

## Consolidation Analysis by the Extended Taylor Method (ETM)

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### ABSTRACT

The Taylor  $\sqrt{t}$  method is extended for improving the estimation of the end-of-primary (EOP) settlement  $\delta_p$  and coefficient of vertical consolidation  $c_v$ . The extended Taylor method (ETM) utilizes the same graphical procedure of the conventional Taylor method, but this procedure is repeated twice at any two  $U$  values greater than 52.6% instead of the one used in the Taylor method at 90%;  $U$  is the average degree of consolidation. The ETM explicitly relates  $c_v$  to the slope of the initial linear portion of the  $\delta_t - \sqrt{t}$  curve as well as the EOP settlement  $\delta_p$ ;  $\delta_t$  is the settlement at time  $t$  during consolidation. The ETM can be applied with a minimum of four consolidation data points for estimating  $\delta_p$  and  $c_v$ . Two types of EOP settlements are distinguished; firstly, local EOP settlement  $\delta_{pi}$  obtained by matching experimental results to theory at a particular  $U$  value; secondly, global EOP settlement  $\delta_p$  obtained by extrapolation using two or more local EOP settlements. The ETM explains the observed differences in  $c_v$  values estimated by the existing methods in terms of the differences in  $\delta_p$  values obtained by these existing methods. Experimental results are used to validate and compare the extended Taylor method with existing methods.

**KEYWORDS:** Taylor method, Casagrande method, Coefficient of vertical consolidation, End of primary settlement, Initial compression, Secondary compression.

### INTRODUCTION

Consolidation analysis utilizing the Terzaghi one-dimensional consolidation theory along with the results of oedometer tests is still widely used in settlement analysis of soils. The Terzaghi theory was developed only for the primary consolidation assuming constant coefficient of vertical consolidation and ignoring time compressibility during primary consolidation; whereas the observed compression-time curve exhibits initial compression, primary consolidation and secondary compression (Terzaghi et al., 1996). Hence, to properly apply the Terzaghi theory in settlement analysis, the primary consolidation must be recognized by identifying the initial and secondary compressions and

then matched with the Terzaghi theory at a particular  $U$  value or over a range of  $U$ , where  $U$  is the average degree of vertical consolidation.

The application of the Terzaghi theory to settlement analysis requires reliable values for the coefficient of vertical consolidation  $c_v$  and end-of-primary (EOP) settlement  $\delta_p$ . Numerous methods were developed for estimating  $c_v$  and  $\delta_p$  values (Casagrande and Fadum, 1940; Taylor, 1948; Scott, 1961; Cour, 1971; Sivaram and Swamee, 1977; Asaoka, 1978; Parkin, 1978; Sridharan and Rao, 1981; Sridharan et al., 1987; Robinson and Allam, 1996; Robinson, 1997, 1999; Mesri et al., 1999a; Feng and Lee, 2001; Al-Zoubi, 2008a, 2008b, 2010, 2014). The most widely used methods for estimating  $c_v$  and  $\delta_p$  are the standard  $\log t$  method (Casagrande and Fadum, 1940) and the standard  $\sqrt{t}$  method (Taylor, 1948). The  $\log t$  method

computes  $c_v$  at 50% consolidation and requires the determination of the initial and final compressions that correspond to 0% and 100% consolidation, respectively. The  $\sqrt{t}$  method computes  $c_v$  at 90% consolidation and requires the determination of the initial compression. The two standard  $\log t$  and  $\sqrt{t}$  methods and most existing methods utilize the same theoretical basis for evaluating the initial compression, but these methods differ in the way in which the EOP consolidation is identified. The  $\sqrt{t}$  method generally yields lower  $\delta_p$  and higher  $c_v$  values as compared to the  $\log t$  method; the differences in  $c_v$  values estimated by the existing methods for a particular pressure increment could chiefly be explained by the differences in  $\delta_p$  values obtained by these different methods (Al-Zoubi, 2008b, 2010).

In this paper, the coefficient of vertical consolidation  $c_v$  is expressed as a function of slope  $m$  of the initial linear portion of the  $\delta_t - \sqrt{t}$  curve and the EOP settlement  $\delta_p$ . The graphical construction of the conventional Taylor method is extended for improving the estimation of  $\delta_p$  (and thus  $c_v$ ) as compared to the Casagrande method that yields almost identical  $\delta_p$  values to those obtained from pore water pressure measurements (Mesri et al., 1999b; Robinson, 1999). Experimental results of oedometer tests on clayey soils are used to validate and compare the extended Taylor method (ETM) with existing methods.

## ANALYTICAL BACKGROUND

The relationship between average degree of consolidation  $U$  and time factor  $T$  of the Terzaghi theory may, depending on the range of  $U$ , be given by the following two expressions (Terzaghi, 1943):

$$U = \sqrt{\frac{4}{\pi}} \sqrt{T} \quad \text{for } U \leq 52.6 \% \quad (1)$$

$$\ln(1-U) = \ln \frac{8}{\pi^2} - \frac{\pi^2}{4} T \quad \text{for } U \geq 52.6 \% \quad (2)$$

In the Terzaghi theory, the consolidation time  $t$  is

expressed as a function of the time factor  $T$ , longest drainage path  $H_m$  and coefficient of vertical consolidation  $c_v$ , as follows:

$$t = \frac{T H_m^2}{c_v} \quad (3)$$

The settlement  $\delta_t$  may be given in terms of  $U$  and  $\delta_p$  by the following expression:

$$\delta_t = U \delta_p \quad (4)$$

where  $\delta_p = R_p - R_o$ ;  $R_p$  is the dial reading at the EOP consolidation and  $\delta_t$  is the settlement at time  $t$  during consolidation that is equal to  $R_t - R_o$ ;  $R_t$  is the dial reading at time  $t$  and  $R_o$  is the initial compression defined as the dial reading that corresponds to 0% consolidation. The initial compression  $R_o$  may be given as follows:

$$R_o = \frac{R_2 - R_1 \sqrt{t_2/t_1}}{1 - \sqrt{t_2/t_1}} \quad (5)$$

where  $R_1$  and  $R_2$  are the dial gauge readings at time  $t_1$  and time  $t_2$ , respectively, and are selected such that these two points are on the initial linear portion of the  $R_t - \sqrt{t}$  curve. The Casagrande and Taylor methods use the same basis for obtaining the initial compression  $R_o$ . The Casagrande method arbitrarily takes  $t_2 = 4t_1$ , therefore,  $\Delta = R_1 - R_o = R_2 - R_1$  or  $R_o = 2R_1 - R_2$  whereas, in the Taylor method,  $R_o$  is determined graphically as the intercept of the initial linear portion of the  $R_t - \sqrt{t}$  curve. Hence, the Taylor, Casagrande and extended Taylor methods are similarly affected by the factors that influence the initial portion of the consolidation curve. However, these methods differ in the way by which the primary consolidation range (or EOP  $\delta_p$ ) is obtained as shown later.

The coefficient of consolidation may be expressed based on Eqs. (1), (3) and (4), as follows:

$$c_v = \frac{\pi}{4} \left( \frac{m H_m}{\delta_p} \right)^2 \quad (6)$$

where  $m$  is the slope of the initial linear portion of the experimental  $\delta_t - \sqrt{t}$  curve that may be expressed as follows:

$$m = \frac{R_2 - R_1}{\sqrt{t_2} - \sqrt{t_1}} \quad (7)$$

Because Eq. (6) involves three unknowns (i.e.,  $R_0$ ,  $R_p$  and  $m$ ; where  $\delta_p = R_p - R_0$ ), the coefficient of vertical consolidation may not be obtained from only the initial linear portion. Therefore, at least one additional data point ( $t_i, R_{ii}$ ) must be selected from the later stages of consolidation beyond the initial linear portion (theoretically, at  $U \geq 52.6\%$ ) along with the two data points ( $t_1, R_1$ ) and ( $t_2, R_2$ ) required for obtaining the initial compression  $R_0$  (Eq. 5) and the slope  $m$  (Eq. 7).

Equation (6) can, however, be used for assessing the coefficient of vertical consolidation independently of the procedure in which the EOP  $\delta_p$  is obtained; for example, at  $U$  of 90% (used in the Taylor method), the slope  $m$  and EOP settlement  $\delta_p$  can be expressed in terms of  $\delta_{90}$  and  $t_{90}$ , respectively, as follows:

$$m = 1.153 \frac{\delta_{90}}{\sqrt{t_{90}}} \quad (8)$$

$$\delta_p = \frac{\delta_{90}}{0.90} \quad (9)$$

Substituting Eqs. (8) and (9) into Eq. (6), the coefficient of consolidation may be given by the following expression:

$$c_v = \frac{\pi}{4} \left( \frac{1.153 \delta_{90} H_m 0.90}{\sqrt{t_{90}} \delta_{90}} \right)^2 = \frac{0.848 H_m^2}{t_{90}} \quad (10)$$

Equation (10) is the same as that used by the  $\sqrt{t}$  method (Taylor, 1948) to estimate  $c_v$ .

On the other hand, at  $U$  of 50% (used in the Casagrande method), the EOP settlement  $\delta_p$  can be expressed as follows:

$$\delta_p = 2 \delta_{50\%} \quad (11)$$

The slope  $m$  can also be expressed as the secant slope of the observed  $\delta_t - \sqrt{t}$  curve at 50% consolidation as follows:

$$m = \frac{\delta_{50\%}}{\sqrt{t_{50}}} \quad (12)$$

Substituting Eqs. (11) and (12) into Eq. (6), the coefficient of consolidation may be given by the following expression:

$$c_v = \frac{\pi}{4} \left( \frac{\delta_{50\%} H_m}{\sqrt{t_{50}} 2 \delta_{50\%}} \right)^2 = \frac{0.196 H_m^2}{t_{50}} \quad (13)$$

Equation (13) is the same as that used by the  $\log t$  method (Casagrande and Fadum, 1940) to estimate  $c_v$ .

Equation (6) is thus valid for estimating the coefficient of consolidation  $c_v$  regardless of the procedures used to estimate  $\delta_p$ . It should, however, be emphasized that the  $c_v$  value depends on these procedures that may yield different  $\delta_p$  values. Hence, the differences in  $c_v$  values obtained by the various existing methods for a particular pressure increment can be explained by the differences in  $\delta_p$  values obtained either explicitly or implicitly by these existing methods. Therefore, any improvements that can be made on the estimation of the EOP settlement  $\delta_p$  will improve the estimation of  $c_v$  values as may be deduced from Eq. (6).

The extended Taylor method (ETM) is introduced in the following section to improve the estimation of  $\delta_p$  and  $c_v$  values and compared to existing methods by using experimental results of clayey soils (Table 1).

These soils cover a wide range of liquid limit and plasticity; the testing procedures of these soils were

described in detail by Al-Zoubi (2008b, 2010, 2013, 2014).

**Table 1. Basic properties of the clayey soils utilized in the present study**

Soil	Particle size			Liquid limit %	Plastic limit %	Specific gravity G
	Sand %	Silt %	Clay %			
Azraq Green Clay (AGC-5; AGC-6 and AGC-8)	8	23	69	108	42	2.76
Mutah Clay (Mutah-0)	15	60	25	44	26	2.73
Chicago Blue Clay <sup>a</sup> (CBC-3)	4	64	32	29	17	2.73
Chicago Blue Clay <sup>b</sup> CBC (Taylor, 1948)	-----	-----	-----	-----	-----	2.73
Madaba Clay (Madaba-6)	14	41	45	55	25	2.78
Treated Madaba Clay (Mad-t1, 2% cement)	-----	-----	-----	-----	-----	-----

<sup>a</sup> This specimen of CBC was tested by the Author.  
<sup>b</sup> This specimen of CBC was reported by Taylor (1948).

### The Extended Taylor Method (ETM)

In the conventional Taylor  $\sqrt{t}$  method, a factor of 1.15 is used along with the initial linear portion of the  $\delta_t - \sqrt{t}$  curve to compute  $\delta_p$  and  $c_v$  at 90% consolidation. This factor (i.e., 1.15) can be interpreted as the ratio of the secant slope  $m_{50}$  at 50% consolidation to the secant slope  $m_{90}$  at 90% consolidation. Assuming that the experimental and theoretical ratios are the same, the following expression can be obtained for this ratio.

$$\begin{aligned}
 ROSS(U = 90\%) &= \frac{m_{50}}{m_{90}} = \frac{\delta_{50}/\sqrt{t_{50}}}{\delta_{90}/\sqrt{t_{90}}} = \frac{M_{50}}{M_{90}} \\
 &= \frac{50\sqrt{T_{90}}}{90\sqrt{T_{50}}} = 1.15
 \end{aligned} \quad (14)$$

where  $ROSS$  is the ratio of secant slopes;  $\delta_{50} = 0.50\delta_p$ ; and  $\delta_{90} = 0.90\delta_p$ .

In the conventional Taylor method, the 90%

consolidation was selected arbitrarily such that it is as close as possible to the EOP consolidation but least affected by secondary compression. The conventional Taylor method can, therefore, be extended for obtaining  $\delta_{pi}$  and  $c_{vi}$  at other  $U_i$  values than 90%. In other words, the same graphical procedure of Taylor (1948) can be used such that the third point is arbitrarily selected at different  $U_i$  values for obtaining  $\delta_{pi}$  and  $c_{vi}$  at those selected  $U_i$  values. This extension of the Taylor method requires a factor at each selected  $U_i$  value to be used for obtaining  $\delta_{pi}$  and  $c_{vi}$  in the same way the factor of 1.15 is used in the conventional Taylor method at  $U_i$  of 90%. The ratio of secant slopes ( $ROSS_i$ ) at 50% and at any other  $U_i$  value can be generalized for this purpose as follows:

$$ROSS(U_i, \%) = \frac{m_{50}}{m_U} = \frac{\delta_{50}/\sqrt{t_{50}}}{\delta_U/\sqrt{t_U}} = \frac{M_{50}}{M_U} = \frac{50/\sqrt{T_{50}}}{U_i/\sqrt{T_U}} = \frac{112.65\sqrt{T_{U_i}}}{U_i} \quad (15)$$

**Table 2. Results of direct analytical method (DAM) and extended Taylor method (ETM) using consolidation data from Taylor (1948) for Chicago blue clay (CBC)**

<b>Early stages of consolidation (theoretically, <math>U &lt; 52.6\%</math> )</b>										
Time (min)	0	0.25	1	2.25	4	6.25	9	12.25	16	
Dial Reading, R (x 10 <sup>-4</sup> in) 1 in = 25.4 mm	1500	1451	1408	1354	1304	1248	1197	1143	1093	
Slope $m$ (mm/min <sup>-1/2</sup> )	0.274									See Fig. 1
$R_0$ 25.4 x 10 <sup>-4</sup> (mm)	1516									
<b>Later stages of consolidation (theoretically, <math>U &gt; 52.6\%</math> )</b>										
Time (min)	20.25	25	30.25	36	42.25	60	100	200	400	1440
R (x 10 <sup>-4</sup> in) 1 in = 25.4 (mm)	1043	999	956	922	892	830	765	722	693	642
Settlement $\delta_{ti}$	1.201	1.313	1.422	1.509	1.585	1.742	1.908	2.017	2.090	2.220
<b>Extended Taylor method (ETM)</b>										
EOP $\delta_{pi}$ (mm)	1.665	1.717	1.777	1.792	1.804	1.863	1.940	-----	-----	-----
$c_v / H_m^2$ (10 <sup>-3</sup> min <sup>-1</sup> )	21.3	20.2	18.7	18.4	18.2	17.0	15.7	-----	-----	-----
Local $U_i = \frac{\delta_{ti}}{\delta_{pi}}$ , %	72.1	76.5	80.0	84.2	87.9	93.5	98.4	-----	-----	-----
Global $U = \frac{\delta_{ti}}{\delta_p}$ , %	61.8	67.6	73.2	77.7	81.6	89.7	98.2	-----	-----	-----
<b>Direct analytical method (DAM)</b>										
EOP $\delta_{pi}$ (mm)	1.674	1.717	1.780	1.791	1.806	1.864	1.911	2.018	2.092	2.220
$c_v / H_m^2$ (10 <sup>-3</sup> min <sup>-1</sup> )	21.1	20.1	18.7	18.4	18.1	17.0	16.2	-----	-----	-----
Local $U_i = \frac{\delta_{ti}}{\delta_{pi}}$ , %	71.7	76.5	79.9	84.3	87.8	93.5	99.8	-----	-----	-----
Global $U = \frac{\delta_{ti}}{\delta_p}$ , %	61.8	67.6	73.2	77.7	81.6	89.7	98.2	-----	-----	-----

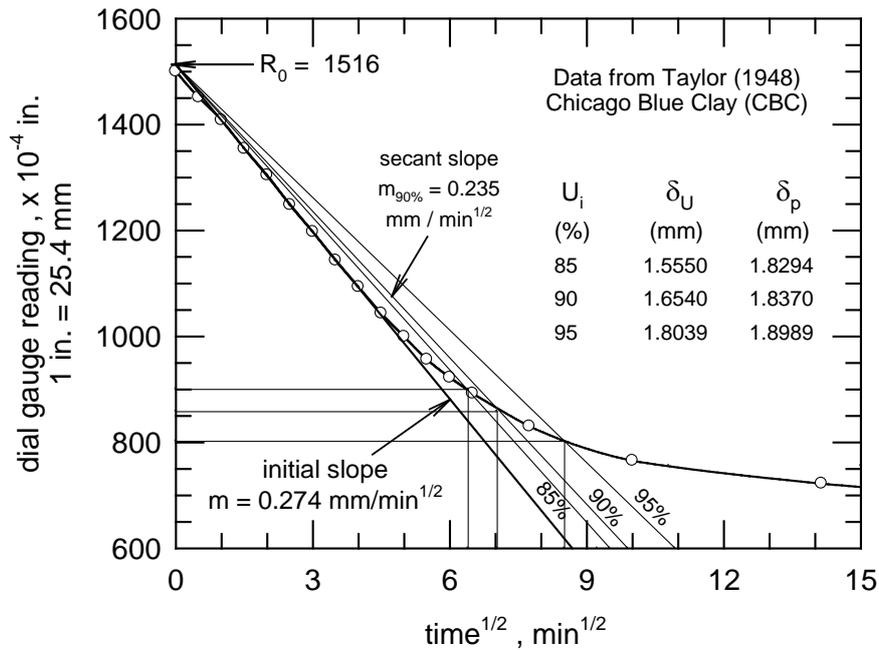
The corresponding EOP settlement  $\delta_{pi}$  at the arbitrarily selected  $U_i$  value can be given by the following expression:

$$\delta_{pi} = \frac{100}{U_i} \delta_U \tag{16}$$

where  $\delta_{ti} = R_{ti} - R_0$  is the settlement at time  $t_i$  and  $\delta_p = R_{pi} - R_0$ . The subscript  $i$  is added to  $\delta_p$  and  $U$

because of the dependence of  $\delta_p$  on the arbitrarily

selected  $\delta_{ii}$  value as shown in the following analysis.



**Figure (1): Graphical construction for the extended Taylor method (ETM); data from Taylor (1948) for Chicago blue clay**

The settlement  $\delta_U$  ( $=\delta_{ii}$  at  $U_i$ ) at any arbitrarily selected  $U_i$  value can be obtained by utilizing the same graphical procedure of Taylor (1948) at that  $U_i$  value as demonstrated in Fig. 1, which shows the determination of three EOP  $\delta_{pi}$  values at three arbitrarily selected  $U_i$  values. The estimated EOP  $\delta_{pi}$  values are plotted against the selected  $\delta_U = \delta_{ii}$  values in Fig. 2, which was plotted by using additional data points (Table 2). These results show that the estimated EOP  $\delta_{pi}$  depends on the third point ( $t_i, \delta_{ii}$ ) selected for the analysis. Figure 2, however, shows that the estimated EOP  $\delta_{pi}$  value increases linearly with the selected  $\delta_{ii}$  value during primary consolidation. This linear relationship between  $\delta_{pi}$  and  $\delta_{ii}$  can be expressed as follows:

$$\delta_{pi} = a + b\delta_{ii} \tag{17}$$

where  $a$  and  $b$  are the intercept and slope of the

linear  $\delta_{pi} - \delta_{ii}$  relationship, respectively.

Figure 2 and Table 2 show that as the compression-time curve approaches the EOP consolidation, the estimated  $\delta_{pi}$  value approaches the arbitrarily selected  $\delta_{ii}$  value; therefore, the following expression can be suggested at the EOP consolidation:

$$\delta_{pi} = \delta_{ii} \tag{18}$$

Hence, a unique EOP  $\delta_p$  value can be defined for any particular pressure increment where the estimated  $\delta_{pi}$  values become equal to the arbitrarily selected  $\delta_{ii}$  values. Therefore, the unique EOP  $\delta_p$  may be expressed by equating Eq. 17 and 18 as follows:

$$\delta_p = \frac{a}{1-b} \tag{19}$$

Equation 19 shows that the unique EOP  $\delta_p$  value can be obtained from the linear relationship between  $\delta_{pi}$  and  $\delta_{ii}$  observed in the primary consolidation range by forward extrapolation without the need to use secondary compression (Fig. 2). This extrapolation requires at least two compression-time data points in the range  $U \geq 52.6\%$  to obtain EOP  $\delta_p$  in addition to

the two data points  $(t_1, R_1)$  and  $(t_2, R_2)$  required for back-calculating the initial compression  $R_0$  and the slope  $m$  of the initial linear portion; the coefficient of vertical consolidation can then be computed by Eq. 6. Similar trend is observed in Fig. 3 for another specimen of treated Madaba clay (Mad-t1).

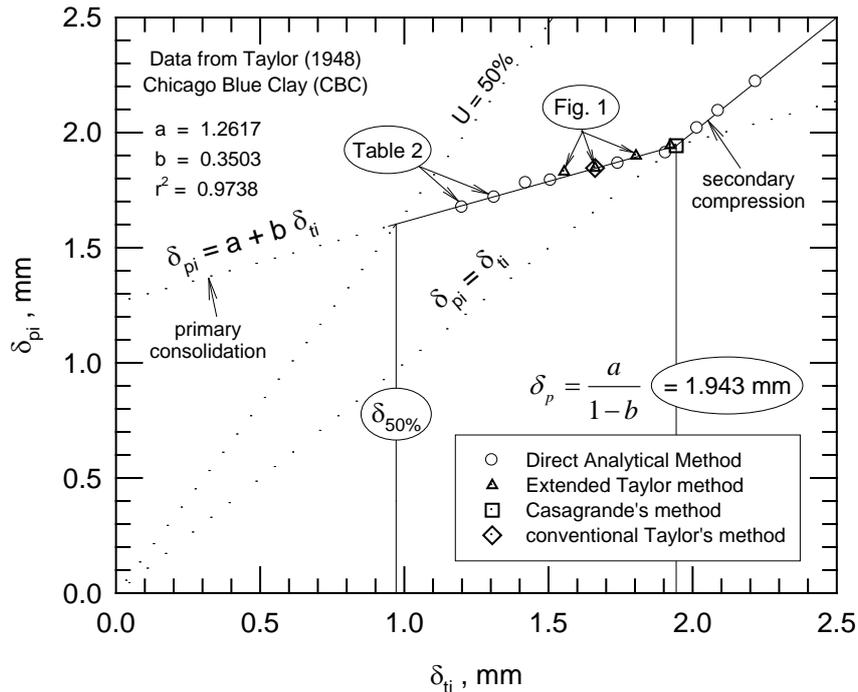


Figure (2): The  $\delta_{pi} - \delta_{ii}$  relationships obtained by the extended Taylor method (ETM) and direct analytical method (DAM); data from Taylor (1948) for Chicago blue clay

Based on Figs. 2 and 3, two types of end-of-primary (EOP) settlement are distinguished. The first type is called the local EOP settlement  $\delta_{pi}$  (Eq. 16), which is the settlement determined by matching the observed compression-time curve to the Terzaghi theory at a single  $U_i$  value by using three consolidation data points (two points at  $U \leq 52.6\%$  and one point at  $U \geq 52.6\%$ ) as is the case in the Taylor method at  $U_i$  of 90%. The second type is called the global EOP settlement  $\delta_p$  (Eq. 19), which is the settlement determined by forward extrapolation using four or more consolidation data points (two points at  $U \leq 52.6\%$  and two points at  $U \geq 52.6\%$ ). The extended

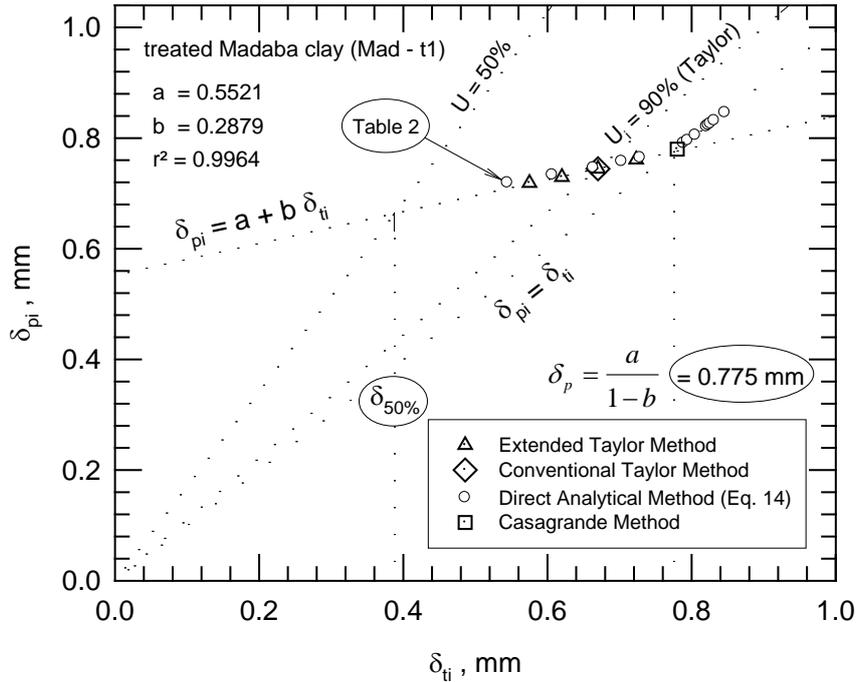
Taylor method (ETM) yields both local and global EOP settlements.

**Validity of the Proposed ETM Method**  
**Comparison with Experimental Compression-Time Curves**

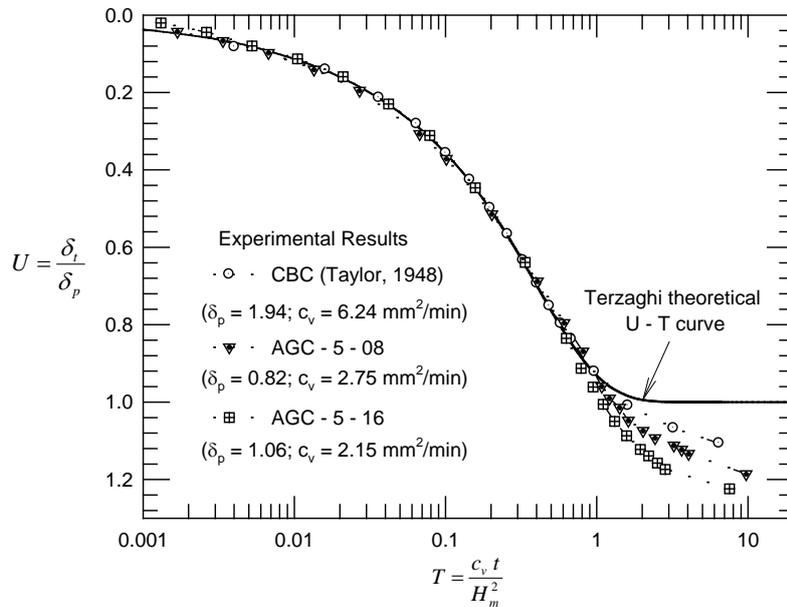
The validity of the extended Taylor method (ETM) is verified by comparing the experimental results with the Terzaghi theory throughout the entire primary consolidation stage using  $c_v$  and  $\delta_p$  values obtained by the extended Taylor method. Excellent agreement exists between the experimental and theoretical  $U - T$  curves in the primary consolidation stage as shown in

Fig. 4 for three clays (Azraq green, Madaba and Chicago blue clays). It should be pointed out that the experimental curves deviate from each other and from

the Terzaghi theoretical relationship in the secondary compression stage.

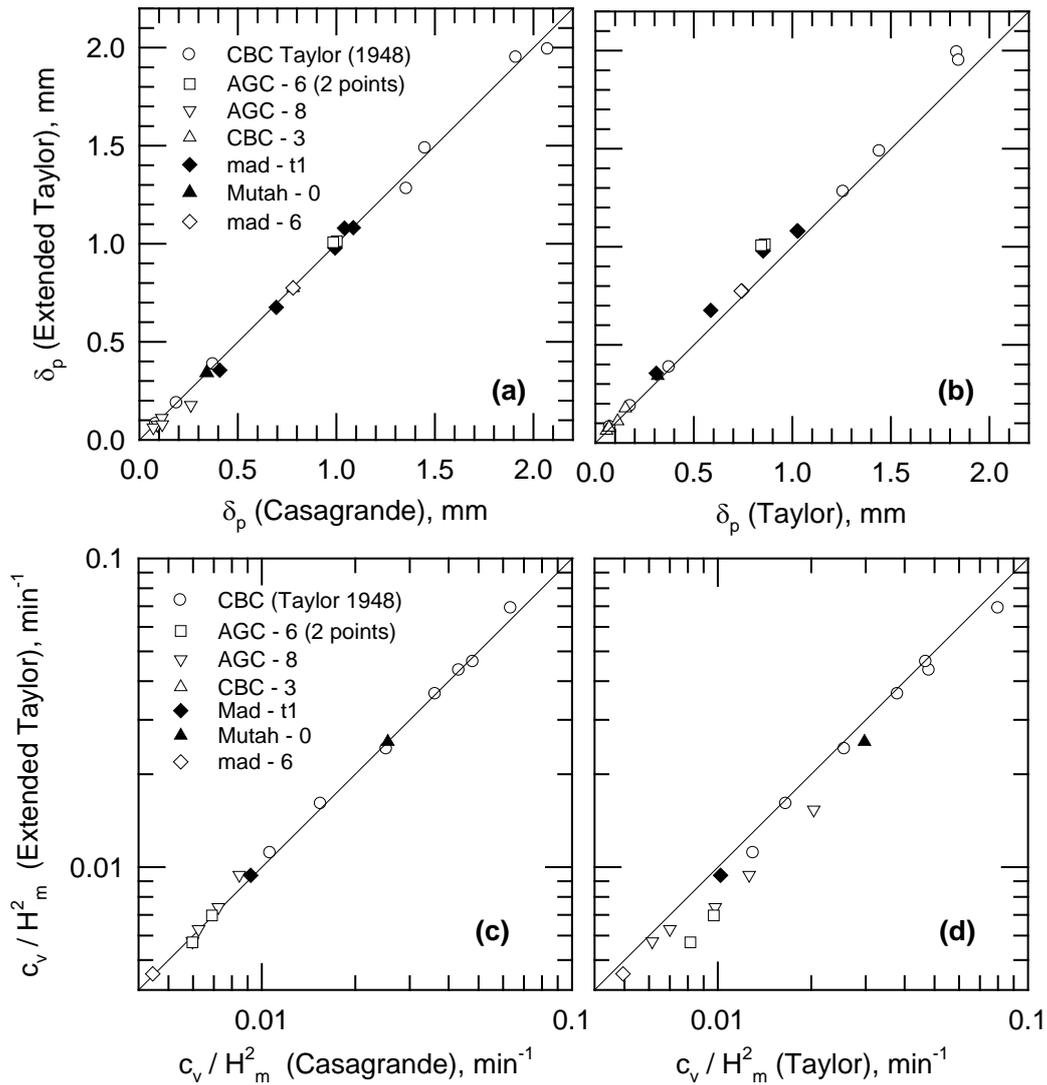


**Figure (3): The  $\delta_{pi} - \delta_{ti}$  relationships obtained by the extended Taylor method (ETM) and direct analytical method (DAM); treated Madaba clay (Mad-t1)**



**Figure (4): Comparison of the experimental  $U - T$  curves obtained using the extended**

**Taylor method (ETM) for three soil specimens with the Terzaghi theoretical relationship**



**Figure (5): (a) and (b) Comparison of  $\delta_p$  values of the extended Taylor method with those of the Casagrande and Taylor methods; (c) and (d) Comparison of  $c_v$  values of the extended Taylor method with those of the Casagrande and Taylor methods; for the cases where  $c_v$  is practically constant**

**Comparison with the Direct Analytical Method (Al-Zoubi, 2008a)**

Al-Zoubi (2008a) showed that the EOP settlement  $\delta_{pi}$  can be given, based on Eqs. 2 to 6, by the following rigorous expression (Eq. 20):

$$f(\delta_{pi}, \delta_{ti}, t_i) = \ln\left(1 - \frac{\delta_{ti}}{\delta_{pi}}\right) - \ln\frac{8}{\pi^2} + 1.94 \frac{m^2}{\delta_{pi}^2} t_i = 0 \quad (20)$$

In order to solve Eq. 20 for  $\delta_{pi}$ , three data points {i.e.,  $(t_1, R_1)$ ,  $(t_2, R_2)$  and  $(t_i, R_{ii})$ } must be selected from the consolidation data. The first two data points

$(t_1, R_1)$  and  $(t_2, R_2)$  are required for obtaining the initial compression  $R_0$  (Eq. 5) and the slope  $m$  of the initial linear portion of the  $\delta_i - \sqrt{t}$  curve (Eq. 7). The third data point  $(t_i, R_{ii})$  can be taken at any time beyond the initial linear portion (theoretically, at  $U \geq 52.6\%$ ).

The solution of Eq. 20 using the selected three data points requires iterations for obtaining the EOP  $\delta_{pi}$  (and then  $c_{vi}$  by Eq. 6); this solution can be obtained graphically or numerically by using any method for finding the roots of an equation (Al-Zoubi, 2008a). It should, however, be mentioned that Al-Zoubi (2014) developed a non-iterative procedure for solving Eq. 20.

Figures 2 and 3 show that the linear relationships between EOP  $\delta_{pi}$  and  $\delta_{ii}$  are practically the same for both the direct analytical method (DAM) and the extended Taylor method (ETM). Consequently, the global EOP  $\delta_p$  and  $c_v$  values obtained by the extended Taylor method (ETM) are almost identical to those of the direct analytical method (DAM).

The conventional Taylor procedure can be shown to be a graphical solution of the direct analytical method (DAM). Substituting  $\delta_{ii} = \delta_{90}$  and  $t_i = t_{90}$  into Eq. 20 yields a value of 1.15 for the ratio of the secant slope at 50% to that at 90% used by the Taylor method, where  $\delta_{90} = 0.90\delta_{pi}$ . The EOP  $\delta_{pi}$  values obtained by the Taylor method in which the third point  $(t_i, \delta_{ii})$  is taken at  $U = 90\%$  are shown in Figs. 2 and 3. These  $\delta_{pi}$  values of the Taylor method (open diamond) are on the linear relationships between  $\delta_{pi}$  and  $\delta_{ii}$  in primary consolidation obtained by the direct analytical and extended Taylor methods.

Hence, the conventional Taylor  $\sqrt{t}$  method (Taylor, 1948) is theoretically correct when only three data points are used. However, the Taylor method inherently includes limitation due to the fitting of the experimental compression-time curve in which the actual time to EOP consolidation exhibits a definite value (i.e.,  $t_p$ ) to the Terzaghi theory in which the theoretical time to EOP consolidation is infinity (Al-Zoubi, 2014). This limitation of the standard Taylor method is overcome in this study by introducing the

extended Taylor method (ETM) that uses forward extrapolation to obtain the global EOP  $\delta_p$ .

On the other hand, the EOP  $\delta_p$  values of the standard  $\log t$  method (open square) are quite comparable to those (global EOP  $\delta_p$  values) of the extended Taylor and direct analytical methods (Figs. 2 and 3). The  $\log t$  method yields global EOP settlement  $\delta_p$  as it is obtained by extrapolating the primary consolidation data to the secondary compression range.

### Comparison of the Proposed Extended Taylor Method (ETM) with the Taylor and Casagrande Methods

The conventional Taylor  $\sqrt{t}$  method (Taylor, 1948) generally yields lower  $\delta_p$  and higher  $c_v$  values than those of the  $\log t$  method (Casagrande and Fadum, 1940) as reported in the geotechnical engineering literature (Lambe and Whitman, 1969; Hossain, 1995; Sridharan and Prakash, 1995; Robinson, 1999; Al-Zoubi, 2010, 2013). The  $\delta_p$  values of the Taylor method may range from 0.5 to 1 of those of the  $\log t$  method, whereas the  $c_v$  values of the Taylor method may range from 1 to 4 times those of the  $\log t$  method (Al-Zoubi, 2008b).

The differences in  $c_v$  values estimated by the existing methods for a particular pressure increment may chiefly be attributed to two main reasons. Firstly, the different procedures used to estimate the EOP settlement may yield different  $\delta_p$  values and thus different  $c_v$  values are obtained. Secondly, the use of pressure increments in which  $c_v$  is not constant, contrary to the assumption of the Terzaghi theory, also contributes to the differences observed in the computed  $c_v$  values. The extended Taylor method (ETM) was developed for improving the procedure of estimating the EOP settlement  $\delta_p$  using the  $\sqrt{t}$  method as compared to the  $\log t$  method that yields almost identical  $\delta_p$  values to those obtained from pore water pressure measurements (Mesri et al., 1999b; Robinson, 1999). Moreover, only cases where  $c_v$  is practically constant are considered in this study to validate and compare the ETM with the standard  $\log t$  and  $\sqrt{t}$

methods to obtain more reliable results for evaluating and comparing these methods developed based on the Terzaghi theory that assumes constant  $c_v$ .

Experimental results of oedometer tests show that the  $\delta_p$  and  $c_v$  values obtained by the extended Taylor method (ETM) are quite similar to those of the standard  $\log t$  method as demonstrated in Figs. 5(a) and (c) for cases where  $c_v$  is practically constant (data points in this case mostly correspond to the normally consolidated range). These results also show that the EOP  $\delta_p$  values of the conventional Taylor method are lower than those of the extended Taylor and Casagrande methods as shown in Fig. 5(b), whereas the  $c_v$  values of the Taylor method are higher than those of the extended Taylor and Casagrande methods as shown in Fig. 5(d).

#### Graphical Procedure for the Extended Taylor Method (ETM)

The extended Taylor method (ETM) can be performed by using the experimental  $\delta_t - \sqrt{t}$  curve in the same graphical procedure as the conventional Taylor method (Fig. 1), but the procedure is repeated twice using any two different  $U_i$  values instead of the one used in the Taylor method at 90%. The procedure of ETM can be summarized as follows:

- A. Plot the compression - root time ( $R_t - \sqrt{t}$ ) curve.
- B. Obtain the initial compression  $R_0$  and slope  $m$  either graphically as shown in Fig. 1 or select two data points  $[(t_1, R_1), (t_2, R_2)]$  such that these two points are on the initial linear portion of the  $\delta_t - \sqrt{t}$  curve, and then compute  $R_0$  from Eq. 5 and  $m$  from Eq. 7.
- C. Arbitrarily select  $U$  value (say,  $U_1$ ) from the later stages of consolidation beyond the initial linear portion (e.g., 80%, 85%, 90%,... etc.) and compute the settlement  $\delta_{t1}$  ( $=\delta_{U1}$ ) that corresponds to the selected  $U_1$  value using Fig. 1 and the ratio ( $ROSS_i$ ) by Eq. 15 in the same way the 90% consolidation of the conventional Taylor method is obtained.
- D. Compute a first local EOP settlement

$$\delta_{p1} = \delta_{t1} / U_1.$$

- E. Repeat C and D; using another  $U$  value (say,  $U_2$ ).
- F. Compute a second local EOP settlement  $\delta_{p2} = \delta_{t2} / U_2$ .
- G. Repeat E and F using other  $U$  values if needed, for example, to plot Fig. 2.
- H. Calculate  $a$  and  $b$  using the linear  $\delta_{pi} - \delta_{ti}$  relationship (Fig. 2) or using the two points computed in D and F [i.e.,  $(\delta_{p1}, \delta_{t1}), (\delta_{p2}, \delta_{t2})$ ] as follows:

$$b = \frac{\delta_{p2} - \delta_{p1}}{\delta_{t2} - \delta_{t1}} \quad (21)$$

$$a = \delta_{p1} - b\delta_{t1} \quad (22)$$

- I. Calculate the global EOP settlement  $\delta_p$  by Eq. 19.
- J. Calculate  $c_v$  by Eq. 5.

An illustrative example is provided in the Appendix for the pressure increment of Fig. 1.

#### SUMMARY AND CONCLUSIONS

The Taylor graphical procedure is extended for evaluating the coefficient of vertical consolidation  $c_v$  and EOP settlement  $\delta_p$ . The extended Taylor method (ETM) utilizes the same graphical construction as the conventional Taylor method, but computes  $c_v$  and  $\delta_p$  by using at least two arbitrarily selected  $U_i$  values instead of the single value used in the conventional Taylor method at  $U_i = 90\%$ . The extended Taylor method (ETM) can be applied by using a minimum of four compression-time data points for estimating the coefficient of vertical consolidation; at least two data points are required from the early stages of consolidation ( $U \leq 52.6\%$ ) for back-calculating the initial compression (by backward extrapolation) and the initial slope of the  $\delta_t - \sqrt{t}$  curve and at least two data points from the later stages of consolidation ( $U \geq 52.6\%$ ) for computing the EOP settlement (by forward extrapolation).

In this study, two types of EOP settlement are distinguished; the first type is called the local EOP settlement  $\delta_{pi}$  obtained by matching the compression-

time curve to the Terzaghi theory at a specific  $U_i$  value as is the case in the conventional Taylor method; the second type is called the global EOP settlement  $\delta_p$  obtained by matching the compression-time curve to the Terzaghi theory at two or more  $U_i$  values as is the case in the extended Taylor method (ETM) and direct analytical method (DAM). The local EOP settlement  $\delta_{pi}$  is generally lower than the global EOP  $\delta_p$ . Therefore, the available methods that yield local EOP  $\delta_{pi}$  (e.g., Taylor method) generally give higher  $c_v$  values than those methods that yield global EOP  $\delta_p$  (e.g., Casagrande method, direct analytical method and extended Taylor method).

The extended Taylor method computes  $c_v$  and  $\delta_p$  values without the need to use secondary compression. Hence, the extended Taylor method requires as much testing time as the conventional Taylor method but yields  $c_v$  and  $\delta_p$  values quite similar to those of the Casagrande method for the case of constant  $c_v$ . The Casagrande method, however, requires longer testing time than both the Taylor and extended Taylor methods. Therefore, the proposed method has the advantages of both the Taylor and Casagrande methods.

#### Appendix (Example: Graphical Approach)

The graphical procedure of the extended Taylor method (ETM) can be summarized in a few steps.

Firstly, the initial compression  $R_0$  and the slope  $m$  are estimated from the initial linear portion of the  $\delta_t - \sqrt{t}$  curve (from Fig. 1,  $R_0 = 1516$  and  $m = 0.274$ ). Secondly, an arbitrarily selected  $U_i$  value is used to obtain a first local EOP settlement in a way similar to the graphical procedure used in the Taylor method at  $U_i = 90\%$  (from Fig. 1, for  $U_1 = 85\%$ ,  $ROSS = 1.096$  by Eq. 15,  $\delta_{t1} = 1.5550$  mm, and thus  $\delta_{p1} = 1.8294$  mm by Eq. 16). Thirdly, another arbitrarily selected  $U_2$  value is used to obtain a second local EOP settlement (from Fig. 1, for  $U_2 = 95\%$ ,  $ROSS = 1.260$ ,  $\delta_{t2} = 1.8039$  mm,  $\delta_{p2} = 1.8989$  mm). This step can be repeated using additional  $U_i$  values ( $U_3, U_4, \dots$  etc.) if needed to plot Fig. 2. Fourthly, the  $a$  and  $b$  values can be calculated by Eq. 16 if only two  $U_i$  values are used or can be obtained graphically from a plot similar to Fig. 2 if more than two  $U_i$  values are used. For the pressure increment of Fig. 1,  $a = 1.2617$  and  $b = 0.3503$  (Fig. 2) and therefore the global EOP settlement  $\delta_p$  can be obtained by Eq. 19 as  $\delta_p = a/(1-b) = 1.942$  mm. Finally, the  $c_v/H_m^2$  value can be computed by Eq. 6 as  $\pi(0.274/1.942)^2/4 = 0.0156 \text{ min}^{-1}$ . The corresponding values of the Casagrande and Taylor methods for this particular pressure increment (Fig. 1) are 0.0155 and 0.0173, respectively; the  $c_v$  value computed by the ETM is quite similar to that of the Casagrande method.

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