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Analysis on Three-dimensional Stress Distribution in Subgrade during the Vibrating Compaction

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ABSTRACT

Study on stress distribution in subgrade during the vibrating compaction process is of great value to find out the compaction mechanism. In order to study the three-dimensional distribution characteristics of stress in subgrade, the basic kinetic equation of subgrade compaction system is derived by using the finite element method. Next, the physical model of "vibratory roller-subgrade" system is established. Then, the three-dimensional stress distribution in subgrade is simulated by COMSOL Multiphysics. The numerical results are reliable, since they are in accordance with the test results and are consistent with the theory of the vibrating compaction in road construction. The simulation results are as follows: First, the compaction stress decreases gradually from the upper to the lower layer of the subgrade. Second, along the width direction, the stress has maximum value in the middle area, but decreases symmetrically towards both sides of the roller.

KEYWORDS: Subgrade soil, Vibrating compaction, Finite element method, Three-dimensional stress distribution, Compaction mechanism.

INTRODUCTION

The existing kinetic models of subgrade vibrating compaction are almost with two degrees of freedom (Yoo, 1975; Yoo and Selig, 1979; MacHet and Morel, 1977; Pietzsch and Poppy, 1992), in which the exciting force is of a linear distribution, so it is theoretically a plane strain problem. But, in the process of vibrating compaction, there is a contact area between roller and subgrade. The exciting force loaded in this area is a surface pressure instead of a linear pressure. It is difficult to obtain a numerical solution of this problem. Liu and Wang (2008) used LS-DYNA dynamics software to simulate the interaction process of impact-like vibratory roller-subgrade system and analyzed the influence of different parameters on the impact-like vibrating compaction effects. Guo et al. (2007) used SIMULINK simulation software to study the relationship between the compaction effect and both machine parameters and soil parameters. Qin et al. (2001) studied the dynamic property of vibratory roller-subgrade system and analyzed the response characteristics of vibratory roller. Zhou and Zhou (2000) identified soil parameters during the process of vibrating compaction. It can be seen that there is no literature reports on the real-time three-dimensional stress distribution of "vibratory roller-subgrade" system.

In this paper, the subgrade is considered as pure, uniform density and elastomeric soil material. The three-dimensional physical model of subgrade vibrating compaction system is presented by FEM (finite element method). Based on COMSOL

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Multiphysics, numerical simulation and calculation analysis of the model are carried out and the three-dimensional stress distribution characteristics of subgrade during vibrating compaction are studied. This paper is of great theoretical significance to supplement and perfect the vibrating compaction theory, reveal the mechanism of vibrating compaction and guide the engineering practice.

Kinetic Equation of Subgrade Vibrating Compaction System

In the process of subgrade vibrating compaction, elastic or plastic deformation of subgrade occurs due to the exciting force. There is also a viscous damping force during the deformation, so that the "vibratory roller-subgrade" compaction model can be simplified as a "spring-viscous damping" system. According to the D'Alembert principle, the motion equation of the system can be written as:

$$[M][\ddot{\delta}(t)] + [C][\dot{\delta}(t)] + [K][\delta(t)] = [F(t)]_s; \qquad (1)$$

where:

[M] = the mass matrix.

[C] = the damping matrix.

[K] = the stiffness matrix.

$[\delta(t)]$ = the displacement of subgrade unit node.

$[F(t)]_s$= the exciting force that the vibratory roller loads on the subgrade.

This motion equation can be derived as follows. Since the exciting force F(t) varies with time, the displacement $[f(t)]^e$ of subgrade unit, the subgrade comprehensive damping coefficient c(t), the comprehensive stiffness coefficient k(t) and the density of subgrade soil ρ(t) vary with time, accordingly. These parameters mainly vary with the compaction times. In the process of subgrade vibrating compaction, there are two components of load vector loading on each subgrade unit e. One is the dynamic load $[F(t)]_s^e$ produced by vibratory roller and the other load is composed of the inertial force$[F(t)]_I^e$ and the damping force$[F(t)]_c^e$.

The inertial force of each unit volume of subgrade unit is:

$$[p(t)]_I = -\rho(t)[N]^e[\ddot{\delta}(t)]^e. \qquad (2)$$

So, the power of inertial force of subgrade unit e can be given as:

$$W_I = \iiint_e [f(t)]^{eT}[p(t)]_I dxdydz = -[\delta(t)]^{eT} \iiint_e [N]^{eT}\rho(t)[N]^e[\ddot{\delta}(t)]^e dxdydz. \qquad (3)$$

As the power is equal to force multiplied by displacement, the inertial force of unit e can be given as:

$$[F(t)]_I^e = -\iiint_e \rho(t)[N]^{eT}[N]^e dxdydz[\ddot{\delta}(t)]^e. \qquad (4)$$

Let the element mass matrix be:

$$[M]^e = \iiint_e \rho(t)[N]^{eT}[N]^e dxdydz. \qquad (5)$$

So, the inertial force of subgrade unit e can be given as:

$$[F(t)]_I^e = -[M]^e[\ddot{\delta}(t)]^e. \qquad (6)$$

Supposing c(t) is the soil viscous damping coefficient, then the damping force of each point in the subgrade is (Xu, 2006):

$$[p(t)]_c = -c(t)[N]^e[\dot{\delta}(t)]^e. \qquad (7)$$

So, the power of damping force of subgrade unit e can be given as:

$$W_c = \iiint_e [f(t)]^{eT}[p(t)]_c dxdydz$$
$$= -[\delta(t)]^{eT} \iiint_e c(t)[N]^{eT}[N]^e dxdydz[\dot{\delta}(t)]^e. \qquad (8)$$

Similarly, the damping force vector of subgrade unit is:

$$[F(t)]_c^e = -\iiint_e c(t)[N]^{eT}[N]^e dxdydz[\dot{\delta}(t)]^e. \quad (9)$$

Let the element damping matrix be:

$$[C]^e = \iiint_e c(t)[N]^{eT}[N]^e dxdydz. \quad (10)$$

So, the damping force vector of subgrade unit is:

$$[F(t)]_c^e = -[C]^e[\dot{\delta}(t)]^e. \quad (11)$$

Assuming that the subgrade is meshed into m elements, then the global matrices are as follows:

$$[K] = \sum_{n=1}^{m}[K]^e \quad (12)$$

$$[M] = \sum_{n=1}^{m}[M]^e \quad (13)$$

$$[C] = \sum_{n=1}^{m}[C]^e \quad (14)$$

$$[F(t)]_s = \sum_{n=1}^{m}[F(t)]_s^e. \quad (15)$$

The global load vector of visco-elastoplastic subgrade can be expressed as:

$$[F(t)]_s - [M][\ddot{\delta}(t)] - [C][\dot{\delta}(t)]. \quad (16)$$

According to the D'Alembert principle, the equation can be written as follows:

$$[K][\delta(t)] = [F(t)]_s - [M][\ddot{\delta}(t)] - [C][\dot{\delta}(t)]. \quad (17)$$

It can be rewritten as:

$$[M][\ddot{\delta}(t)] + [C][\dot{\delta}(t)] + [K][\delta(t)] = [F(t)]_s. \quad (18)$$

The equation above is the basic kinetic equation based on the finite element method in the process of vibrating compaction of subgrade soil. It is consistent with the equation derived by Zhang (2010).

3-D Simulation of the Progress of Subgrade Vibrating Compaction

Three-dimensional Model of Subgrade Soil during Vibrating Compaction

The exciting kinetic models of subgrade vibrating compaction are almost with two degrees of freedom (Yoo, 1975; Yoo and Selig, 1979; MacHet and Morel, 1977; Pietzsch and Poppy, 1992), in which the exciting force is of a linear distribution, so that it is theoretically a plane strain problem, which should be simulated by two-dimensional finite element method. But in the actual process of compaction, the contact between roller and subgrade soil is a surface contact owing to the deformation of subgrade. Thus, the three-dimensional simulation of vibratory roller-subgrade system in this paper is more reliable and scientific than the two-dimensional one. The physical three-dimensional compaction model is shown in Fig.1 and the surface contact area is shown as the shadow area $[x_1, x_2]$ in Fig. 1. The exciting force is loaded in different contact areas as time passes by. When the time step is small, the exciting force can be approximated as a continuous force. Then, the actual vibrating compaction process can be simulated. The parameters used in this model are as follows:

The size of subgrade is 10m×0.75m×0.35m (length× width× height); the width of vibratory roller is equal to the width of subgrade soil. The mass of vibratory roller $m_d = 309$kg. The vibratory frequency f=30Hz and the nominal amplitude is A=1.2mm. The roller moves along the x direction at a constant speed of v_0=1.2km/h. So, after a time t, its displacement is $x_1=v_0t$. It is assumed that $x_2=v_0+B$, where B=R sin β is the chord length of contact area and β is the angle between horizontal plane and the tangent of the contact area.The time step in the simulation is Δt=0.005. Therefore, it needs $T=10/v_0$=30 seconds for each compaction, so the time interval in the simulation is [0.30] second.

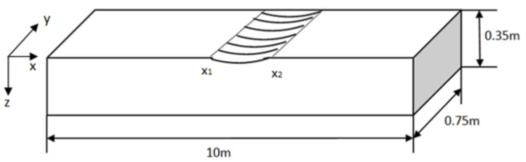

Figure (1): Model of subgrade during vibrating compaction

According to the theory of vibrating compaction, the exciting force of vibratory roller can be calculated as follows:

$$F = A \cdot m_d \cdot w^2; \quad (19)$$

where:
A= the nominal amplitude.
m_d= the mass of vibratory roller.
w = 2πf is the angular frequency of vibration.
f = the vibratory frequency of the vibratory roller.

At time t, the distributed exciting force on the contact area can be expressed as:F_B=F/(BL). Therefore, distributed surface pressure applied for the model can be written as:

$$F(t,x) = \begin{cases} F_B = F/(BL), & if \quad x \in [vt, vt+B] \\ 0, & else \end{cases}; \quad (20)$$

where:
L= the width of vibratory roller.

This is the exciting force equation of vibrating compaction in the solution region.

According to the actual compaction situation, the boundary constraints of the model are set as follows: The front, back, left and right boundaries of subgrade are plane constraints; normal displacement of each plane and the rotation around the tangential axis are restricted. The bottom of subgrade is set as fixed constraint; while the top of subgrade is free and the exciting force F(t,x) is distributed over the contact area $[x_1, x_2]$ on the top of subgrade.

Determination of Soil Constitutive Model

Since the subgrade deformation behavior is very complex and difficult to describe correctly, hundreds of approximate soil constitutive models have been put forward. Based on the experience and practice of civil engineering, the D-P yield criterion (Deng et al., 2006) of elastoplasticity constitutive model is selected in this paper to simulate the stress-strain relationship in subgrade. The model can not only accurately simulate the interaction between the vibratory roller and soil, but is also convenient for numerical calculation.

The yield function of the series D-P criterion can be represented as a general formula:

$$f = \alpha I_1 + \sqrt{J_2} - k = 0; \quad (21)$$

where:
I_1 = the first stress invariant.
J_2 = the second deviation stress invariant.

For a given stress state, I_1 and J_2 are constants. I_1 and J_2 can be calculated as follows:

$$I_1 = \sigma_x + \sigma_y + \sigma_z. \quad (22)$$

$$J_2 = 1/6[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]. \quad (23)$$

The series D-P criterion varies for different parameter assignments α and k, where α and k are constants for a given material and can be calculated by soil cohesion c and internal friction angle φ that could be obtained from field measurements. The formula is as follows:

$$\alpha = \frac{\sin\varphi}{\sqrt{3}\sqrt{3+\sin^2\varphi}}, k = \frac{3c\cdot\cos\varphi}{\sqrt{3}\sqrt{3+\sin^2\varphi}}. \quad (24) \text{ and } (25)$$

Determination of Finite Element Model Parameters

The exciting force F (t, x) and the D-P yield function required by the model are given directly by mathematical equations in the software COMSOL

Multiphysics (William and Zimmerman, 2007). The main parameters used in the simulation can be seen in Table 1.

Table 1. The main parameters needed in the model

Name	Expression	Value	Description
φ	8.836/180*π	0.1542	Friction angle
c	69e3[Pa]	69000[Pa]	cohesion
alpha	sinφ /(sqrt(3)*sqrt (3+(sinφ)^2))	0.051	Drucker-Prager material constant
K	3*c*cosφ /(sqrt(3)*sqrt (3+(sinφ)^2))	67914.56108 [Pa]	Drucker-Prager material constant
V0	1.2*1000/3600	0.3333	Step velocity in x-dir of roller
RollerR	0.28	0.28	Radius of roller
beta	8.8*π/180	0.1536	degree
B	RollerR*sinβ	0.0428	Longitude of roller-ground interface
l	0.75	0.75	
area	B*l	0.0321	

Simulation Results of Stress Distribution in Subgrade

Stress Distribution in Subgrade at a Given Time

Since the exciting force loading on the subgrade can be seen as constant, but applied on different places of the upper layer of subgrade, the stress distribution in subgrade at a given time can reflect the three-dimensional stress distribution characteristics in subgrade. The stress distribution at 15s is shown in Fig.2, where 100 parallel sections with equal intervals are taken along the X direction. From Fig. 2, we can see that:

At t=15s, the exciting force loads on the area between x=5m and x=5.043m. The stress in subgrade distributes nearly uniform; that is to say that the subgrade can be uniformly compacted with the exciting force.

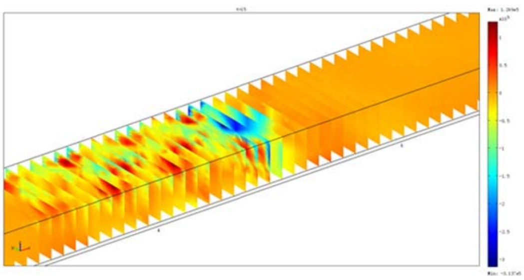

Figure (2): Stress distribution at 15 s

Fig.3 shows the stress distribution at the section of x=5m. From Fig. 3, we can see that the stress distribution is not uniform and decreases towards both sides of the roller. Meanwhile, the stress decreases from the upper layer to the lower layer along the depth direction of subgrade.

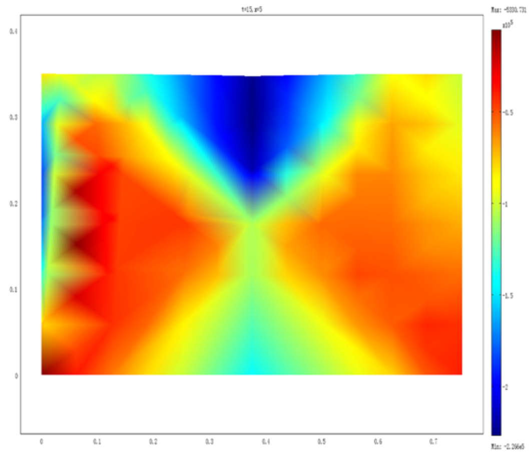

Figure (3): Stress distribution at the section of x= 5m

Stress Distribution Characteristics at the Depth Direction of Subgrade

In order to investigate the distribution characteristics of compressive stress along the depth direction of subgrade, three detection points are set in each layer (the upper layer, middle layer and lower layer). The stress curves in each layer (take the section of x=5m for example) are shown in Fig.4. It can be seen from Fig.4 that the compressive stress decreases from the upper layer to the lower layer of subgrade. That is to say, during compaction, the maximal stress appears in the upper layer, the middle stress appears in the middle layer and the minimal stress appears in the lower layer. Fig. 5 shows the stress curve in every layer measured during the vibrating compaction. AL7, AL9 and AL5 are the upper layer signal, middle layer signal and lower layer signal, respectively. By comparing Fig. 4 and Fig. 5, it can be seen that the simulation result is the same as the experimental result in literature (Zhang et al., 2010), which indicates that the proposed model in this paper is reasonable.

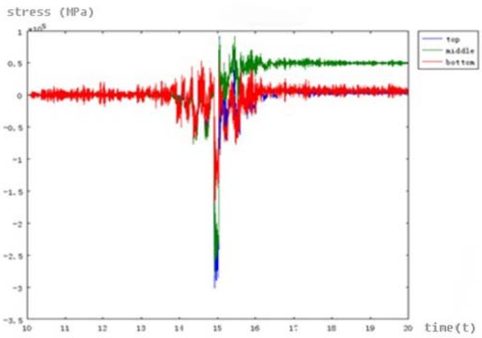

Figure (4): Stress at different depths

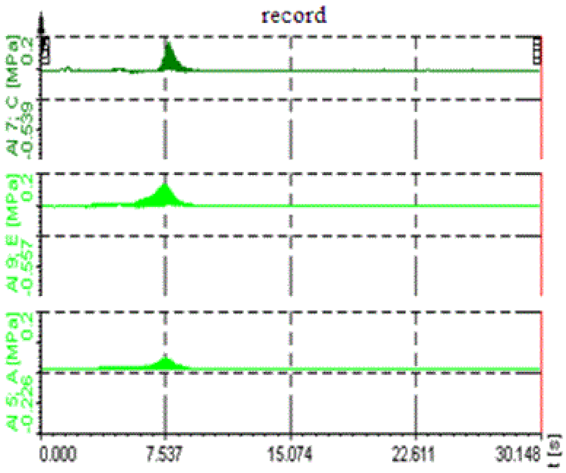

Figure (5): The stress curve in every layer measured during the vibrating compaction

Stress Distribution Characteristics at the Width Direction

In order to investigate the stress distribution along the width direction of subgrade, three detection points are set along the width of roller (one is set at the middle of subgrade, the other two are set symmetrically near both sides of the roller). The stress curves along the width of subgrade for different layers are shown in Figs. 6,7 and 8, respectively.

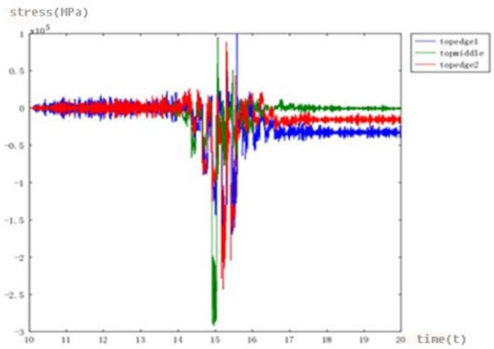

Figure (6): Stress of upper layer at the width direction

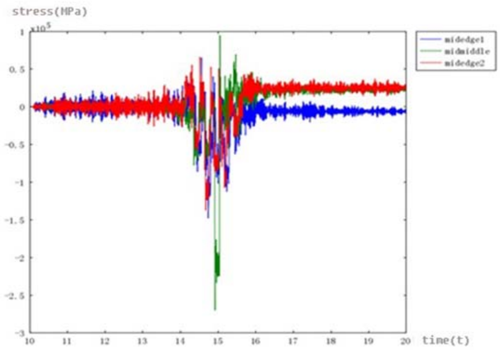

Figure (7): Stress of middle layer at the width direction

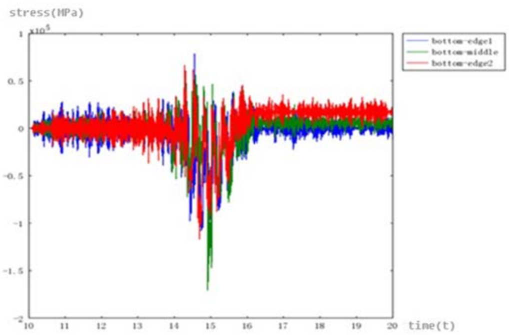

Figure (8): Stress of lower layer at the width direction

It can be seen from Figs. 6, 7 and 8 that for each layer of subgrade, the maximum stress occurs in the center of subgrade, while the stress becomes smaller towards both sides of the roller. This result indicates that compaction at width direction is not uniform. The middle part of subgrade has best compaction effect, while both sides of subgrade are less effective. This explains the reason why the overlap should exist in the two adjacent rolling tracks.

CONCLUSION

In this paper, the basic kinetic equation of subgrade compaction is derived by using the finite element method. The physical model of "vibratory roller-subgrade" system is established and the process of vibrating compaction conducted with the exciting force is simulated by COMSOL Multiphysics. The results show that the stress distribution is uniform at a given time when the exciting force is loaded on the subgrade, but it is not uniform along the width direction of subgrade. The middle area of the roller has the best compaction effect, while both sides of the roller have less effective compaction. The stress decreases along the depth direction of subgrade. The simulation result is consistent with the experimental results in literature (Xu, 2006) and with the practice of construction, which shows that the finite element model proposed in this paper and the simulation results are reasonable.

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