suction head at the wetting front (m) and K_s = the coefficient of saturated hydraulic conductivity.

The infiltration rate (f) and the cumulative infiltration (FF) and its corresponding depth of wetting front (z_w) at any time T can be calculated using the following equations.

$$FF = F_p + K_s(T - T_p) + \psi_f \nabla\theta_i \ln\left(\frac{FF + \psi_f \nabla\theta_i}{F_p + \psi_f \nabla\theta_i}\right) \tag{4}$$

$$f = K_s\left(1 + \frac{\psi_f \nabla\theta_i}{FF}\right) \tag{5}$$

$$z_w = \frac{FF}{\nabla\theta_i} \tag{6}$$

Deterministic Approach - Factor of Safety

Once the depth of wetting front (which is also the depth of assumed potential slip surface) with time has been determined, the modified 'infinite slope' model enables the calculation of the factor of safety (FS) at each depth.

Model-I: Only ground surface data is obtained. The model description is shown in Fig. 1. In this case, the possible slip surface is the wetting front.

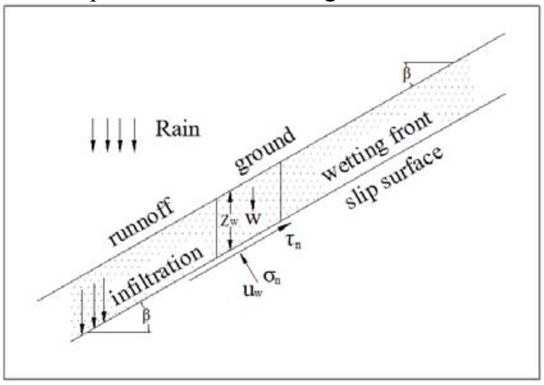

Figure (1): The 'infinite slope' model used in the study for Model-I

The safety factor is calculated using Eq. (7) (Crosta, 1998; Cho and Lee, 2002, Collins and Znidarcic, 2004).

$$FS = \frac{c'_w + \left\{\gamma_{sat} z_w \cos^2\beta - u_w\right\}\tan\phi_w'}{\gamma_{sat} z_w \sin\beta\cos\beta} \quad (7)$$

Model-II:

A rocked distribution is available. The model description is shown in Fig. 2. In this case, the possible slip surface is either the wetting front or the rocked interface. So, three limit equilibrium equations may arise for three different cases. The safety factor is calculated using Eqs. (8-10) (Rahardjo et al., 1995; Crosta, 1998).

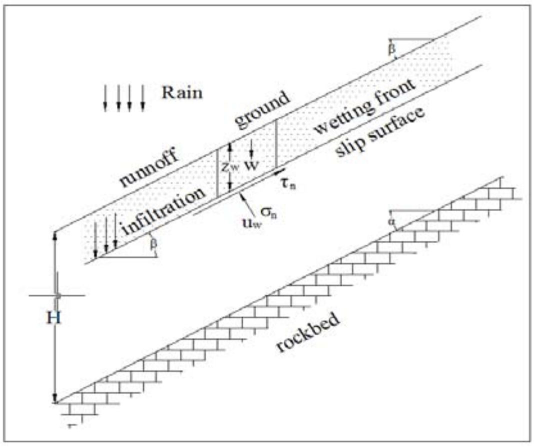

Figure (2): Rocked distribution is available for Model-II

Case-I: For slip surface along the wetting front: ($Z_w < H$)

$$SF = \frac{C'_W + \left\{\gamma_{sat} Z_W \cos^2\beta - u_w\right\}\tan\phi_w'}{\gamma_{sat} Z_W \sin\beta\cos\beta} \quad (8)$$

Case-II: For slip surface along the rocked surface: ($Z_w \geq H$)

$$SF = \frac{C'_W + \left\{\gamma_{sat} H \cos^2\alpha - u_w\right\}\tan\phi_w'}{\gamma_{sat} H \sin\alpha\cos\alpha} \quad (9)$$

Case-III: For slip surface along the rocked surface: ($Z_w \leq H$)

$$SF = \frac{C'_W + \left\{\gamma_i H \cos^2\alpha + FF\gamma_W \cos^2\alpha\right\}\tan\phi_i'}{\left(\gamma_i H + FF\gamma_w\right)\sin\alpha\cos\alpha} \quad (10)$$

The value of u_w in Eqs. (7-10) depends upon the exact location of the assumed failure plane. Thus, three situations may arise, as follows.

Case-I: Potential failure plane is just above the wetting front.

In this case, $u_w = \gamma_w z_w \cos^2\beta$

Case-II: Potential failure plane is just below the wetting front.

In this case, $u_w = -\psi_f \gamma_w$.

Case-III: When pore pressure is zero. $u_w = 0$.

The two models mentioned have been used as 'infinite slope' models in this study as shown in Fig. 1 and Fig. 2, wherein the potential slip surface is the wetting front for model-I and for model-II as is given by Eq. (8) and Eq. (9).

Probabilistic Approach

For probabilistic analysis, the simple Mean-Value First-Order Second-Moment method (MVFOSM) has been used. Taking the performance function and limit state as FS-1 = 0, the reliability index based on the MVFOSM model is given by:

$$\beta_{FS} = \frac{E[FS]-1}{\sigma[FS]}$$

$$= \frac{FS(\mu_{X_i})-1}{\sqrt{\sum_{i=1}^{n}\left(\frac{\partial FS}{\partial X_i}\right)^2 \sigma^2[X_i] + 2\sum_{i,j=1}^{n}\left(\frac{\partial FS}{\partial X_i}\right)\left(\frac{\partial FS}{\partial X_j}\right)\rho\sigma[X_i]\sigma[X_j]}} \quad (11)$$

where n = number of soil parameters (c'_w, ϕ_w, γ_t, K_s); E[FS] = expected value of FS; σ[FS] = standard deviation of FS; μ_{xi} = mean value of soil parameter X_i ; $\sigma[X_i]$ = standard deviation of X_i and ρ = correlation coefficient between X_i and X_j.

Calculation of Probability of Failure

Corresponding to the value of the reliability index β_{FS} assuming normal distribution, the probability of failure P_f (Ang and Tang, 1975) is calculated by the following equation.

$$P_f = \Phi(-\beta_{FS}) \quad (12)$$

where Φ(·) is the cumulative standard normal distribution function.

ILLUSTRATIVE EXAMPLE

Description

For the sake of numerical study, an infinite slope section previously analyzed by Xie et al. (2004) having the following data is considered in this paper: initial moisture deficit ($\Delta\theta_i$) = 0.28; suction head at the wetting front (ψ_f) = 55 cm; cohesion (c'_w) = 3 kPa; saturated unit weight (γ_{sat}) = 18 kN/m^3; angle of internal friction (ϕ'_w) = 25˚. The rainfall data are as shown in Table 1. The analyses have been carried out considering the value of hydraulic conductivity (Ks) = 1.87 cm/hr. To study the effect of variation of slope inclinations, the analyses have been carried out for three different slope inclinations, β = 30°, 40° and 45°.

Table 1. Rainfall data for the illustrative example

Time (hr)	Rainfall Intensity (cm/hr)	Time (hr)	Rainfall Intensity (cm/hr)
1	3.8	16	4
2	3.8	17	4
3	3.5	18	4.4
4	3.5	19	4.1
5	4.6	20	4.1
6	4.6	21	4.0
7	4.2	22	3.7
8	4.2	23	4.2
9	4.2	24	4.4
10	4.4	25	3.6
11	4.8	26	4.4
12	4.8	27	3.9
13	4.9	28	4.2
14	4.8	29	4.2
15	4.5	30	4.8

RESULTS AND DISCUSSION

Deterministic Analysis

The worst possible pore pressure has been considered and the factor of safety has been calculated using the developed computer program RISSA (Rainfall Induced Slope Stability Analysis). Fig. 3 shows that the factor of safety decreases with the increase in the depth of wetting front for each slope inclination. It is quite obvious that pore water pressure increases due to the increase in depth of wetting front. This causes a reduction in the effective stress, which, in turn, reduces shear strength of soil leading to slope instability.

For example, for a particular variation of wetting front depth, such as from 177.0632 to 196.8009, the factor of safety varies from 1.06422% to 0.943396% and 0.874036% respectively, for 30˚, 40˚ and 45˚, respectively. Moreover, it can also be stated that beyond a certain depth (1.5m), there is a sudden drop in the factor of safety irrespective of the considered slope angle.

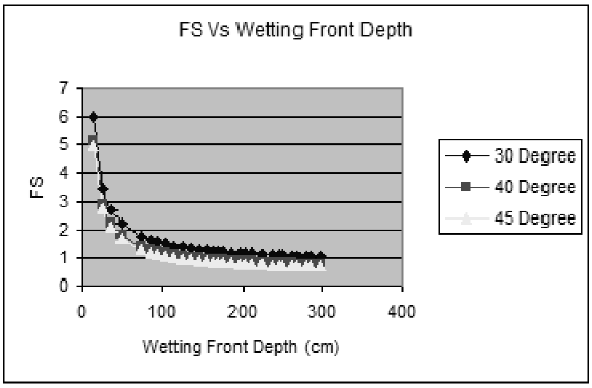

Figure (3): Variation of factor of safety with wetting front depth

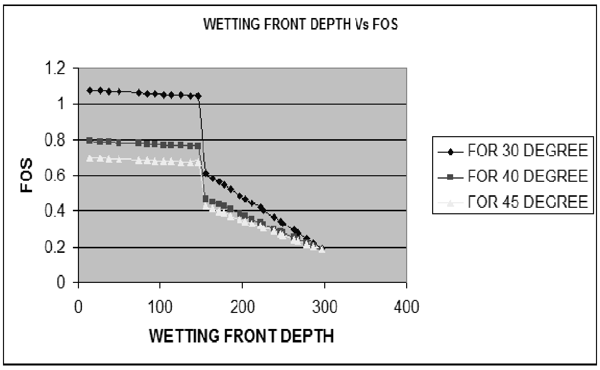

Figure (4): Variation of factor of safety with wetting front depth for steep and flat slopes

Fig. 4 presents a plot of factor of safety *versus* wetting front depth, where in case of steep slope the variation of factor of safety is less than that of flat slope as depth of wetting front is increased.

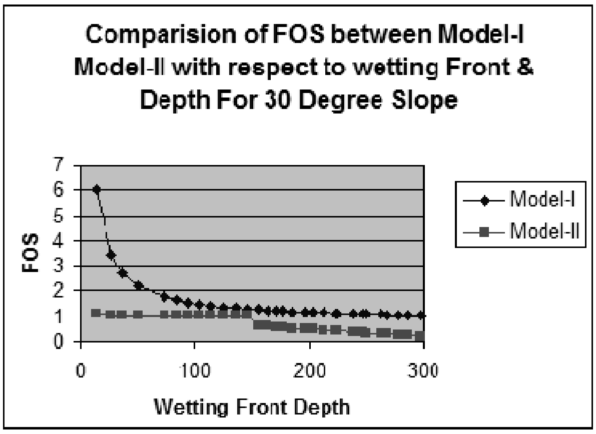

Figure (5): Variation of FOS with wetting front depth for model-I and Model-II

Fig. 5 shows that due to the presence of rockbed, the factor of safety becomes less in comparison with Model-I and that beyond a certain depth (say 150cm), it becomes significantly much less as the parameters of the given slope are of concern. As in case of model-II, the decrement rate ranges from 0.196% to 41%.

Probabilistic Analysis

As already stated, in view of the fact that the geomechanical parameters involved in the stability analysis are uncertain, an attempt has been made in this paper to determine the probability of failure of the slope complementary to the conventional factor of safety. For this purpose, a computer program; RRISSA (Reliability Analysis for Rainfall Induced Slope Stability Analysis) has been developed using the Mean Value First Order Second Moment (MVFOSM) method, assuming the random variables to be normally distributed and uncorrelated. The developed computer program has been used to determine the reliability index and the associated probability of failure for various depths of wetting front and for a uniform c.o.v. of 0.2, as presented in Figs. (6-10).

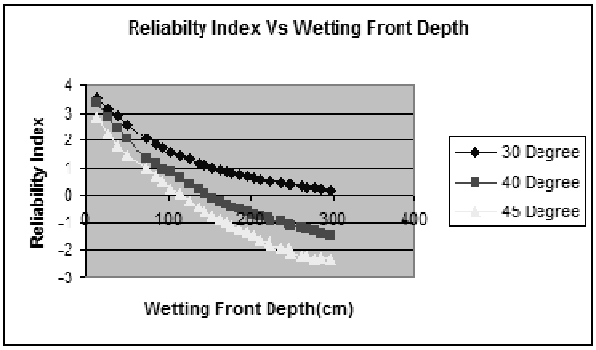

Figure (6): Variation of reliability index with wetting front depth

Fig. 6 presents a plot of reliability index *versus* depth of wetting front based on the values obtained by the developed program. From Fig. 6, it can be observed that reliability index decreases with increasing depth of wetting front, as expected. It is also observed that reliability index decreases with increment of slope steepness as per expectation.

Fig. 7 presents a plot of probability of failure *versus* depth of wetting front using the developed program. From detailed results relevant to Figs. 6 and 7, it is seen that while corresponding to a depth of wetting front of 171 cm for the 30 degree slope, the value of factor of safety is 1.22 and the corresponding probability of failure is about 20% which is rather high. This observation clearly demonstrates the necessity and importance of reliability analysis under a probabilistic framework. Further, as expected, the probability of failure increases with the increase in wetting front depth for each slope angle. For example, if the depth of wetting front increases from 249 cm to 268 cm, the probability of failure increases from about 9% to 51%, respectively, for 30 degree slope and 45 degree slope.

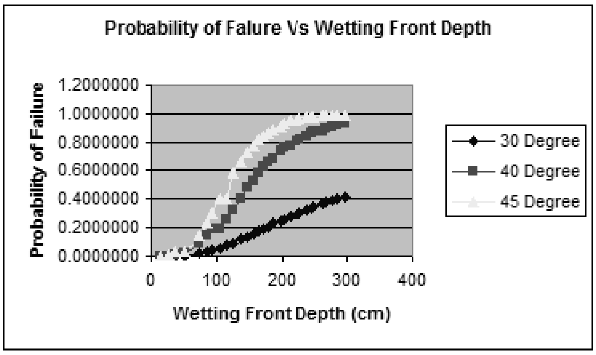

Figure (7): Variation of failure probability with wetting front depth

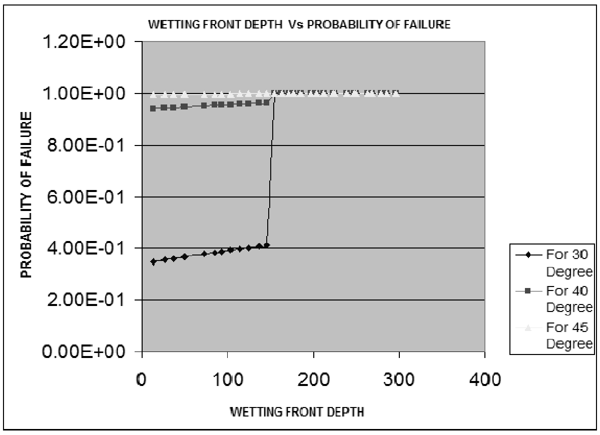

Figure (8): Variation of failure probability with wetting front depth

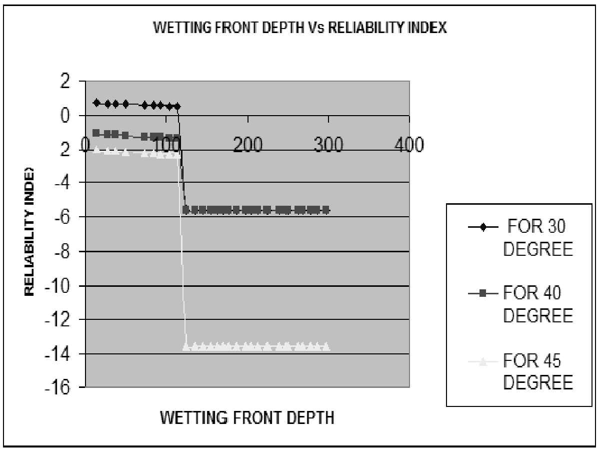

Figure (9): Variation of reliability index with wetting front depth

Fig. 8 represents a plot of probability of failure *versus* wetting front depth, which shows that in case of steep slope, the variation probability of failure is negligible, as expected. Fig. 9 represents a plot of reliability index *versus* wetting front depth, which shows that in case of steep slope the reliability index decreases as expected.

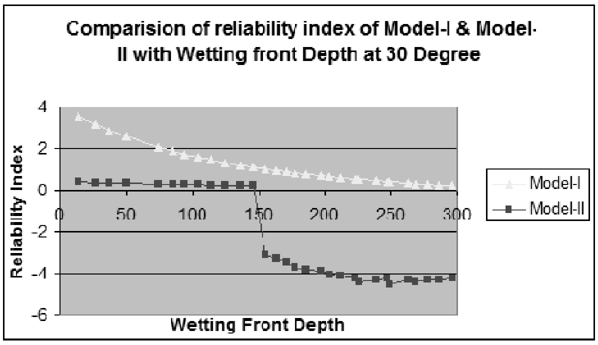

Figure (10): Variation of reliability index with wetting front depth for model-I and model-II

CONCLUSIONS

The following concluding remarks can be drawn on the basis of the observations made in the study undertaken in this paper.

1. The developed computer program for (MRISSA) deterministic analysis and (MRRISSA) for

probabilistic analysis of rainfall induced slope instability have been found to be useful computing tools for the evaluation of the status of stability of slopes subjected to rainfall infiltration. The programs are developed on the basis of infinite slopes of unsaturated soils, considering the widely used Green-Ampt model of infiltration and unsteady rainfall.

2. When rainfall induced instability is modeled such that the wetting front is assumed as a potential failure plane, two distinctly different possibilities arise in respect of magnitude of the developed pore water pressure: one when the failure plane is located just above the wetting front, and the other when it is just below the wetting front. In the former case, the failure plane being located in a saturated zone (as per Green-Ampt assumption) is subjected to positive pore water pressure, while in the latter, the failure plane being located in an unsaturated zone is subjected to negative pore water pressure or suction. In the former case, obviously, the value of safety factor is lower, and, therefore, this case is crucial.
3. Failures of unsaturated slopes often occur as a consequence of water infiltration after intense or prolonged rainfall. Catastrophic failures can occur in slopes composed of loosely compacted soil.
4. With the help of developed computer program (MRISSA), a plot of FOS *versus* rainfall duration can be easily obtained, which should be of great practical value to keep and watch on the safety status as time progresses.
5. The results obtained from the probabilistic analysis using the developed computer programs based on MFOSM method depict the necessity and importance of adopting a probabilistic approach. Even in that case in which the values of the factor of safety (obtained from the deterministic approach) are above unity (1.045797) (which indicates critical equilibrium), the corresponding propability of failure values are found to be (nearly 41%). This simply demonstrates that a deterministic approach does not indicate the true stability status of the slopes.
6. The observations noted in this study are based on one slope case. Several such cases should be analyzed before arriving at generalized conclusions with regard to the above mentioned points.

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