

## Damage Identification in Truss Structures Using Finite Element Model Updating and Imperialist Competitive Algorithm

*Hosein Ghaffarzadeh<sup>1)</sup> and Farzad Raeisi<sup>2)</sup>*

<sup>1)</sup> Associate Professor, University of Tabriz, Iran. E-Mail: ghaffar@tabrizu.ac.ir

<sup>2)</sup> Engineer, University of Tabriz, Iran

### ABSTRACT

In this paper, Finite Element Model (FEM) updating based on a new heuristic algorithm is proposed for damage identification in truss structures. This method updates the dynamic properties of the damaged and undamaged state of a structure to identify the presence, location and magnitude of the damage in structural members. Imperialist Competitive Algorithm (ICA), which is one of the most efficient heuristic methods, is used to minimize the objective function which is based on dynamic properties of the structure. Damage in structures is caused by the reduction in stiffness of specific members, especially in Young's modulus. The capability and efficiency of this method to identify damage using frequencies and mode shapes are demonstrated by means of several numerical examples. Results show the superiority and effectiveness of the proposed method.

**KEYWORDS:** Damage identification, Modal data, Finite element model (FEM) updating, Imperialist competitive algorithm.

### INTRODUCTION

Most of structures experience deterioration and damage during their lifetime. The various reasons of damage in structures include manufacturing processes, fatigue failure, buckling,... etc. Sometimes, the presence and location of the damage can be specified by visual inspection. But, this method has some weak points in assessing the quantity of the damage and even in detecting it, especially when the damage is inside the structure and is not visible from outside. Therefore, an effective and appropriate approach in damage identification will be a valuable tool in the determination of damage in structural members.

During the past few decades, a variety of non-destructive methods have been carried out on damage

detection in structures. Among the most frequent methods are modal-based approaches based on the variation of modal data including natural frequencies and mode shapes. Damage causes a change in fundamental properties of a structure, mainly in stiffness and damping at the damaged location. This change will result in variation in natural frequencies, mode shapes and other dynamic characteristics. Several researchers have used dynamic properties to identify damages. Pandey et al. (1991) showed that absolute changes in the curvature mode shapes are localized in the region of damage and hence can be used to detect damage in a structure. The change in the curvature mode shapes increases with increasing the size of damage. This parameter can be used to obtain the amount of damage in the structure. Cawley and Adams (1979) used changes in natural frequencies to identify damage in composite materials. To compute the ratio

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between frequency shifts for two modes, they regarded a grid between likely damage points and created an error term that related measured frequency shifts to those predicted by a model based on local stiffness reduction. Farrar et al. (1994) implemented the shifts in natural frequencies to identify damage on an I-40 bridge and noted that shifts in the natural frequencies were not adequate for detecting the damage of small faults. To improve the accuracy of the natural frequency technique, it was found more practical to carry out the experiment in controlled environments where the uncertainties of measurements were comparatively low. Recently, FEM updating, which determines the damage by updating the measured data, achieved through experimental data and those obtained from finite element (FE) modeling, has become a promising method in damage identification. Jaishi and Ren (2006) applied a sensitivity-based FEM updating for damage detection. They used an objective function consisting of modal flexibility residual and its gradient was derived. The updated parameters were used as a damage indicator. They verified the modal flexibility which is sensitive to damage. The proposed procedure is promising for damage detection. Wu and Li (2006) investigated a two-stage eigen-sensitivity-based FEM updating for damage detection of a steel structure. In the first stage, the weighted least squares and Bayesian estimation methods are adopted for identification of the connection stiffness of beam-column joints. Then, the damage detection is conducted *via* the FEM updating for detecting damaged braces. Teughels and Roeck (2005) applied damage functions to approximate the stiffness distribution in order to reduce the number of unknowns. They used Gauss-Newton method for local optimization and the method of Coupled Local Minimizers (CLM) for global optimization of functions.

In recent years, the application of the heuristic optimization methods, also known as stochastic or intelligent techniques with promising performance, has been successfully used for damage detection and FEM updating. Jafarkhani and Masri (2011) studied the

performance of an evolutionary strategy based on covariance matrix adaption in FEM updating for damage detection of a quarter-scale two-span reinforced concrete bridge. They concluded that the applied FEM updating could accurately detect, localize and quantify the damage in the tested bridge columns. A multi-stage scheme for damage detection of large structures based on experimental modal data and FEM updating methods was developed by Perera and Ruiz (2008). In the first stage, occurrence and approximate location of damage is performed by using damage functions in order to decrease the number of parameters to be updated. In the second stage, the specific damaged members and damage extent are identified considering only the members belonging to the regions detected as damage in the first stage. To improve identification, the optimization procedure was formulated in a multiobjective context solved by using a genetic algorithm. Levin and Lieven (1998) introduced genetic algorithms and simulated-annealing methods for FEM updating. These algorithms were tested on several objective functions for model updating in both modal and frequency domains, using simulated data. An adaptive real-parameter hybrid of simulated annealing and genetic algorithm to detect damage occurrence in beam-type structures was implemented by He and Hwang (2006). Begambre and Laier (2009) presented a hybrid of the particle-swarm optimization (PSO) and the Nelder-Mead simplex method for structural damage identification. This method was used to minimize the objective function that used frequency response functions. The hybrid method directed the PSO parameters using the Nelder-Mead simplex method. This ensured that the convergence of the PSO method is independent of the heuristic constants and that the stability and confidence of the method were improved. They found that this hybrid method performs better than the simulated annealing and PSO. In many practical conditions, more than one objective function may be optimized simultaneously. Marwala and Heyns (1998) introduced a multiple-criterion method (MCM) that

minimized the Euclidian norm of the error based on modal properties and frequency-response function data. They applied this method for damage detection in structures. They found that the multiple-criterion updating method predicted well the presence, the position and the extent of damage. Recently, the global search heuristic method; Imperialistic Competitive Algorithm (ICA) which is inspired by imperialistic competition and human's socio-political evolution was proposed (Atashpaz-Gargari and Lucas, 2007; Atashpaz-Gargari et al., 2008). Similar to other evolutionary algorithms, the gradient of the function is not necessary in this optimization process.

In this study, a simple, but effective, method of FEM updating based on ICA is introduced for damage identification in structures. The formulation of the optimization of the objective function is based on dynamic properties (frequencies and mode shapes) of damaged and undamaged structures. FEM updating is essentially an optimization method. Its objective is to minimize the distance between the FE predicted data and the measured data. ICA, as a new efficient heuristic method, is used to minimize the objective function. The damage in the structure is considered by a reduction in the stiffness at the damaged location for the evaluation of the objective function. Numerical examples of plane truss and space truss structures are implemented to validate the accuracy of the current model compared to experimental data obtained from literature.

#### Imperialist Competitive Algorithm (ICA)

ICA is a new progressive algorithm for optimization, simulating the social political process of imperialism and imperialistic competition. This algorithm starts with an initial population in which each individual is called a country. Some of the best countries, countries with lower cost, are selected to be the imperialists and the rest form the colonies of these imperialists. In this algorithm, the more powerful the imperialists, the more colonies they have. The power of each empire, the counterpart of fitness value, is

inversely proportional to its cost. The imperialists and their colonies form some empires.

The objective of optimization is to reach an optimal solution in terms of the variables of the problem. In ICA, each country is formed of an array of variable values and the related cost of a country is found by the evaluation of the cost function  $f_{cost}$  of the corresponding variables considering the related objective function. Total number of initial countries is labeled as  $N_{country}$ , and the number of the most powerful countries to form the empires is set to  $N_{imp}$ . The remaining initial countries will be the colonies each of which belongs to an empire. The number of colonies of an empire should be directly proportional to its power. To have a proportional division of colonies among imperialists, the normalized cost of an imperialist is defined as:

$$C_n = c_n - \max_i \{c_i\}; \quad (1)$$

where  $c_n$  is the cost of the  $n^{\text{th}}$  imperialist and  $C_n$  is its normalized cost. The initial colonies are divided among empires based on their power or normalized cost, and for the  $n^{\text{th}}$  empire it follows that:

$$NC_n = \text{Round} \left( \left| \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \right| \cdot N_{col} \right); \quad (2)$$

where *Round* is the function that rounds a number to the nearest integer,  $NC_n$  is the initial number of colonies corresponding to the  $n^{\text{th}}$  empire and  $N_{col}$  is the number of all colonies. To divide the colonies, for each imperialist we randomly choose  $NC_c$  of the colonies and give them to it. These colonies along with the  $n^{\text{th}}$  imperialist form the  $n^{\text{th}}$  empire.

In ICA algorithm, by moving all the colonies toward the imperialist, the assimilation policy can be modeled, which is pursued by some of former imperialist states. This movement is illustrated in Figure 1(a) in which a colony moves toward the imperialist by  $x$  which is a random variable with uniform distribution between 0 and  $\psi \times d$ ; where  $\psi$  is a number greater than 1 and  $d$  is the distance

between colony and imperialist.  $\psi > 1$  causes the colonies to get closer to the imperialist state from both sides.

To increase the searching around the imperialist, a random amount of deviation is added to the direction of movement. Figure 1(b) shows the new direction in which  $\theta$  is a random number with uniform distribution

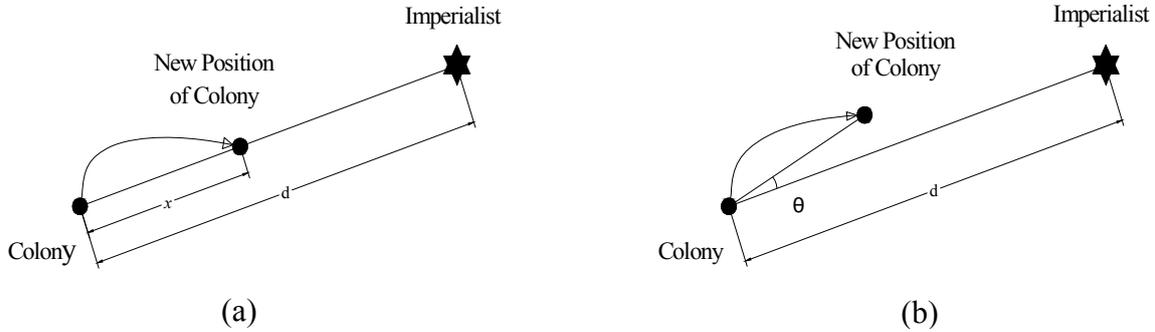


Figure (1): Movement of colonies to their new locations: (a) toward their relevant imperialist (b) in randomly deviated directions

During a colony's movement toward an imperialist, if the colony reaches a better position than that of the corresponding imperialist, the imperialist replaces the colony and the new location with lower cost is allotted to the imperialist. Then, the other colonies move toward this new position.

Imperialistic competition is another strategy utilized in the ICA methodology. All empires try to take the possession of colonies of other empires and control them. The imperialistic competition gradually reduces the power of weaker empires and increases the power of more powerful ones. The imperialistic

$$TC_n = Cost(imperialist_n) + \xi \times mean\{Cost(colonies\ of\ empire_n)\} \quad (3)$$

where  $TC_n$  is the total cost of the  $n^{th}$  empire and  $\xi$  is a positive number which is considered to be less than 1. A small value for  $\xi$  causes the total power of the empire to be determined by just the imperialist, while increasing it will add the role of the colonies in determining the total power of the corresponding empire. Similar to Eq.1, the normalized total cost is defined as:

$$NTC_n = TC_n - max_i\{TC_i\} \quad (4)$$

between  $-\gamma$  and  $+\gamma$ , where  $\gamma$  is a parameter that adjusts the deviation from the original direction. In most of the implementations, a value of about 2 for  $\psi$  and a value of about  $\frac{\pi}{4}$  (rad) for  $\gamma$  result in good convergence of the countries to the global minimum.

competition is modeled by just picking some (usually one) of the weakest colonies of the weakest empires and making a competition among all empires to possess these colonies. Based on their total power, in this competition, each of the empires will have a likelihood of taking possession of the mentioned colonies.

Total power of an empire is mainly affected by the power of imperialist country. But, the power of the colonies of an empire has an effect, albeit negligible, on the total power of that empire. This fact is modeled by defining the total cost as:

where  $NTC_n$  is the normalized total cost of the  $n^{th}$  empire. Having the normalized total cost, the possession probability of each empire is evaluated by:

$$P_n = \left| \frac{NTC_n}{\sum_{i=1}^{N_{imp}} NTC_i} \right| \quad (5)$$

When a powerless empire loses all its colonies, it is regarded as a collapsed one. Hereby, corresponding colonies will be distributed among the other empires in

the model implementation. Moving colonies toward imperialists is continued and imperialistic competition and implementations are performed during the search process. When the number of iterations reaches a pre-defined value, the search process is stopped.

### Damage Formulation

Modal properties; i.e., natural frequencies and mode shapes, which are obtained by solving eigenvalue problem, can be used as a basis for FEM updating and damage assessment.

Equation of motion in an undamped or lightly damped system is described through the following expression:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\}, \quad (6)$$

In which  $[M]$  and  $[K]$  are mass and stiffness matrices, respectively, and the vector  $\{u\}$  is the displacement. Considering Eq.6, the eigenvalue equation is associated as follows:

$$(K - \lambda_i M)\varphi_i = 0 \quad (7)$$

where  $\lambda_i$  and  $\varphi_i$  are eigenvalues and mode shapes, respectively. The existence of damage in the structural members affects stiffness and probably mass matrices. Therefore, in order to study the damage in stiffness, the eigenvalue equation will be expressed as:

$$((K + \Delta K) - (\lambda_i + \Delta\lambda_i)M)(\varphi_i + \Delta\varphi_i) = 0 \quad (8)$$

$\Delta K$  is the matrix which includes the damage parameters in the global form:

$$\Delta K = -\sum_{i=1}^m \beta_i [k]_i; \quad (9)$$

where  $\beta_i$  is the vector including the dimensionless value of damage in each of  $m$  members which times  $[k]_i$  is the local stiffness matrix of each element. The term  $\beta_i$  can be introduced by reduction in stiffness mainly in Young's modulus at the damaged location without any change in mass matrices of the structure.

These values vary from 0 to 1, in which 0 and 1 indicate undamaged and damaged state, respectively. Therefore, comparison between damaged and undamaged natural frequencies and mode shapes will lead to damage identification.

Since damage assessment is an inverse problem and can probably be ill-posed, in order to overcome this limitation the following equation will be used as an objective function (Marvala, 2010; Friswell et al., 1995):

$$E = \sum_{i=1}^N \eta_i \left( \frac{\omega_i^m - \omega_i^{calc}}{\omega_i^m} \right)^2 + \varpi \sum_{i=1}^N [1 - \text{diag}(\text{MAC}(\varphi_i^{calc}, \varphi_i^m))] \quad (10)$$

where  $m$  indicates a measured variable,  $calc$  indicates a calculated variable,  $\omega_i$  is the  $i^{\text{th}}$  natural frequency,  $\varphi_i$  is the  $i^{\text{th}}$  mode shape and  $N$  is the number of measured modes.  $\eta_i$  is the weighting factor that measures the relative distance between the initial estimated natural frequencies for mode  $i$  and the target frequency of the same mode,  $\varpi$  is the weighting function on the mode shapes and MAC is the modal assurance criterion for more correlation between the mode shapes of the measured and updated finite element models:

$$MAC = \frac{|\varphi_i^{mT} \varphi_i^{calc}|^2}{\left( \varphi_i^{mT} \varphi_i^m \right) \left( \varphi_i^{calcT} \varphi_i^{calc} \right)}. \quad (11)$$

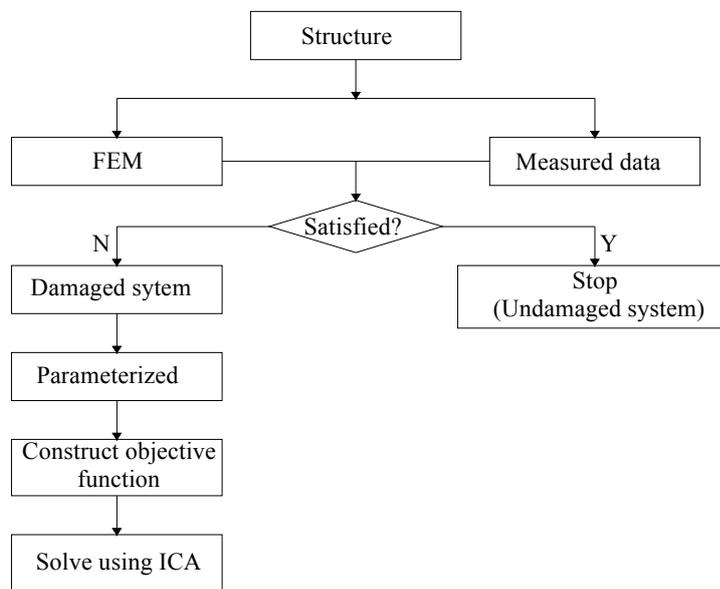
Practically, the parameters associated to measured variables are obtained by experimental data. However, in this study these data are obtained numerically from eigen solution of assumed damaged state. On the other hand, parameters related to calculated variables are implemented in the objective function dependent on  $\beta_i$  as the damage index:

$$E = E(\beta_1, \beta_2, \dots, \beta_n) \quad , \quad 0 \leq \beta_i \leq 1 \quad (12)$$

**PROPOSED METHOD**

The outline of the proposed method to identify damage and its location in an entire structure is demonstrated in Figure 2. Modal properties of the structure including frequencies and mode shapes are calculated using an FE model of the structure for undamaged and damaged states. Since the damage is involved by damage index,  $\beta$ , the constructed FE model of the damaged state will depend on this parameter. Finite element model updating process is implemented based on the objective function defined

through equation 10 for updating the data to achieve damage index. As noted, damage is modeled by change in stiffness matrix of an element and linear finite element analysis is used to solve eigenvalue problem in each stage of stiffness reduction. In this stage, ICA is applied as a powerful optimization method to obtain an optimal solution in terms of the variable of the objective function. By solving the optimization problem, the proper magnitude of the damage index is reached out. The magnitude of damage index shows damage in the members of the structure.



**Figure (2): Outline of finite element model updating in damage identification based on ICA**

**Numerical Analysis**

In this section, to show the effectiveness and efficiency of the proposed strategy in damage identification, two well-known truss structures are considered and the method is tested. The models include:

- A nine-bar plane truss subjected to one damaged member.
- A twenty five-bar space truss subjected to two damaged members.

**Plane Truss with One Damaged Member**

The first example is a plane truss including nine bars. This is a standard structure used previously by many researchers such as Kwon and Bang (2000). Figure 3 shows the geometry of the nine-bar truss structure. The material properties and cross-sectional area of the truss are  $E=200\text{GPa}$ ,  $\rho = 7860 \frac{\text{kg}}{\text{m}^3}$  and

$A=0.0025 \text{m}^2$ . Finite element modeling of the truss was constructed and the associated eigenvalue problem of the truss was solved. The accuracy of the model is

compared with the result obtained by Kwon and Bang (2000) and tabulated in Table 1.

After insuring the accuracy of FE modeling to investigate the efficiency of the proposed method, damage is considered in bar number 2 by a given reduction in the member stiffness. FE modeling of the damaged state is reconstructed and FEM updating

developed based on ICA optimization process as shown in the flowchart of Figure 2. After trying sufficient iterations, the result of frequencies updating for the damaged truss is tabulated in the fourth column of Table 1.

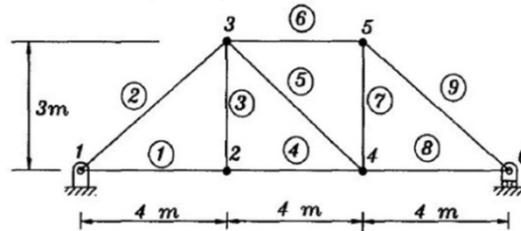


Figure (3): Geometry of plane truss

Table 1. Natural frequencies (rad/sec) of damaged and undamaged structures

Mode	Undamaged state [Kwon and Bang]	Undamaged state [Present study]	70% damage at element 2
1	240.9	240.87	206.6
2	467.9	467.94	377.37
3	739.8	739.85	628.55
4	1243	1243.4	1210.4
5	1633	1633.4	1601.6
6	-	2102.2	2102.1
7	-	2180.1	2134
8	-	2310.1	2308.6
9	-	2802.1	2797.6

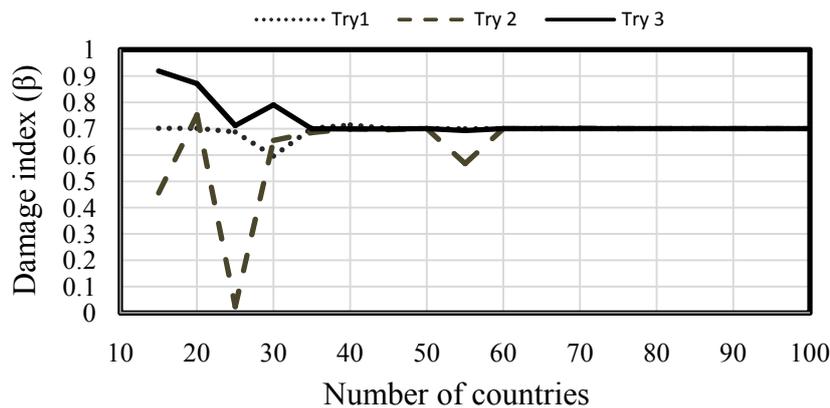


Figure (4): Variation of damage index relative to number of countries in ICA

In this study, to investigate the sensitivity of the process, the effect of ICA parameters including the number of countries ( $N_{country}$ ), the number of initial imperialist ( $N_{imp}$ ) and the number of decades ( $N_{dec}$ ) are investigated. After investigation on each parameter separately, the best parameters have been obtained. Figure 4 represents the effect of ( $N_{country}$ ) regarding  $N_{imp} = 6$  and  $N_{dec} = 20$ . This figure is derived from the optimization of objective function introduced in Eq.10 which is based on the damage index ( $\beta$ ). As shown in the mentioned figure, the damage state is

equal to 0.70 at element 2. Moreover, in order to represent the random performance of ICA algorithm for best value ( $\beta$ ), three trials are carried out.

It is observed that the value of  $\beta$  varies up to  $N_{country} = 60$  and its value reaches the constant state which is equal to the applied damage value  $\beta=0.7$ .

Figure 5 is related to the variation of number of decades in which ICA parameters are assumed as  $N_{country} = 25$  and  $N_{imp} = 5$ . Similar to Figure 4, the randomness behavior can be observed. In this manner, the value of constant and stable limit is 20. By increasing  $N_{dec}$ , the values will be constant.

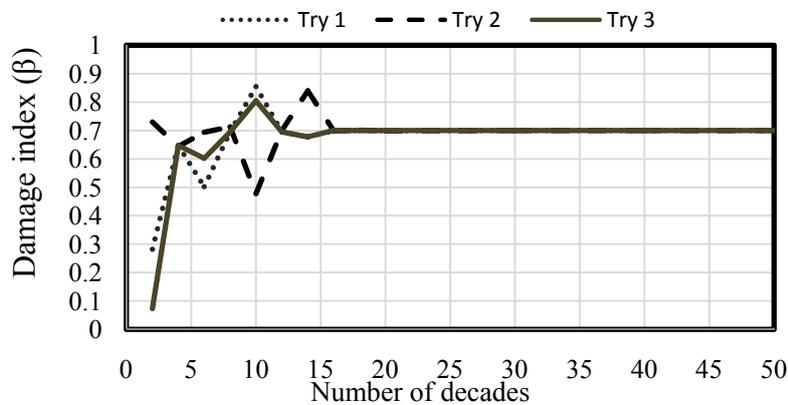


Figure (5): Variation of damage index relative to number of decades in ICA

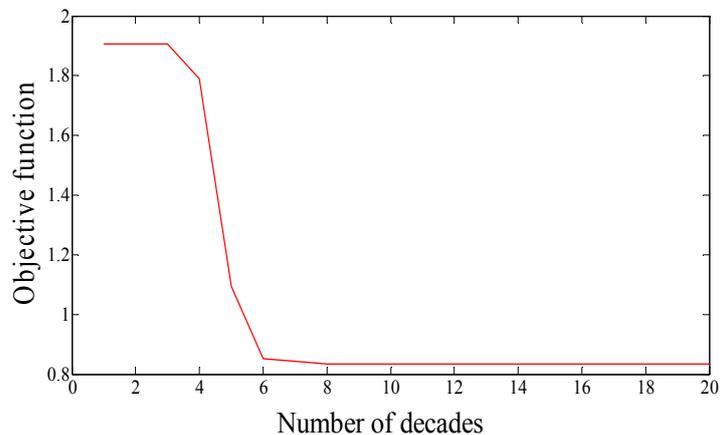


Figure (6): Optimization process for plane truss

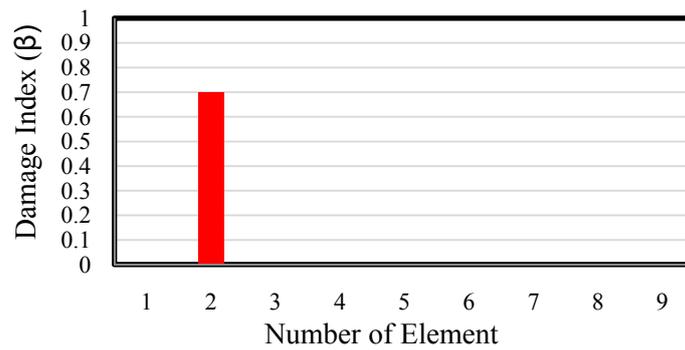


Figure (7): Damage assessment based on ICA algorithm in plane truss

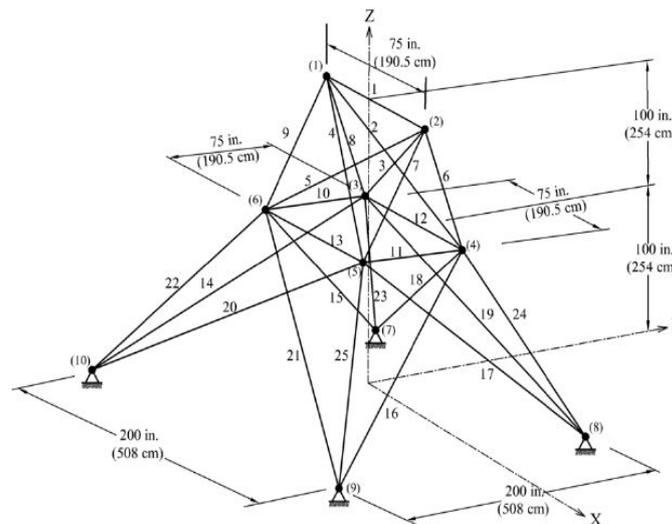


Figure (8): Geometry of space truss

Table 2. Natural frequencies (rad/sec) of damaged and undamaged structures

Mode	Undamaged state [Present study]	30% damage at element 6 + 50% damage at element 22
1	374.86	345.79
2	396.72	388
3	485.39	481.2
4	639.89	632.36
5	650.94	647.83
6	670.07	660.89

Therefore, in order to determine the extent of damage and its related location in the plane truss, the ICA parameters are considered as  $N_{country} = 35$ ,  $N_{imp} = 5$  and  $N_{dec} = 20$ . The results show that these parameters are quite sufficient to damage assessment

including detection, localization and quantification. Figure 6 shows that the objective function is minimized by ICA and the result of this optimization is illustrated in Figure 7.

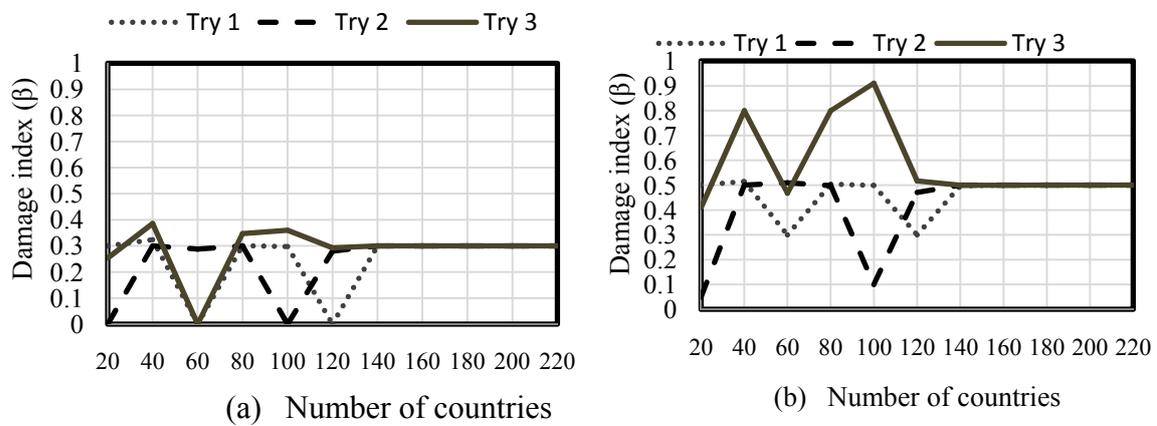
**Space Truss with Two Damaged Members**

As a second example, a 25-bar-space truss is considered to demonstrate the efficiency and robustness of the method in damage identification. The geometry of the truss is shown in Figure 8. The material properties and cross-section of members for this model are the same as those considered for the plane truss. The damage state has been simulated by reducing the stiffness of the 6<sup>th</sup> and 22<sup>nd</sup> elements by 30% and 50%, respectively. Table 2 shows the natural frequencies of the damaged and undamaged states.

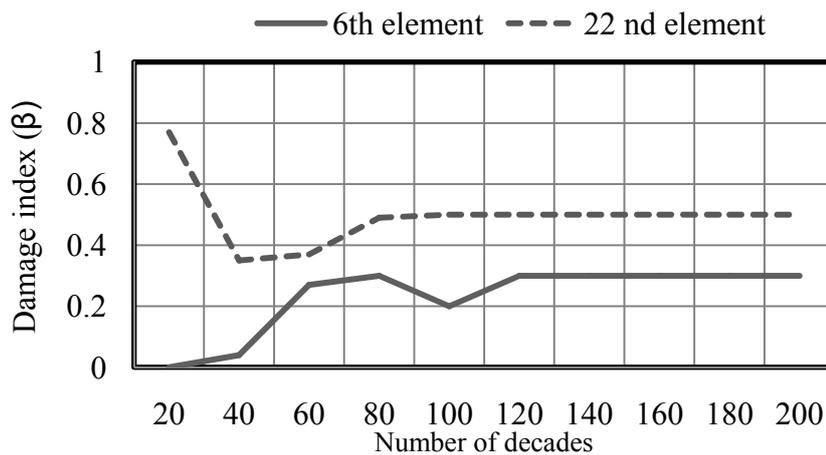
Similar to plane truss, in order to implement ICA and investigate the effect of each ICA parameter,

$N_{imp} = 8$  and  $N_{dec} = 100$  are considered for a survey on the number of countries. Figure 9 represents the variation of damage index relative to number of countries. In order to represent the random behavior of ICA, three tries are carried out. The stable limit which leads to the constant value for  $N_{country}$  is 160 in all three tries and both 6<sup>th</sup> and 22<sup>nd</sup> elements.

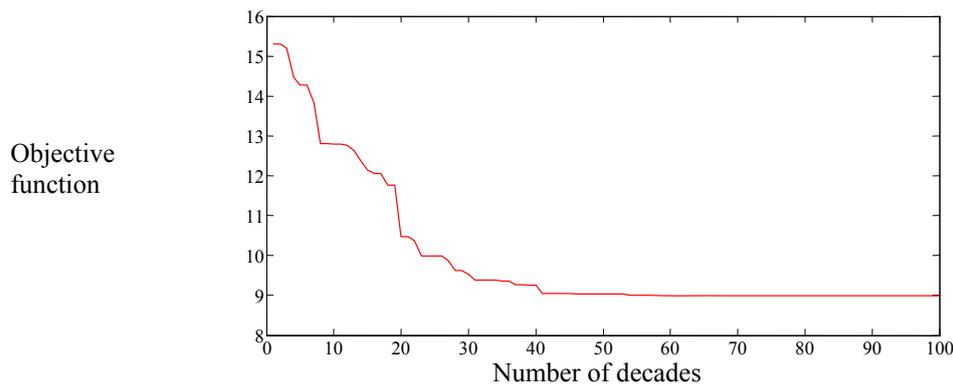
For studying the effect of the number of decades,  $N_{imp}$  and  $N_{country}$  are considered as 10 and 100, respectively. Figure 10 shows that after  $N_{dec} = 120$ , the value of  $\beta$  will be stable and constant.



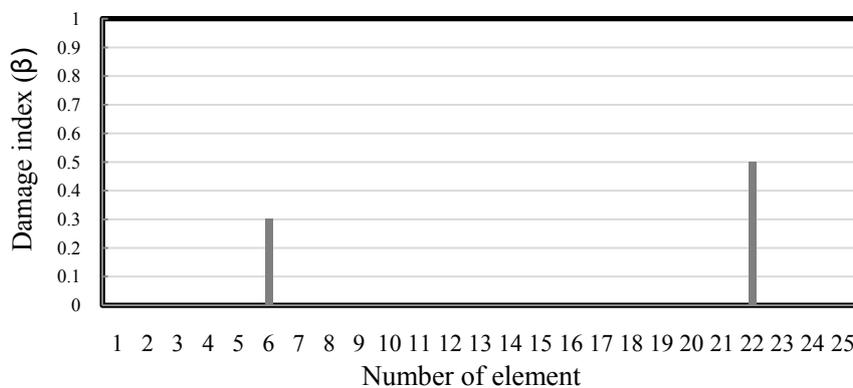
**Figure (9): Variation of damage index relative to number of countries in ICA, a) 6<sup>th</sup> element, b) 22<sup>nd</sup> element**



**Figure (10): Variation of damage index relative to number of decades in ICA**



**Figure (11): Optimization process for space truss**



**Figure (12): Damage assessment based on ICA algorithm in space truss**

In order to determine the damage index and its related location in this space truss, the ICA parameters are considered as  $N_{country} = 120$ ,  $N_{imp} = 9$  and  $N_{dec} = 100$ . Results show that these parameters are quite sufficient to detect the damaged members and their percentages of damage. Figure 11 shows that the objective function is minimized by ICA and the result of this optimization is illustrated in Figure 12.

## CONCLUSIONS

In this study, we implemented imperialist competitive algorithm (ICA) as a new heuristic approach to damage identification of structures based on finite element model updating method. With regard to the fact that finite element model updating is essentially the optimization process, ICA was

introduced as a simple and robust methodology for damage assessment. The efficiency of the proposed method is validated by detecting the pre-defined damages in two well-known and benchmark truss structures including 9-bar plane truss and 25-bar space truss. According to the objective function which was constructed by regarding the ill-posed condition in inverse problems, the optimization using ICA was carried out to decrease the discrepancies between the modal data of the damaged and undamaged structures. The results showed that reduction in stiffness of the elements in both structures causes slight changes in their modal properties. After trying sufficient iterations in the objective function by using ICA, the number of countries was selected respectively as 35 and 120 for plane and space trusses, which was sufficient to

identify the magnitude and the location of damages in the members. The obtained results showed that the

proposed method is a viable method for the detection and estimation of damages in structures.

## REFERENCES

- Atashpaz-Gargari, E., and Lucas, C. (2007). "Imperialist competitive algorithm: an algorithm for optimization inspired by imperialistic competition." In: IEEE Congress on Evolutionary Computation, Singapore 27, 4661-4667.
- Atashpaz-Gargari, E., Hashemzadeh, F., Rajabioun, R., and Lucas, C. (2008). "Colonial competitive algorithm: a novel approach for PID controller design in MIMO distillation column process." *International Journal of Intelligent Computing and Cybernetics*, 1 (3), 1337-1355.
- Begambre, O., and Laier, J.E. (2009). "A hybrid particle swarm optimization-simplex algorithm (PSOS) for structural damage identification." *Advances in Engineering Software*, 40, 883-891.
- Cawley, P., and Adams, R.D. (1979). "The location of defects from measurements of natural frequencies." *Journal of Strain Analysis*, 14, 49-57.
- Farrar, C.R., Baker, W.E., Bell, T.M., Cone, K.M., Darling, T.W., Duffey, T.A., Eklundand, A., and Migliori A. (1994). "Dynamic characteristics and damage detection in the I-40 bridge over the Rio Grande." Los Alamos National Laboratory Report LA-12767-MS.
- Friswell, M.I., and Mottershead, J.E. (1995). "Finite element model updating in structural dynamics". Kluwer Academic Publishers Group, Norwell.
- He, R., and Hwang, S. (2006). "Damage detection by an adaptive real-parameter simulated annealing genetic algorithm." *Computers and Structures*, 84, 2231-2243.
- Jafarkhani, R., and Masri, S.F. (2011). "Finite element model updating using evolutionary strategy for damage detection." *Computer-Aided Civil and Infrastructure Engineering*, 26, 207-224.
- Jaishi, B., and Ren, W.X. (2006). "Damage detection by finite element model updating using modal flexibility residual." *Journal of Sound and Vibration*, 290, 369-387.
- Kwon, Y.W., and Bang, H. (2000). "The finite element method using matlab". CRC Press.
- Levin, R.I., and Lieven, N.A.J. (1998). "Dynamic finite element model updating using simulated annealing and genetic algorithms." *Mechanical Systems and Signal Processing*, 12, 91-120.
- Marwala, T. (2010). "Finite-element-model updating using computational intelligence techniques: applications to structural dynamics". Springer Verlag, London.
- Marwala, T., and Heyns, P.S. (1998). "A multiple criterion method for detecting damage on structures." *AIAA Journal*, 195, 1494-1501.
- Pandey, A.K., Biswas, M., and Samman, M.M. (1991). "Damage detection from changes in curvature mode shapes". *Journal of Sound and Vibration*, 145 (2), 321-332.
- Perera, R., and Ruiz, A. (2008). "A multi-stage FE Updating procedure for damage identification in large-scale structures based on multi-objective evolutionary optimization." *Mechanical Systems and Signal Processing*, 22, 970-991.
- Teughels, A., and De Roeck, G. (2005). "Damage detection and parameter identification by finite element model updating." *Archives of Computational Methods in Engineering*, 12, 123-164.
- Wu, J.R., and Li, Q.S. (2006). "Structural parameter identification and damage detection for a steel structures using a two-stage finite element model updating method". *Journal of Constructional Steel Research*, 62, 231-239.