Digital Image Processing Based System for the Characterization of Coarse Aggregates

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ABSTRACT

Aggregate characteristics have a significant effect on the properties of concrete in the fresh state and the hardened state. They also influence the quantity of cement paste required to fill the voids between aggregate particles. The manual methods suggested for the measurement of aggregate characteristics are laborious, time consuming and approximate. This paper presents the development of a Digital Image Processing (DIP) based system for the measurement of sphericity, shape factor, elongation ratio and flatness ratio of coarse aggregate particles. The system is calibrated using standard objects such as marbles, coins and then used for the measurement of coarse aggregate particles having varied characteristics. Samples of rounded gravels and crushed aggregates from different crushers are considered for the study. The results indicated that the system can be used for the accurate measurement of aggregate characteristics.

KEYWORDS: Aggregate, Shape, Surface texture, Particle packing, Digital Image Processing (DIP).

INTRODUCTION

Concrete is a widely used construction material. Aggregates occupy maximum volume of concrete and their characteristics such as size, grading and shape have significant influence on properties of concrete in fresh state and hardened state. They also provide dimensional stability and cost effectiveness.

Several methods are being used to characterize the aggregate shape. These methods can be classified into indirect (BIS 812 Part-I (1975), IS-2386 Part-I (1963), ASTM D3398 (2000), ASTM-C1252 (1998), AASHTO TP-56 (1999) and ASTM D5821 (2003)), direct manual (ASTM-D4791) and automated methods. The manual methods are tedious, labor intensive, time consuming and prone to errors. With the rising requirements of civil infrastructure worldover, automated methods of aggregate characterization have become more relevant for research purposes and implementation.

In this paper, a novel method is proposed and a system is designed for automatically characterizing the shape of aggregates using digital image processing (DIP). The proposed system improvises on the existing methods of automated shape characterization and shows promising results.

LITERATURE REVIEW

Automated methods for shape characterization can be divided into three categories: (a) Tomographic techniques, (b) Laser scanning techniques and (c) Digital image processing techniques.

Techniques based on computer aided tomography

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divide an entire object or body into slices, so that capturing the entire shape of the body or object is possible (Karbozi). However, on a large scale, these techniques are time consuming, computationally and financially expensive. Kim et al. (2002) and Illerstorm (1998) have studied laser scanning based techniques which are able to capture the entire shape of the aggregate. However, it is observed that these techniques face the problem of occlusions during the process of scanning.

Digital image processing techniques have been studied by several researchers. Some studies have focused on quantifying the internal structure of compacted asphalt mixes in terms of aggregate orientation (Yue et al., 1995; Gharaybeh et al., 1998; Masad et al., 1998; Masad et al., 1999 a, b). Others have been devoted to the development of procedures to describe the shape of aggregates with emphasis on elongation (Barksdale, 1991; Kuo et al., 1996; Masad et al., 1999 a and b; Brzezicki and Kasperkiewicz, 1999; Weingart and Prowell, 1999; Maertz and Zhon, 1999; Rao and Tutumluer, 2000), angularity (Yudhbir and Abedinzadeh, 1991; Li et al., 1993; Wilson and Klotz, 1996; Yeggoni et al., 1996; Masad et al., 2000; Kuo and Freeman, 2000; Masad et al., 2001) and surface texture (Hryciw and Raschke, 1996; Wang and Lai, 1998; Masad and Button, 2000; Masad et al., 2001). Taleb Al-Rousan et al. (2007) discussed the image analysis techniques used by most of the available imaging systems that utilize different mathematical procedures for the analysis of aggregate shape characteristics.

It is seen from literature that only three mutually perpendicular faces have been considered for shape characterization. It can be, therefore, said that there is an assumption of two parallel faces being similar for a single aggregate. However, in the real world, aggregate shapes are irregular. Moreover, the existence of shadows during the process of observation cannot be ruled out.

It is, therefore, required that (a) more directions of observation are considered and (b) proper illumination is given during the process of observation. Therefore, the research questions addressed in this paper are as follows:

(a) Is it possible to develop a cheaper alternative for characterizing shape?
(b) How many faces are required to be observed for complete characterization?
(c) What arrangements can be made for proper illumination of the aggregate to avoid the presence of shadows?

Measurement of Aggregate Shape Using DIPAM

Coarse Aggregates

The coarse aggregates considered for the study are: (a) crushed basalt from two crushing plants (b) gravels and (c) elongated and flaky aggregate particles from two crushing plants. All the aggregates are washed, cleaned, dried and sieved into closer size fractions of 20mm-16mm, 16mm-12.5mm, 12.5mm-10mm, 10mm-6.3mm and 6.3mm-4.75mm. An example of the aggregates under study is presented in Figure (1).

Figure (1): Different shapes of aggregates

The various dimensions of the aggregates are demonstrated in Figure (2).

Figure (2): Various dimensions of aggregates

System Design

Digital Image Processing based Aggregate Measurement System (DIPAMS) is developed and described in the following sections.

The system consists of four components: (a) Image acquisition, (b) Turntable arrangement, (c) Illumination arrangement and (d) Conveyor arrangement.

Image Acquisition

In digital photogrammetry (Kasser and Egels, 2002), it is suggested that two different perspectives can generate the 3D model of a terrain or object. However, in order to capture the 3D structure of an aggregate, Tutumleur et al. (2005) suggested the use of at least 3 cameras. Therefore, in the proposed system, 3 cameras have been used. Moreover, all the cameras are arranged in an orthogonal manner, so that all the three-dimensional images of the aggregate can be acquired. In the proposed system, Logitech Pro 9000 web camera is used. The camera has 8 Mega-pixel resolution.

Turntable Arrangement

The camera arrangement proposed by Tutumleur et al. (2005) has a limitation that only three faces of the aggregate are captured. Therefore, in order to capture more faces of the aggregate, a turntable arrangement is added to the system. The turntable arrangement is a device which rotates the aggregate by a predefined angle and enables the camera arrangement to record more than three views for the aggregate on a horizontal plane. As shown in Figure (3), a "semi-transparent milky sheet" is mounted on the shaft of the stepper motor, where the stepper motor used here is of 200 step size which gives maximum angular resolution of 0.5º. The electronic component of the turntable was centred on an ATMEL mega series controller, which provided adequate real-time capabilities to interface with the computer. A proposed method used a Serial to USB Line Driver FT232R chip from FTDI in order to interface the ATmega328 with the USB port of a computer. A Low Dropout voltage regulator was used to power both the ATmega328 and the FT232R chip. To illuminate the "semi-transparent milky sheet", there is an arrangement of LED module underneath. This arrangement is illustrated in Figure (3).

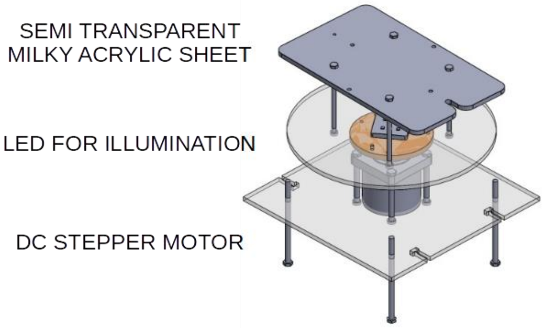

Figure (3): The turntable arrangement

Illumination Arrangement

Up till now, all the methodologies use a setup facing the camera and the focus light in the same direction, which is towards the aggregate, but the proposed system is designed in such a way that camera and focus light are both facing each other, and camera exposure is set to maximum with minimum gain.

A white semi-transparent acrylic sheet is mounted in between the camera and the focus light. Focus light concentrates all the light toward this white acrylic sheet and this sheet is then uniformly illuminated. This kind of setup provides a well illuminated white background for the target aggregate. The silhouetting algorithm required an evenly illuminated object and background, so LED lights are placed on opposite side of each webcam.

The illumination arrangement is shown in Figure (4).

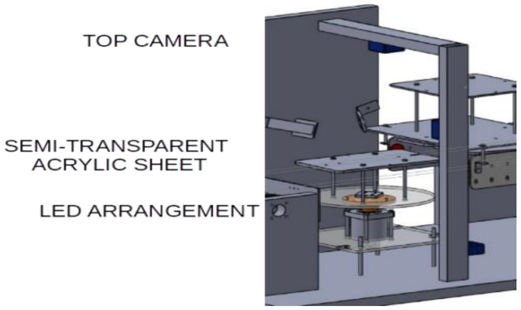

Figure (4): The illumination arrangement

Conveyor Arrangement

The conveyor arrangement consists of: (a) two shafts, (b) four pulleys, (c) two threads of equal length, (d) an encoder disc, (e) a DC motor and (f) a tray for carrying the aggregate.

Each of the shafts is mounted with two pulleys. The two threads are mounted on the pulleys. One of the shafts is equipped with the encoder disc. The pulleys have 50mm diameter each and the encoder disc has 32 slots. Thus, the minimum linear displacement that can be measured is 4.90mm.

The DC motor rotates one of the shafts and the entire arrangement of pulleys, threads and shafts moves. A tray, named as the aggregate tray is kept over the threads. This tray is used to keep the aggregate for which the dimensions have to be measured. An opto-sensor is used along with the encoder disk to send the pulses to the micro-controller. Each pulse is mapped as 4.9mm linear displacement in the micro-controller firmware. There is an anchor tied to the thread which can lock with the notch on the aggregate tray for moving the tray from one position to another. A DC motor driver is used in between DC motor and micro-controller to amplify the PWM signal from the micro-controller. PWM helps in controlled acceleration of the DC motor.

The conveyor arrangement is depicted in Figure (5).

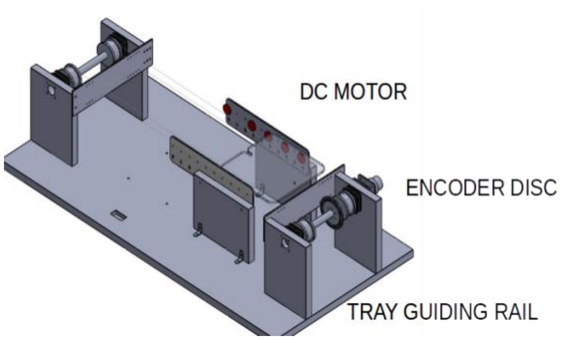

Figure (5): The conveyor arrangement

Integration of the Various Arrangements

The proposed system is an integration of the conveyor, turntable, lighting arrangement and embedded control system. The controlling device is connected with: (a) stepper motor for turntable, (b) DC motor for linear movement of the aggregate tray, (c) optical sensor for encoder disk, (d) home position limit switch and (e) computer connected on USB port of the controller.

The control system takes commands from the computer through USB port and executes them accordingly. These commands direct the micro-controller to linearly move or rotate the aggregate tray. Also, the commands expose the aggregate under study to the camera arrangement to capture the different images. The final integrated system is shown in Figure (6).

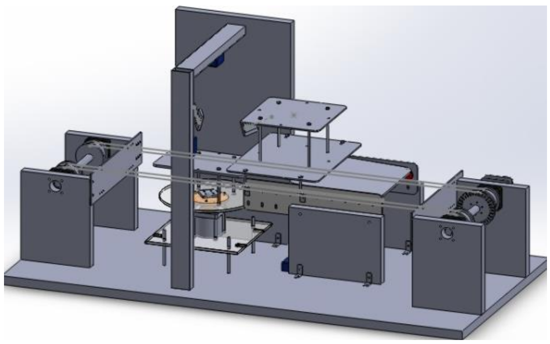

Figure (6): Integration of various arrangements

Implementation of the System

In order to measure the various characteristics of an aggregate, it is placed on the aggregate tray. The conveyor arrangement moves the tray linearly to the first position, where the first LED lamp illuminates the aggregate. The bottom camera captures the first view. The aggregate tray then moves to a second position, where a second image is captured by the top camera in the camera arrangement. The third camera also captures an image. Thereafter, the aggregate tray captures 3 different additional perspectives using the third imaging device by repeatedly turning the tray by 90 degrees clockwise each time, using the turntable arrangement. The turntable arrangement rotates the tray anticlockwise by 270 degrees and finally the tray is returned to the original position by the conveyor arrangement. This flow is explained in Figure (7).

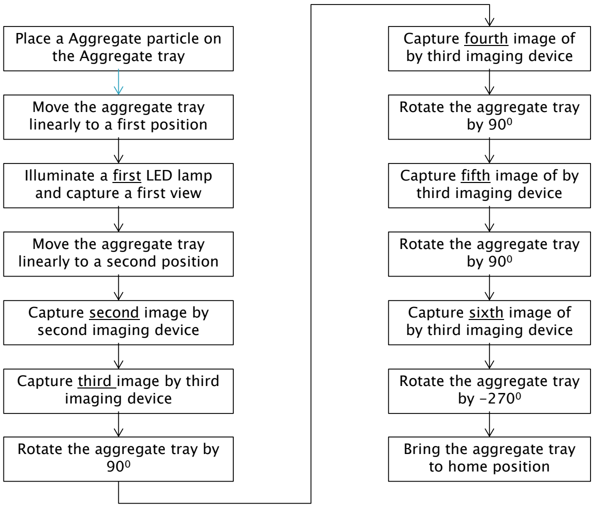

Figure (7): Implementation of the system

Taking Measurements Using the System

The entire process described in the previous subsection results in 6 different images. The MATLAB® image processing toolbox (Mathworks, Inc., New York) is used to compute the different dimensions of the aggregate.

Capturing Different Dimensions of the Aggregate

Pre-processing is applied on each of the images by using MATLAB® image processing toolbox (Mathworks, Inc., New York). The processing of each of these images is carried out in the following steps:

(a) *Conversion to grayscale:* Each of the images is first converted to grayscale. This allows conventional digital image processing algorithms to be applied easily on the images.

(b) *Detection of boundary:* The boundary of the aggregate should be determined in order to determine its dimensions. Therefore, the images are binarized using a threshold value.

(c) *Noise filtering:* In general, the noise in an image is any undesired information that contaminates that image. It can appear in images from a variety of sources. One of the primary processes by which noise appears in digital images is the acquisition process. In this process, an optical image is converted into a continuous electrical signal that is then sampled. Fluctuations caused by natural phenomena are present at every step of this process. They add a random value to the exact brightness value for a given pixel. Environmental conditions, such as temperature, are also affecting the noise already present in the electronics, as it can vary during the acquisition of an image database. Some periodic noise may be introduced during the acquisition process as a result of the physical systems involved. As a result of the above-mentioned causes, many noise points exist in the binary image, both inside and/or outside the aggregate domain. Because of this noise, the final profile of the analyzed aggregate particle is strongly affected. In the third step of image processing, using the differentiation algorithm from the Fourier analysis incorporated in the MATLAB program, the noise points are completely and accurately removed from the binary image.

(d) *Computation of the dimensions of the aggregate:* From the denoised binary image, the dimensions of the aggregate are determined using the *regionprops* command from the MATLAB Image Processing Toolbox. These dimensions are labelled as: longest dimension (d_L), intermediate dimension (d_I) and shortest dimension (d_S). Therefore, shape descriptors; namely, sphericity, shape factor, elongation ratio and flatness ratio (see Table 1) are computed by writing a MATLAB® script.

Table 1. Shape descriptors for aggregates

Descriptor	Formula	Description
Sphericity	$\psi = \sqrt[3]{\frac{d_S \cdot d_I}{d_L^2}}$	Barksdale et al. (1991)
Shape factor	$SF = \frac{d_S}{\sqrt{d_L \cdot d_I}}$	Barksdale et al. (1991)
Elongation ratio	$ER = \frac{d_L}{d_I}$	Kuo et al. (1998)
Flatness ratio	$FR = \frac{d_I}{d_S}$	Kuo et al. (1998)

RESULTS AND DISCUSSION

DIPAMS

Conventional studies on aggregates using Digitial Image Processing (DIP) have taken images of aggregates in three mutually perpendicular directions. The system designed in this work is able to capture images from more than three directions. In all, the system is able to capture six different faces of each aggregate. Also, for each aggregate, the time for capturing these images is about 5 seconds.

Analysis of the Images

The images for all the aggregates were captured in normal color, which were then converted to grayscale. For each of the aggregates, a set of six such images are captured. Each image in this set denotes one face of the aggregate.

The MATLAB Image Processing Toolbox is used to generate the various shape descriptors for each of these images. Therefore, the maximum, intermediate and minimum lengths of the aggregates are computed.

Verification of the New System

Calibration of the Camera

In order to measure the dimensions of the objects obtained from the images, a calibration was performed on the camera. The camera used in the system is an 8 megapixel camera with dimensions of 1600 pixels x 1200 pixels. The calibration of the camera used in the system was carried out by capturing photographs of coins of known dimensions. 6 coins of known dimensions, measured using vernier callipers, were placed at a distance of 10cm from the lens of the camera. 5 photographs for each of the coins were clicked at an interval of 5 seconds. Similar to the process described in the above paragraph, for each of the photographs, dimensions are computed in pixels. Based on the experiments conducted on the camera, it was found that 1600 pixels corresponded to 62.4 mm. In this way, calibration was performed by placing the objects of known dimensions under the camera and computing the corresponding number of pixels. Thus, it was found out that the calibration factor was 0.039 mm/pixel.

Comparison of Dimensions

To verify the applicability of the proposed method, the results obtained by digital image processing were compared by measuring the dimensions of same aggregates with vernier callipers, with a precision of 0.1mm. The dimensions measured using vernier callipers and the DIPAMS are compared and the results are presented in Figure (8).

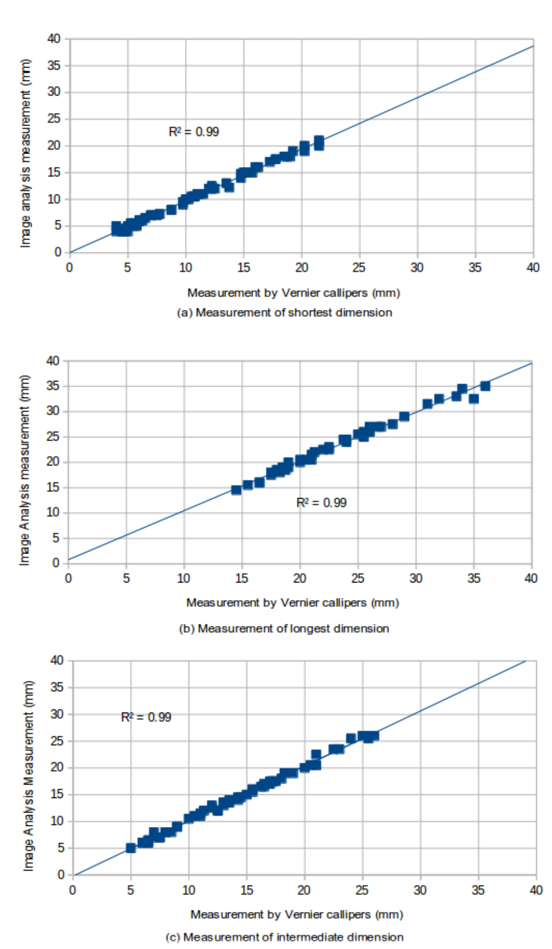

Figure (8): Comparison of dimensions measured through manual and digital modes

It is seen that the goodness of fit values (R^2) are very close to 1 for all the dimensions; hence, it can be concluded that there is a fair degree of agreement between the two readings. However, it is also seen that there is lesser agreement for measuring the shortest dimension (see Table 2), but this can be attributed to the difficulty in measuring the shortest dimension with the venier arrangement.

Computation of the Shape Measures

The dimensions obtained from the images, as well as the images, are then used to compute the sphericity, shape factor, elongation ratio and flatness ratio for each of the aggregates. These shape characteristics of the aggregates computed from the images are presented in Table (3).

Table 2. Error statistics for aggregates; manual *vs.* DIP-based

	Mean Error (mm)	Standard Deviation (mm)
Longest	-0.0956	0.3744
Shortest	-0.1545	0.2124
Intermediate	0.0802	0.3562

Table 3. Summary of shape measures of aggregates

Aggregate	Size	d_L	d_I	d_S	Sphericity	Shape Factor	Elongation	Flatness	Specification
Type-A	6.3-4.75	16.1	5.4	4.3	0.4474421	0.4611674	2.9814815	3.744186	Elongated
	10-6.3	16.6875	10.75	6.3125	0.6246102	0.4713044	1.5523256	2.6435644	
	12.5-10	20.57143	10.75	8.714286	0.6049281	0.5859969	1.9136214	2.3606558	
	16-12.5	27.4	12.75	10.875	0.5694813	0.5818337	2.1490196	2.5195402	
	20-16	43.7	23	12.5	0.5319755	0.3942806	1.9	3.496	
	25-20	30.1	23.3	14	0.7114051	0.5286483	1.2918455	2.15	
Average					0.5816404	0.5038719	1.9647156	2.8189911	
Type-B	6.3-4.75	9.2	5.425	5.275	0.6966514	0.74667	1.6958525	1.7440758	Angular
	10-6.3	12.15	6.075	5.9	0.6238525	0.6867375	2	2.059322	
	12.5-10	16.65	9.675	9.475	0.6915134	0.7465289	1.7209302	1.7572559	
	16-12.5	20.4	11.825	11.5	0.6887785	0.7404269	1.7251586	1.773913	
	20-12.5	27.8	16.3	15.7	0.6918297	0.7375364	1.7055215	1.7707006	
	25-20	31.25	19.65	18.85	0.7238656	0.7606854	1.5903308	1.6578249	
Average					0.6860818	0.7364308	1.7396323	1.7938487	
Type-C	6.3-4.75	9.4	5.25	5.2	0.6760347	0.7402182	1.7904762	1.8076923	Cubical
	10-6.3	10.75	6.325	6.075	0.6927821	0.7367358	1.6996047	1.7695473	
	12.5-10	15.15	9.675	9.525	0.7377291	0.7867435	1.5658915	1.5905512	
	16-12.5	22.9	14.475	14.1	0.730111	0.7744477	1.582038	1.6241135	
	20-12.5	26.3	14.475	14.1	0.6657463	0.7226566	1.8169257	1.8652482	
	25-20	30.1	18.275	17.8	0.7107466	0.7589411	1.6470588	1.6910112	

Average					0.7021916	0.7532905	1.6836658	1.724694	
	6.3-4.75	9.3	8.4	6.183333	0.9340763	0.6995867	0.9032258	1.3584906	
	10-6.3	13.45	13.6	9.133333	0.8822152	0.675303	0.9889706	1.4726278	
	12.5-10	15.825	15.75	10.83333	0.8799372	0.6861986	1.0047619	1.4607697	
Type-D	16-12.5	19.975	20.95	12.65	0.8725031	0.6183795	0.9534606	1.5790514	Rounded
	20-12.5	21.5	22.75	15.03333	0.9044542	0.6797438	0.9450549	1.4301555	
	25-20	28.8	28.3	20.66667	0.8900696	0.7239041	1.0176678	1.3935482	
Average					0.8938759	0.6805193	0.968857	1.4491072	

The shape characteristics computed from the images are also compared with the shape characteristics obtained from the measurements from vernier callipers. Results are presented in Figure (9), where high values of goodness of fit (R^2) are seen.

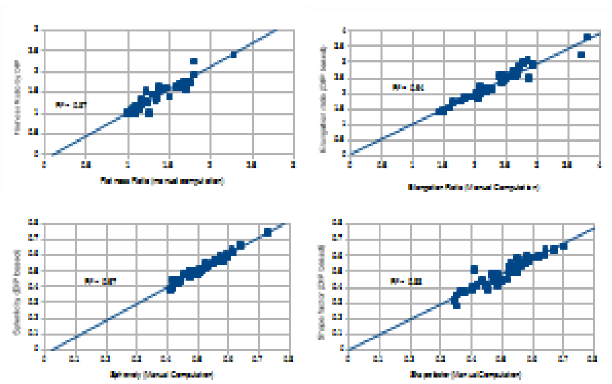

Figure (9): Comparison of shape measures computed from manually and digitally obtained dimensions

Answers to the Research Questions

More faces for observation: It was mentioned earlier in this paper that pictures from more than one perspective can give three-dimensional information about an object. Researchers till date have attempted to capture the three-dimensional structure of the aggregate by using three different cameras. In this paper, the system uses three cameras as well as a turntable arrangement. Thus, a more accurate three-dimensional structure is generated. Observations made from the cameras as well as the vernier callipers are highly consistent.

Illumination of the aggregate to avoid the presence of shadows: The illumination arrangement in the proposed system uses a semi-transparent milky white sheet to diffuse the illumination. This ensures uniform lighting and avoids shadows.

A cheaper alternative to the existing techniques: Tomography and laser scanning- based techniques have been studied till date for the characterization of the shape of the aggregate. It is observed that these techniques are highly expensive and also impractical for use in a large-scale environment. This paper has presented an accurate and cheaper alternative DIPAMS, based on digital image processing techniques for capturing the 3D structure of the aggregates as well as derive the shape descriptors. These shape descriptors help in characterizing the shape of the aggregate.

Conclusions and Future Directions of Work

The shape of aggregates forms an important part in controlling the void space in a concrete mix. Quantification of aggregate properties is very important for rationalizing various concrete mix proportioning methods. Irregular shapes, such as flat, elongated..., affect the fresh and hardened concrete properties negatively. In the absence of constitutive information with respect to the shape and surface texture, the mix proportioning method has become more or less a trial and error process. In a real world scenario, the crushers are responsible for the shape of the aggregates. Changes in the shapes of the aggregates happen owing to wear and tear of the crusher jaw. Thus, it is important to have a system which can study the shapes of the aggregates coming out of the crusher, characterize them and issue a warning if the shapes would not lead to appropriate compressive strength of the concrete. Various studies have been conducted to develop systems which characterize the shapes of the aggregates. Such systems are based on various technologies; namely: (a) tomography, (b) laser scanning and (c) digital image processing. This paper has demonstrated the development of a digital image processing based system named DIPAMS which uses: (a) a camera arrangement, (b) a turntable arrangement, (c) an illumination arrangement and (d) a conveyor arrangement, in order to capture the various facets of an aggregate and thus characterization of its shape parameters is possible. Results show that dimensions of the aggregate and the corresponding shape characteristics, obtained from manual and digital image processing techniques, are consistent. Also, the system is cheaper compared to the tomography and laser scanning- based arrangement. In addition, since the system is fast (ability to capture all the facets of the aggregate in about 5 seconds), it is also implementable at a large scale. However, in this study, only coarse aggregates were used. It would be also needed to see the effect of DIPAMS using fine aggregates. Of course, for such an arrangement, cameras of better resolution have to be used. The authors are currently working on the refinement of the DIPAMS system focussing on finer aggregates and better camera resolutions.

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