A Multi-objective Stochastic Programming Approach for Project Time, Cost and Quality Trade-off Problem (TCQTP)

Behrooz Ahadian [1)*]***, Omid Veisy*** [2)] ***and Vahid Azizi*** [3)]

[1)] Department of Industrial Engineering, Najaf Abad Branch, Islamic Azad University, Isfahan, Iran.
* Corresponding Author. E-Mail: Ahadian.Behrooz@gmail.com
[2)] Department of Industrial Engineering, Tehran University, Tehran, Iran.
[3)] Faculty of Industrial Engineering, Khajeh Nasir Toosi University of Technology, Tehran, Iran.

ABSTRACT

Generally, in project time, cost and quality trade-off problems, the decision maker (DM) has to take different conflicting objectives into account in order to find an optimal solution. In project management literature, many models have been developed for project crashing and project time-cost trade-off, but only a few of them consider quality as an objective. This article employs a chance constrained compromise programming approach (CCCP) to solve the time, cost and quality trade-off problem (TCQTP). The purpose of this paper is to present and test a new model for the project time, cost and quality problem. In fact, we try to find the optimal decrease in cost and duration of activities while maximizing quality under uncertainty conditions. In order to expect more realistic outcomes for the problem of uncertainties in project activities, time should be taken into account. Therefore, we assume the activity duration as a random and normally distributed parameter. An example of a project network, consisting of 6 nodes and 7 activities, is analyzed under different uncertainty levels.

KEYWORDS: Project management, Chance-constrained compromise programming, Multi-objective programming, Time, cost and quality trade-off, Linear programming.

INTRODUCTION

Project time, cost and quality trade-off (TCQTP) is one of the most important and useful topics in project management. The main purpose of time- cost trade-off in a project is the analysis of the effective costs of the project. This is compared to the changes in the duration of activities for obtaining the most possible reduction in project completion time. The aim is to minimize the whole expenditures of the project which include both direct and indirect expenditures. In fact, time-cost trade-off models try to find optimum solutions for allocating resources to activities in which both time and cost will have the least amount for each activity.

The importance of time-cost trade-off has been recognized for five decades; from the very start of the development of the critical path method (CPM) (Kelly and Walker, 1959). Kelley (1961) was the first who used linear programming for solving this problem. His model gave managers the ability to answer questions, such as required resources, budget and communication in transportation. Perera (1980) indicated that by assuming a linear relation between time and cost, one can find the minimum overall costs for a project within a specific finishing time. Hendrickson et al. (1987) presented mathematical models for project scheduling. Patterson et al. (1974) used integer programming to

Received on 18/8/2015.
Accepted for Publication on 16/4/2016.

solve the project time-cost trade-off problem. A model was developed by Shatub et al. (1994) by taking into account the indirect costs in objective function and discrete time-cost relations. Chassiakos et al. (2000) designed an integer programming optimization method. This method emphasizes critical path activities and suggests two optimum and a compromise solution. In 1995, Liu et al. presented a model in which integer programming and linear programming were mixed together in order to enhance the accuracy of the results. Permachandra (1993) has used goal programming for activity crashing in a project network. The above mentioned methods mainly take place in certain environments. However, most of the recent studies usually concentrated on uncertain and probabilistic environments. These models are categorized in literature into two divisions: probabilistic and fuzzy models. Projects are planned before they are executed. Therefore, due to unclear activity completion time and many unforeseen events that may occur in the duration of a project, it is obvious that uncertainty is part of the nature of a project. Issues related to uncertainty in projects are generally evaluated by stochastic programming models. Charnes and Cooper (1959) presented a two-stage stochastic approach for solving the problem. But, Freeman (1960) was the first who used the concept of probability in project scheduling and programming. Coskunoglu (1984) presented a linear programming model to reach an optimized project finishing time. Dodin (1985) developed an approach in which the probability distribution of project completion time was determined. El-Rayes and Kandil (2005) developed a multi-objective genetic algorithm optimization method to search for optimal resource utilization plans that minimize construction time and cost while maximizing quality. Seifi and Tavakoli-Mogaddam (2008) presented a new bi-objective model for multi-mode resource-constrained cash flows. Their model maximizes the net present value (NPV) of a project and minimizes the holding cost of project activities while considering four different payment models. They finally used a meta-heuristic algorithm based on simulated annealing (SA) to solve the proposed model. Azaran and Tavakoli-Mogaddam (2007) developed a multi-objective model for time-cost trade-off in a dynamic PERT network with dedicated resources. In this model, the classical dynamic PERT network is a transformed network of queues. Mokhtari et al. (2010) constructed a non-linear mathematical program with the activity mean durations as the decision variables, where the objective function was concerned with minimizing the project crashing direct cost. Afterwards, they solved the proposed model using a hybrid approach based on the Cutting Plane (CP) method and Monte Carlo (MC) simulation. Kim et al. (2012) proposed a mixed integer linear programming model that accounts for both non-conformance risks and potential quality loss cost (PQLC) of project activities. By identifying these non-conformance risk activities in the process of project scheduling, they showed that the project manager can take preventive actions that eliminate the need for rework or modification.

Although since the emergence of time-cost trade-off problem (TCTP), time and cost have been the basic criteria, quality assurance has become a vital criterion in all projects. Moreover, by taking into account the increasing importance of quality and competition for winning the operational contracts in recent years, the quality criterion was added to TCTP and changed it into TCQTP. For the first time, Babu and Suresh (1996) surveyed the impact of project crashing on project quality. This method was applied by Khang and Myint (1999) in an actual cement factory construction project and the applicability of the method was evaluated. Tareghian and Taheri (2006) presented a framework for project time, cost and quality trade-off in which three inter-related integer programming models were developed, so that each model would optimize one of the given entities by assigning desired bounds on the other two models. Sonmez and Bettemir (2012) presented a hybrid strategy using genetic algorithms (GAs), simulated annealing (SA) and quantum simulated annealing (QSA) techniques for the discrete time–cost trade-off problem (DTCTP). Szmerekovsky and Venkateshan (2012) explored four formulations for solving project scheduling problems with irregular time-cost trade-offs (PSIC) which can capture the importance of including financial objective functions in time-cost trade-off project scheduling. Salmasnia et al. (2012) suggested a three- dimensional TCQTP in a stochastic environment. Their model aimed at minimizing the completion time and overall costs while maximizing the overall quality of a project. In order to tackle this problem, they employed response surface methodology (RSM) as a statistical tool which consists of three steps: design of experiment (DoE) for data collection, modeling time, cost and quality by the robust RSM approach and optimizing the estimated polynomial surface by minimizing the variance of the three objectives addressed above. Pollack-Johnson and Liberatore (2006) extended the standard discrete time-cost trade-off problem by assuming that each option for each task is evaluated for its duration, cost and quality and provided a mixed integer linear programming model for solving the problem. They provided alternative quality objective functions to show how quality level curves can be a useful management tool in making final project scheduling decisions. Afruzi et al. (2013) expanded a multi-mode resource- constrained discrete time-cost trade-off problem (DTCTP) model. In the proposed model, each activity has multi-modes and each mode requires different resources to perform the activity. Finally, they presented an adjusted fuzzy dominance genetic algorithm to solve the proposed model.

Bagherpour et al. (2012) proposed a fuzzy multi-objective programming (F-MOP) model to balance three simultaneous objectives; completion time minimization, total cost minimization and maximization of the quality of raw materials in an MRP system. Their proposed model can help manufacturers deliver products on time, at low cost and with high quality. Liberatore and Pollack-Johnson introduced the notion of a quality function for individual tasks and used the functional form of the bi-variate normal to model quality at the task level. The specified quality function is then incorporated into a non-linear programming model that allows quality to be explicitly considered in project planning and scheduling. By the suggested method in this paper, it is assumed that the parameters associated with the objectives are random and normally distributed. Therefore, TCQTP is implemented by means of stochastic programming *via* a chance-constrained compromised programming approach.

Generally speaking, the models found in the literature use one of the three parameters; time, cost or quality, as an objective function and the other two parameters as constraints. The advantage of such models is simplicity of the solutions, as there is just one objective function in the model. However, their likely weakness is the existence of a small solution space due to multiple constraints. This weakness can hinder the optimum solution. By considering all three parameters in an objective function, the optimal solution may be improved by enlarging the solution space.

STOCHASTIC PROGRAMMING

Stochastic programming is a science that solves problems in connection with stochastic systems, where the mathematical form of the problem is of an optimized nature (Ruszczyinski and Shapiro, 2003). As in the single objective case, two main approaches are used to solve stochastic programs; namely, the recourse approach and the chance-constrained approach (Ben Abdelaziz et al., 2007). Chance-constrained programming (CCP) consists of fixing a certain level of probability $\alpha_i \in [0, 1]$ for each constraint i and imposing that the constraint i is satisfied with at least this probability. The set of feasible solutions is thus restricted by the following deterministic constraints which are generally non-linear:

$$P(\{w|a_i(w).X \geq b_i(w)\}) \geq \alpha_i, i = 1, \dots, L_1. \quad (1)$$

$$P(\{w|a_i(w).X = b_i(w)\}) \geq \alpha_i, i = L_1 + 1, \dots, L. \quad (2)$$

The multi-objective stochastic program is as outlined below:

$$max \sum_{j=1}^{J} \tilde{C}_{ij} x_j \forall i = 1,2,\dots,M; \tag{3}$$

subject to: P(1)

$$\sum_{j=1}^{J} \tilde{A}_{lj} x_j \le \tilde{b}_l \forall l = 1,2,\dots,L$$

$$x \in X;$$

where $x_j \in \{0,1\}$ for every j, $\tilde{C}_{ij}$, $\tilde{A}_{lj}$ are random matrices and $\tilde{b}_l$ is a random vector. Prékopa (1995) converted the multi-objective stochastic program into a deterministic program in CCP approach as follows:

$$max\, E\left(\sum_{j=1}^{J} \tilde{C}_{ij} x_j\right); \quad \forall i = 1,2,\dots,M;$$

subject to:

$$Prob\left(\sum_{j=1}^{J} \tilde{A}_{lj} x_j \le \tilde{b}_l\right) \ge 1 - \zeta_l; \ \forall l = 1,2,\dots,L$$

$x \in X;$ P(2)

where ζ_l represents the threshold values of the constraints that are specified by the decision maker (DM). $E\left(\sum_{j=1}^{J} \tilde{C}_{ij} x_j\right)$ is the vector of expected values of the objective function with regards to random state of nature. The Compromise Program (CP) model developed by Zeleny (1974) minimizes the difference between the goal value of f_i^* and the achieved value of $f_i(x)$. In case that the higher values of the objective are better, the f_i^* values can be obtained as follows:

$$min \sum_{i=1}^{M} \sigma_i^- ;$$

subject to:

$f_i(x) + \sigma_i^- = f_i^*; \quad \forall i = 1,2,\dots,M$ P(3)

$h_l(x) \le b_l; \quad \forall l = 1,2,\dots,L$

$x \in X, \sigma_i^- \ge 0.$

And if smaller values of the objective are better, then:

$$min \sum_{i=1}^{M} \sigma_i^+ ;$$

subject to:

$f_i(x) - \sigma_i^+ = f_i^* \quad \forall i = 1,2,\dots,M$ P(4)

$h_l(x) \le b_l \quad \forall l = 1,2,\dots,L$

$x \in X, \sigma_i^+ \ge 0.$

CHANCE-CONSTRAINED COMPROMISE PROGRAMMING

Ben Abdelaziz et al. (2007) proposed a deterministic equivalent program to multi-objective stochastic programming and called it "Chance-Constrained Compromise Programming, (CCCP)". Based on the CCP approach and the CP model, they transformed the random constraints:

$$Prob\left(\sum_{j=1}^{J} \tilde{A}_{lj} x_j \le \tilde{b}_l\right) \ge 1 - \zeta_l \ \forall l = 1,2,\dots,L \tag{4}$$

into deterministic constraints as given below:

$$E\left(\sum_{j=1}^{J} \tilde{A}_{lj} x_j - \tilde{b}_l\right) + \phi^{-1}(1-\zeta_l)\sigma\left(\sum_{j=1}^{J} \tilde{A}_{lj} x_j - \tilde{b}_l\right) \le 0 \ \forall l = 1,2,\dots,L; \tag{5}$$

where J is the number of decision variables and $Prob\left(\sum_{j=1}^{J} \tilde{A}_{lj} x_j \le \tilde{b}_l\right) \ge 1 - \zeta_l$ is the chance-constraint related to $\sum_{j=1}^{J} \tilde{A}_{lj} x_j \le \tilde{b}_l$. $\tilde{A}_{lj}$ and $\tilde{b}_l$ are normally distributed parameters. In addition, the part of the chance-constrained compromise program related to the objectives consists of minimizing:

$$\sum_{i=1}^{M} \varepsilon_i ;$$

subject to:

$Prob\left(\sum_{j=1}^{J} \tilde{C}_{ij} x_j \ge f_i^* - \varepsilon_i\right) \ge 1 - \zeta_i; \forall i = 1,2,\dots,M.$

The above mentioned authors also replaced the random objective $\tilde{C}_{ij}$ using a stochastic compromise approach in which they assumed f_i^* to be the best solution for the objective function $\sum_{j=1}^{J} \tilde{C}_{ij} x_j$

subject to system constraints and $\tilde{h}_i(x) = f_i^* - \sum \tilde{C}_{ij}x_j$. Thus, the objectives are transformed as below:

$$Min \sum_{i=1}^{M} \varepsilon_i ; \quad (6)$$

subject to:
$Prob(\tilde{h}_i(x) \leq \varepsilon_i) \geq 1 - \zeta_i;$

where ζ_i represents the threshold values of the objective i. The equivalent program to P(1) named as CCCP as proposed by Ben Abdelaziz et al. (2007) is as follows:

$$min \sum_{i=1}^{M} (\varepsilon_i + \sigma_i^-) + \sum_{l=1}^{L} (\sigma_l^-) ;$$

subject to:

$$E\left(f_i^* - \sum_{j=1}^{J} \tilde{C}_{ij}x_j\right) + \phi^{-1}(1-\zeta_l)\sigma\left(f_i^* - \sum_{j=1}^{J} \tilde{C}_{ij}x_j\right) - \varepsilon_i + \sigma_i^- = 0;\ \forall i = 1,2,\ldots,M \qquad P(5)$$

$$E\left(\sum_{j=1}^{J} \tilde{a}_{lj}x_j \leq \tilde{b}_l\right) + \phi^{-1}(1-\zeta_l)\sigma\left(\sum_{j=1}^{J} \tilde{A}_{lj}x_j \leq \tilde{b}_l\right) + \sigma_l^- = 0;\ \forall l = 1,2,\ldots,L\ x \in X;$$

where:

σ_i^- is the negative deviation between the achievement and the aspiration levels of f_i. σ is the standard deviation. $\phi(y)$ represents the probability distribution function of the standard normal distribution.

In the next section, the model described will be used in order to deal with TCQTP. A numerical example is also discussed.

TCQTP MODEL

In order to formulate a project time, cost and quality trade-off, the raw information required consists of:

1. Activity List
A = {(i, j): there is an activity in the project network from node I to node j }, where the nodes are numbered from1 (the start node) to N (the finish node). r_j = the number of different paths from node 1 to node j, indexed by r , in numerical order.

2. Normal Duration of Each Activity $(\boldsymbol{D_{ij}^N})\forall(i,j) \in A$
Normal time refers to the estimated activity duration used with CPM or PERT in the computation of the earliest (latest) start or finish times.

3.Crash Duration of Each Activity $(\boldsymbol{D_{ij}^C})\forall(i,j) \in A$
Crash duration is a term referring to the shortest possible time for which an activity can be scheduled (Hendrickson et al., 1987).

4. Normal Cost of Each Activity $(\boldsymbol{C_{ij}^N})\forall(i,j) \in A$
Normal cost is the cost of completing an activity in normal time.

5. Crash Cost of Each Activity $(\boldsymbol{C_{ij}^C})\forall(i,j) \in A$
Crash cost is the cost of completing an activity in crash time.

6. Normal Quality Level for Each Activity $(\boldsymbol{Q_{ij}^N})\forall(i,j) \in A$
The performance quality expected under the normal conditions is assumed to be at 100% level for each activity (Khang and Myint, 1999).

7. Crash Quality Level for Each Activity $(\boldsymbol{Q_{ij}^C})\forall(i,j) \in A$
It is the reduced quality of each activity due to crashing activity. We assume that crashing activities may decrease the performance quality of activities. This parameter is estimated by DMs based on their previous experiences in similar projects.

8. Slope of Activity Cost Curve $(\boldsymbol{\alpha_{ij}})\forall(i,j) \in A$

$$\alpha_{ij} = \left|\frac{C_c - C_N}{D_{ij}^C - D_{ij}^N}\right|. \quad (7)$$

9. Slope of Activity Quality Curve $(\boldsymbol{q_{ij}})\forall(i,j) \in A$

$$q_{ij} = \left|\frac{Q_c - Q_N}{D_{ij}^C - D_{ij}^N}\right|. \quad (8)$$

OBJECTIVE FUNCTION OF TIME

$\widetilde{T}_{ij}$ is defined as a normally-distributed (Gaussian) random variable representing the actual normal (non-crashed) duration of activity (*i*, *j*), whose expected value is the normal duration, D^N_{ij}.

T_{ij} is used to represent the actual realized value of $\widetilde{T}_{ij}$. Given these actual values and without the possibility of crashing, one can find the start times at each node on each path (independently) using the formula:

$K_{jr} \geq K_{ir} + T_{ij}$, $\forall j = 2, ..., N$, $\forall r = 1, ..., r_j$, $\forall$ (9)
$(i, j) \in A$ on the r_j different paths from 1 to j

calculated sequentially in the order of the nodes on path *r* .

To allow for crashing, we define X_{ij} to be the decision variable representing the crashing percentage of activity (*i*, *j*), with $0 \leq X_{ij} \leq 1$, meaning the intended percentage of the normal duration that corresponds to the crashed actual duration of the activity (so, X_{ij} = 0.75 would mean that the activity duration is crashed by 25%). The start time for each node on each path then becomes:
$K_{jr} \geq K_{ir} + T_{ij}X_{ij}$, $\forall j = 2, ..., N$, $\forall r = 1, ..., r_j$, $\forall$
$(i, j) \in A$ on the r_j different paths from 1 to j.

The total project duration on each path is then given by:
$K_{Nr} \geq K_{ir} + T_{iN}X_{iN}$, $\forall r = 1, …, r_N$, for all (10)
$(i, N) \in A$ on the r_N different paths from 1 to N.

OBJECTIVE FUNCTION OF TOTAL COST

One of the most important objectives of project managers is to finish projects with the least costs in such a way that both duration and quality of the project are taken into account which is just what the time, cost and quality trade-off problem must solve. The objective is to minimize direct costs, so that indirect costs of the project are not considered in our model. Consequently, the cost function can be written as:

$$\min \alpha_{ij}\left(D_{ij}^N - T_{ij}X_{ij}\right); \forall (i,j) \in A. \quad (11)$$

OBJECTIVE FUNCTION OF QUALITY

In this paper, it was assumed that by crashing project activities, quality will also decrease. Thus, the objective function of quality can be written as:

$$\min q_{ij}\left(D_{ij}^N - T_{ij}X_{ij}\right); \forall (i,j) \in A. \quad (12)$$

In the equation above, q_{ij} is the slope of activity (i,j) for the quality curve, while the performance quality expected under normal conditions is assumed to be at 100% level for each activity (Khang and Myint, 1999). D_{ij}^N is the normal duration time of activity (i,j).

CONSTRAINTS

$D_{ij}^C \leq T_{ij}X_{ij}$
$\leq D\_ij\text{\^{}}N;\ \forall(i,j) \in A$ (13)
$0 \leq X_{ij} \leq 1\ \forall (i,j) \in A.$

Therefore, the model can be written as shown below:

$$Min\left\{\max_{r=1,\dots,r_N} K_{Nr}\right\}$$
$$\min \alpha_{ij}\left(D_{ij}^N - T_{ij}X_{ij}\right); \forall (i,j) \in A$$
$$K_{jr} \geq K_{ir} + T_{ij}X_{ij},\ \forall j = 2, ..., N, \forall r = 1, ..., r_j$$
$$\min q_{ij}\left(D_{ij}^N - T_{ij}X_{ij}\right); \forall (i,j) \in A \quad (14)$$

subject to:
$$D_{ij}^C \leq T_{ij}X_{ij} \leq D_{ij}^N\ \forall (i,j) \in A$$
$$0 \leq X_{ij} \leq 1\ \forall (i,j) \in A$$

CCCP EQUIVALENT MODEL

An example objective is explained below:

$$Min\{max_{r=1,\dots,r_N} K_{Nr}\}. \quad (15)$$

T_{ij} is a normal distributed variable. Assume that $g(\tilde{T}_{ij}) = \tilde{T}_{ij} X_{ij}$ is normally distributed and $E\big(g(\tilde{T}_{ij})\big)$, $Var\big(g(\tilde{T}_{ij})\big)$ are respectively the mean and the variance.

$$Prob\ (g(\tilde{T}_{ij}) \leq C) \geq 1 - \zeta; \quad (16)$$

where ζ is the threshold value of the objective. Then:

$$prob\left(\frac{g(\tilde{T}_{ij}) - E\big(g(\tilde{T}_{ij})\big)}{\sqrt{var\big(g(\tilde{T}_{ij})\big)}} \leq \frac{C - E\big(g(\tilde{T}_{ij})\big)}{\sqrt{var\big(g(\tilde{T}_{ij})\big)}}\right) \geq 1 - \zeta$$

$$\frac{C - E\left(g(\tilde{T}_{ij})\right)}{\sqrt{var\left(g(\tilde{T}_{ij})\right)}} \geq \varphi^{-1}(1-\zeta)$$

$$C \geq E\big(g(\tilde{T}_{ij})\big) + \varphi^{-1}(1-\zeta)\sqrt{var\big(g(\tilde{T}_{ij})\big)}\ .$$

Here, $(z) = P(N(0,1)) \leq Z$ is the standard normal cumulative distribution function.

By broadening (9) to become:
$K_{jr} \geq K_{ir} + T_{ij} X_{ij}$, $\forall j = 2, \ldots, N$, $\forall r = 1, \ldots, r_j$, $\forall\ (i,j) \in A$
on the r_j different paths from 1 to j , then its chance-constrained equivalent would be:

$K_{jr} \geq K_{ir} + E(\tilde{T}_{ij} X_{ij}) + \varphi^{-1}(1 - \zeta)\sqrt{var(\tilde{T}_{ij} X_{ij})}$,
$\forall j = 2, \ldots, N$, $\forall r = 1, \ldots, r_j$, $\forall\ (i,j) \in A$
on the r_j different paths from 1 to j. (17)

In order to transform the minmax problem to a standard linear program, we can use:

Min K
subject to:
$K \geq K_{Nr}$,$\forall r = 1, \ldots, r_N$.

So, the equivalent equation would be:

Min K

subject to:
$K \geq K_{Nr}$, $\forall r = 1, \ldots, r_N$
$K_{jr} \geq K_{ir} + E(\tilde{T}_{ij} X_{ij}) + \varphi^{-1}(1 - \zeta)$ √ (var(T˜_ij X_ij)),
$\forall j = 2, \ldots, N$, $\forall r = 1, \ldots, r_j$, $\forall\ (i, j) \in A$ on the r_j different paths from 1 to j. (18)

So, the equivalent CCCP model for TCQTP is:

$$min\ w_1\left(\sum_{n=1}^{|A|} \sigma_{1n}^{+}\right) + w_2\left(\sum_{n=1}^{|A|} \sigma_{2n}^{+}\right) + w_3 K$$

subject to:

$\propto_{ij}\left(D_{ij}^{N} - E(\tilde{T}_{ij} X_{ij}) + \varphi^{-1}(1 - \zeta)\sqrt{var(\tilde{T}_{ij} X_{ij})}\right) - \sigma_{1n}^{+} = 0, \forall (i,j) \in A$

$K_{jr} \geq K_{ir} + E(\tilde{T}_{ij} X_{ij}) + \varphi^{-1}(1 - \zeta)\sqrt{var(\tilde{T}_{ij} X_{ij})}$,
$\forall j = 2, \ldots, N$, $\forall r = 1, \ldots, r_j$, $\forall\ (i, j) \in A$ on the r_j different paths from 1 to j.

$K_{11} = 0$ P(6)

$K \geq K_{rN} \forall r = 1, 2, \ldots, r_N$

$q_{ij}\left(D_{ij}^{N} - E\big(T_{ij} X_{ij}\big) + \varphi^{-1}(1-\zeta)\sqrt{var\big(T_{ij} X_{ij}\big)}\right) - \sigma_{2n}^{+} = 0\ \forall (i,j) \in A$

$E\left(T_{ij} X_{ij} - \big(D_{ij}^{N}\big)\right) + \varphi^{-1}(1-\zeta) * \sqrt{var\left(T_{ij} X_{ij} - \big(D_{ij}^{N}\big)\right)} \leq 0\ \forall (i,j) \in A$

$E\left(T_{ij} X_{ij} - \big(D_{ij}^{C}\big)\right) + \varphi^{-1}(1-\zeta) * \sqrt{var\left(T_{ij} X_{ij} - \big(D_{ij}^{C}\big)\right)} \geq 0\ \forall (i,j) \in A$

$0 \leq X_{ij} \leq 1\ \forall (i,j) \in A$

$\sigma_{1n}^{+}, \sigma_{2n}^{+} \geq 0; \forall n = 1, \ldots, |A|$.

NUMERICAL EXAMPLE

Consider Figure 1 as a network activity with the following description: *n=7, i=1,2,..,5,r=3 paths, j=2,3,.., 6.*

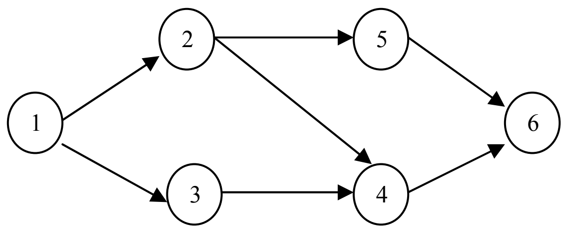

Figure (1): Network activity

Table 1. Project network information

Activity	D_{ij}^N	D_{ij}^C	C_{ij}^N	C_{ij}^C	Q_{ij}^C	α_{ij}	q_{ij}	$e(T_{ij})$	$v(T_{ij})$
(1-2)	15	10	150	200	95%	5	0.01	15	1
(1-3)	20	16	250	280	90%	7.5	0.025	20	4
(2-5)	30	25	500	600	80%	20	0.00625	30	9
(2-4)	30	23	400	490	80%	12.8	0.02857	30	4
(3-4)	45	30	550	700	90%	10	0.00666	45	16
(4-6)	10	8	200	220	95%	10	0.025	10	1
(5-6)	10	8	100	160	%85	30	0.075	10	1

The objective is to minimize project duration, cost and reduced quality due to crashing activities in a probabilistic environment. Objectives are weighted regarding their importance for the DMs. It was assumed that $\widetilde{T}_{ij}, \forall(i,\ j)\in A$ are normally distributed with known mean and variance. The software package Lingo was used to solve the mathematical program. Results are shown in Table 2. The model helps DMs analyze project time, cost and quality trade-offs under different uncertainty levels. By considering higher uncertainty levels, it is expected that activity duration gets higher.

Table 2. Model results

Project parameters	$\zeta = 0.01, w_{1,2}$=*0.1*,w_3=*0.8*	$\zeta = 0.05, w_{1,2}$=**0.2**, w_3=**0.6**
K_{61}, K_{62}, K_{63}	*55,55,55*	*55,55,75*
$\sigma_{11}^+ + \sigma_{12}^+ + \cdots + \sigma_{17}^+$	*835.5351*	*712.4734*
$\sigma_{21}^+ + \sigma_{22}^+ + \cdots + \sigma_{27}^+$	*1.166703*	*1.012149*
Project duration (K)	*55*	*75*
Total costs	$\sum C_{ij}^N$*+835.5351*	$\sum C_{ij}^N$*+712.4734*
Total quality	*(7-1.166703)/7=83%*	*(7-1.012149)/7=86%*

CONCLUSIONS AND FUTURE WORK

As previously mentioned, the importance of project scheduling and project time, cost and quality trade-off as techniques for improving project efficiency among managers is rising ever more. Since project scheduling is a multi-objective problem, a model was proposed with three objectives. These included minimizing project completion time and project direct costs in addition to maximizing the objective function of quality. Additionally, by taking into account the possibility of unpredicted delays occurring for various activities, the activity duration time was assumed to be a normally distributed variable in order to make a more realistic environment in our model. In this paper, a chance-constrained compromise programming approach was used for solving the multi-objective stochastic program. The chance-constrained compromise programming itself is a combination of compromise programming, chance-constrained programming and stochastic programming. The suggested model transforms a stochastic multi-objective program into a deterministic multi-objective program by using two statistical concepts: mean and variance approximations at a specific level of uncertainty.

Future research directions may include various extensions to the present work. For example, new objectives, such as maximization of NPV or project efficiency, or making any changes in the current objective functions could be considered. This model may change into an NP-hard problem if it is used in a large project with numerous activities. Thus, it would be a good case for researchers to develop a meta-heuristic approach to deal with such model. The proposed model is applicable to PERT and activity-on-arrow (AoA) networks which is rarely used. Developing a model that can be used for activity-on-node (AoN) networks would be another possibility for future research. In this article, the activity duration was evaluated as a normal variable. It is possible to analyze the uncertainty of the model under fuzzy situation or using other probability distributions.

APPENDIX

A the set of all activity arcs (i, j) A_r the set of all activity arcs on path r ζ_l the threshold values of the constraints that are specified by the DM. σ_1^+ the positive deviation between the achievement and the aspiration levels of objectives E the average expected value (given data) α_{ij} slope of activity cost curve $\alpha_{ij} = \left\lvert \frac{C_C - C_N}{D_{ij}^C - D_{ij}^N} \right\rvert$ D_{ij}^N normal duration of each activity D_{ij}^C crash duration of each activity K_{rn} Start time of the activity on path r coming out of node n K total project duration L total number of constraints	Mnumber of objectives P probability R number of paths to finish project q_{ij} slope of activity quality curve $q_{ij} = \left\lvert \frac{Q_C - Q_N}{D_{ij}^C - D_{ij}^N} \right\rvert$ $\tilde{T}_{ij}$ a normally distributed (Gaussian) random variable representing the actual normal (non-crashed) duration of activity (i, j), whose expected value is the normal duration, D^N_{ij} T_{ij} the actual realized value of $\tilde{T}_{ij}$ var the variance value (given data) (z) standard normal cumulative distribution function w weight of objective X_{ij} the decision variable representing possible crashing percentage of activity (i,j) and $0 \le X_{ij} \le 1$

Lingo Model

```
MODEL:
MIN = 0.1*(σ+_1n + σ+_2n;)+ 0.8*(K);
75-75*X12+2.33*5*X12-S1=0;
150-150*X13+2.33*15*X13-S2=0;
600-600*X25+2.33*60*X25-S3=0;
384-384*X24+2.33*25.6*X24-S4=0;
450-450*X34+2.33*40*X34-S5=0;
100-100*X46+2.33*10*X46-S6=0;
300-300*X56+2.33*30*X56-S7=0;
S1+S2+S3+S4+S5+S6+S7=σ+_1n;
0.15-0.15*X12+2.33*0.01*X12-R1=0;
0.5-0.5*X13+2.33*0.05*X13-R2=0;
0.1875-0.1875*X25+2.33*0.01875*X25-R3=0;
0.8625-0.8625*X24+2.33*0.05714*X24-R4=0;
0.2997-0.2997*X34+2.33*0.02664*X34-R5=0;
0.25-0.25*X46+2.33*0.025*X46-R6=0;
0.75-0.75*X56+2.33*0.075*X56-R7=0;
R1+R2+R3+R4+R5+R6+R7=σ+_2n;
15*X12-15+2.33*X12<=0;
20*X13-20+2.33*2*X13<=0;
30*X25-30+2.33*3*X25<=0;
30*X24-30+2.33*2*X24<=0;
45*X34-45+2.33*4*X34<=0;
10*X46-10+2.33*X46<=0;
10*X56-10+2.33*X56<=0;
15*X12-10+2.33*X12>=0;
20*X13-16+2.33*2*X13>=0;
30*X25-25+2.33*3*X25>=0;
30*X24-23+2.33*2*X24>=0;
45*X34-30+2.33*4*X34>=0;
10*X46-8+2.33*X46>=0;
10*X56-8+2.33*X56>=0;
K2>=15*X12+2.33*X12;
K3>=20*X13+2*2.33*X13;
K2+30*X25-K5+2.33*3*X25<=0;
K2+30*X24-K41+2.33*2*X24<=0;
K3+45*X34-K42+2.33*4*X34<=0;
K5+10*X56-K61+2.33*X56<=0;
K41+10*X46-K62+2.33*X46<=0;
K42+10*X46-K63+2.33*X46<=0;
K>=T61;
K>=T62;
K>=T63;
X12>0;
x12<=1;
x13>0;
x13<=1;
x25<=1;
x25>0;
x24>0;
x24<=1;
x34<=1;
X34>0;
X46>0;
X46<=1;
X56>0;
X56<=1;
END
```

Solution Values	
Variable	Value
σ^+_{1n}	835.5351
σ^+_{2n}	1.166703
K	55.00000
X12	0.8655511
X13	0.6488240
X25	0.8110300
X24	0.8655511
X34	0.5706922
X46	0.6488240
X56	0.8110300

REFERENCES

Azaran, A., and Tavakoli-Mogaddam, R. (2007). "Multi-objective time-cost trade-off in dynamic PERT networks using an interactive approach". European Journal of Operational Research, 180, 1186-1200.

Babu, A.J.G., and Suresh, N. (1996). "Project management with time, cost and quality considerations". Journal of Operational Research, 88, 320-327.

Bagherpour, M., Feylizadeh, M.R., and Cioffi, D.F. (2012). "Time, cost and quality trade-offs in material requirements planning using fuzzy multi-objective programming". Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, 226 (3), 560-564.

Ben Abdelaziz, F., Aouni, B., and El-Fayedh, R. (2007). "Multi-objective stochastic programming for portfolio selection". European Journal of Operational Research, 177, 1811-1823.

Charnes, A., and Cooper, W.W. (1959). "Chance-constrained programming". Management Science, 6 (1), 73-79.

Chassiakos, A.P, Samaras, C.I., and Theodorakopoulos, D.D. (2000). "An integer programming method for CPM time-cost analysis". Comput. Model Eng. Sci., 4 (1), 8-18.

Coskunoglu, O. (1984). "Optimal probabilistic compression of PERT network". ASCE J. Construction Eng. Manag.,110 (4), 437-446.

Dodin, B.N. (1985). "Bounding the project completion time distribution in PERT networks". Operation Research, 33, 862-881.

Freeman, R.J. (1960). "A generalized PERT". Operation Research, 8 (2), 281.

Hendrickson, C., Zozayz-Gorostiza, D., Rehak, C., Baracco-Miller, E., and Lim, P. (1987). "An expert system for construction planning". Journal of Computing in Civil Engineering, 1 (4), 253-269.

Kelley, J.E. Jr., and Walker, M.R. (1959). "Critical path planning and scheduling". Proc. Eastern Joint Comp. Conf., 16, 160-172.

Kelley, J.E. Jr. (1961). "Critical path planning and scheduling: mathematical basis". Operations Research, 9 (3), 296-320.

Khaled, El-Rayes, and Amr Kandil. (2005). "Time-cost-quality trade-off analysis for highway construction". Journal of Construction Engineering and Management, 131 (4).

Khang, D.B., and Myint, Y.M. (1999). "Time, cost and quality trade-off in project management: a case study". International Journal of Project Management, 17 (4), 249-256.

Kim, J., Kang, C., and Hwang, I. (2012). "A practical approach to project scheduling: considering the potential quality loss cost in the time–cost trade-off problem". International Journal of Project Management, 30 (2), 264-272.

Liberatore, Matthew J., and Bruce Pollack-Johnson. "Improving project management decision making by modeling quality, time and cost continuously".

Liu, L., Burns, S.A., and Fenge, W. (1995). "Construuction time-cost trade-off analysis using LP/IP hybrid method". ASCE J. Constr. Eng. Manag., 121 (4), 446-454.

Mokhtari, H., Aghaie, A., Rahimi, J., and Mozdgir, A. (2010). The International Journal of Advanced Manufacturing Technology, 50, 5-8.

Nabipoor Afruzi, E. et al. (2013). "A multi-mode resource-constrained discrete time-cost trade-off problem solving using an adjusted fuzzy dominance genetic algorithm". Scientia Iranica, 1-14.

Patterson, James H., and Huber, Walter D. (1974). "A horizon-varying zero-one approach to project scheduling". Manag .Sc., 20 (6), 900-998.

Perera. (1980). "Linear programming solution to network compression". ASCE J., Const Div., 106, 315-326.

Permachandra, I.M. (1993). "A goal programming model for activity crashing in project network". International Journal of Project Management, 13 (1), 167-172.

Pollack-Johnson, Bruce, and Matthew J. Liberatore. (2006). "Incorporating quality considerations into project time-cost trade-off analysis and decision making". IEEE Transactions on Eng. Manag., 53 (4), 534-542.

Prékopa, A. (1995). "Stochastic programming". London: Kluwer Academic Publishers, 199.

Ruszczynski, A., and Shapiro, A. (2003). "Stochastic programming models". Handbooks in OR and MS, 10, Elsevier Science B.V.

Salmasnia, A., Mokhtari, H., and Nakhai, I. (2012). "A robust scheduling of projects with time, cost and quality considerations". Int. J. Adv. Manuf. Technol., 60, 631-642.

Seifi, M., and Tavakoli-Mogaddam, R. (2008). "A new bi-objective model for a multi-mode resource constrained project scheduling problem with discounted cash flows and four payment models". International Journal of Engineering, 21 (4).

Shatub, A., Bard, J., and Sholomo, G. (1994). "Project management engineering, technology and implementation", Prentice-Hall, 382-392.

Sonmez, R., and Bettemir, Ö.H. (2012). "A hybrid genetic algorithm for the discrete time-cost trade-off problem". Expert Systems with Applications, 39, 11428-11434.

Szmerekovsky, Joseph G., and Prahalad Venkateshan. (2012). "An integer programming formulation for the project scheduling problem with irregular time–cost trade-offs". Computers and Operations Research, 39 (7), 1402-1410.

Tareghian, H.R., and Taheri, S.H. (2006). "On the discrete time, cost and quality trade-off problem". Applied Mathematics and Computation, 181, 1305-1312.

Zeleny, M. (1974). "A concept of compromise solutions and the method of the displaced ideal". Computers and Operations Research, 1, 479-484.