

A Time Domain Approach for Identifying Dynamic Forces Applied on Structures

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ABSTRACT

A time domain approach based on the structural response is presented for identifying dynamic excitation forces applied on three-dimensional steel trusses. Sub-structure finite element model with a short length of measurement from only four or five accelerometers is required, and an iterative least-squares algorithm is used to identify the dynamic forces applied on the structure. The location of the input force is assumed to be known in the identification. Validity of the method is demonstrated by means of numerical examples using noise-free and noise-contaminated structural response. Both harmonic and impulsive forces are studied. The results show that the proposed approach can identify unknown excitations within very limited iterations with high accuracy and show its robustness even when noise-polluted dynamic response measurements are utilized.

KEYWORDS: Time domain, Dynamic force identification, Limited response and sub-structure.

INTRODUCTION

Accurate identification of static and dynamic forces can be very important to the structural design process. The first step in any structural design is to determine the static and dynamic forces that are applied to the floors and diaphragms of the structures. Usually, structural designers adopt the codes of loads available in each country in order to determine the required applied loads. In addition, in order to identify the location of damages in the structural elements and to determine the amount and importance of the defects on the overall structural behavior, system identification techniques are usually used that require information about the dynamic forces applied.

The system identification techniques that have been

used in the last three decades (Sohn et al., 2004; Kerschen et al., 2006) have three components; input excitation, the system and the output response information. The input excitation is the force that excites the system. The system is a mathematical model of the structure. The output is the response of a structural system due to the input excitation, reflecting the current state of the structure. Knowing the input excitation and the output response information, the system being the third component can be identified. Unfortunately, in most cases (Lam et al., 2004; Beck and Yuen, 2004; Koh et al., 2003), it is impossible to insert force gauges into the force transfer path to directly measure those dynamic forces. Therefore, in order to get better damage detection, it is required to identify the dynamic forces applied to the structures.

There are many methods available in the literature for force identification; the Frequency Response

Function (FRF) - based least-squares approach is the most widely used because it can be applied to a variety of force identification problems. The basic premise of the FRF approach is based on spectral analysis. Given the measured vibrational response at one or more locations and the frequency-domain FRF matrix, one can back-calculate the dynamic excitation forces at each specific frequency by pre-multiplying the measured response vector by the pseudo-inverse of the FRF matrix at that frequency. The pseudo-inverse technique is also known as a least-squares method. An inverse Fourier transform on these computed values provides a time history of the dynamic forces, which is of great interest in many cases such as impact force identification. Depending on the number of measured response points and other important parameters, this technique can be used to find a single force or a set of forces acting on the structure. However, this least-squares approach in the frequency domain can be hindered by the direct inversion of an ill-conditioned FRF matrix at frequencies near the structural resonances. To overcome this inversion instability, Liu and Shepard Jr. (2005) proposed two regularization filters; namely the Truncated Singular Value Decomposition (TSVD) filter and the Tikhonov Filter in conjunction with the conventional least-squares scheme at specific frequencies.

Another efficient technique for identifying the impact force acting on laminated plates was proposed by Hu et al. (2007). Chebyshev polynomial is employed to approximate the impact force history where the coefficients in the polynomial are directly used as unknown parameters. The relation between these unknown parameters and the strain response at the specified positions is formulated through the finite element method and the mode superposition method. After obtaining the impact force history, the impact position is identified by comparing the numerical strains and experimental ones directly.

Lu and Law (2007) proposed a method based on the sensitivity of structural response for identifying both the system parameters and the input excitation force of

a structure. An iterative gradient-based model updating method based on the dynamic response sensitivity was adopted. The poor identification with relatively insensitive parameters in a mixture of parameters with different sensitivities was addressed and solved with another loop of optimization.

Marchesiello and Garibaldi (2008) proposed a method in the time domain for the identification of nonlinear vibrating structures. The method allows for the estimation of the coefficients of the nonlinearities away from the location of the applied excitations and also for the identification of the linear dynamic compliance matrix when the number of excitations is smaller than the number of response locations.

Allen and Carne (2008) proposed an extension of the Inverse Structural Filter (ISF) force reconstruction algorithm that utilizes data from multiple time steps simultaneously to improve the accuracy and robustness of the ISF. The ISF algorithm uses a discrete time system model of a structure and the measured response to estimate the forces causing the response.

Yan and Zhou (2009) proposed a genetic algorithm (GA)-based approach for impact load identification, which can identify the impact location and reconstruct the impact force history simultaneously. In this study, impact load is represented by a set of parameters, thus the impact load identification problem in both space (impact location) and time (impact force history) domains is transformed to a parameter identification problem. A forward model was incorporated to characterize the dynamic response of the structure subject to a known impact force. By minimizing the difference between the analytical response given by the forward model and the measured ones, GA adaptively identifies the impact location and force history with its global search capability.

Lately, Xu et al. (2012) proposed an iterative approach for both structural parameters and dynamic loading identification, referred to as Weighted Adaptive Iterative Least-Squares Estimation with Incomplete Measured Excitations (WAILSE-IME). The accuracy, convergence and robustness of the

proposed approach were demonstrated via numerical simulation on a six-storey shear building model with noise-free and different levels of noise-polluted structural dynamic response measurements.

In the present paper, a time domain approach based on the least-squares method is used to identify the dynamic forces applied on three-dimensional steel trusses using a sub-structure finite element model. A short length of measurement from only four or five accelerometers is required for the identification process. The location of the input force is assumed to be known in the identification. Validity of the method is demonstrated with numerical examples using noise-free and noise-contaminated structural response.

ALGORITHM AND SUB-STRUCTURE MODELING

The sub-structure required for force identification should be selected in a way that the response measurements are available at all Degrees of Freedom (DOF) of the sub-structure and the location of the dynamic excitation force is assumed to be known and included in the sub-structure. These constraints will not affect the accuracy of identification and will use a short length of measurement from only four or five

$$\mathbf{K}_i = \frac{E_i A_i}{L_i} \begin{bmatrix} C_x^2 & C_x C_y & C_x C_z & -C_x^2 & -C_x C_y & -C_x C_z \\ C_x C_y & C_y^2 & C_y C_z & -C_x C_y & -C_y^2 & -C_y C_z \\ C_x C_z & C_y C_z & C_z^2 & -C_x C_z & -C_y C_z & -C_z^2 \\ -C_x^2 & -C_x C_y & -C_x C_z & C_x^2 & C_x C_y & C_x C_z \\ -C_x C_y & -C_y^2 & -C_y C_z & C_x C_y & C_y^2 & C_y C_z \\ -C_x C_z & -C_y C_z & -C_z^2 & C_x C_z & C_y C_z & C_z^2 \end{bmatrix} \quad (2)$$

where E_i , A_i and L_i are the Young's modulus, area of the cross-section and length of the i^{th} element in the sub-structure, respectively. In addition:

$$C_x = \cos \theta_x, \quad C_y = \cos \theta_y, \quad C_z = \cos \theta_z \quad (2a)$$

where θ_x , θ_y and θ_z are the angles between the local

accelerometers instead of the whole structure.

The selection of the sub-structure will start by determining the node where the unknown dynamic force is applied. Then, it is required to determine the nodes and elements that are attached to that node. Accordingly, the sub-structure is now selected and the governing dynamic equation for the sub-structure can be written as:

$$\mathbf{K}_s \mathbf{x}_s(t) + \mathbf{C}_s \dot{\mathbf{x}}_s(t) + \mathbf{M}_s \ddot{\mathbf{x}}_s(t) = \mathbf{f}(t) \quad (1)$$

where \mathbf{K}_s , \mathbf{C}_s and \mathbf{M}_s are the global stiffness, damping and mass matrices for the sub-structure, respectively, $\ddot{\mathbf{x}}_s(t)$, $\dot{\mathbf{x}}_s(t)$ and $\mathbf{x}_s(t)$ are vectors containing the dynamic response in terms of acceleration, velocity and displacement at time t for the sub-structure, respectively, and $\mathbf{f}(t)$ is the unknown dynamic force vector applied on the structure.

The global stiffness matrix for the sub-structure (\mathbf{K}_s) can be assembled by using the method of superposition, the direct stiffness method for the local stiffness matrices of all the elements in the sub-structure. The local stiffness matrix (\mathbf{K}_i) for a three-dimensional truss element of uniform cross-section is given by:

axis \widehat{X} , \widehat{Y} and \widehat{Z} and global axis X , Y and Z , respectively.

The damping matrix \mathbf{C}_s is assumed to be Rayleigh-type damping and can be represented as:

$$\mathbf{C}_s = \alpha \mathbf{M}_s + \beta \mathbf{K}_s \quad (3)$$

where α is the mass-proportional damping coefficient and β is the stiffness-proportional damping coefficient.

The global consistent mass matrix for the sub-structure (\mathbf{M}_s) can be assembled by using the method of superposition for the local mass matrices of all the elements in the sub-structure. The local consistent mass matrix (\mathbf{M}_i) for a three-dimensional truss element of uniform cross-section is given by:

$$\mathbf{M}_i = \frac{\rho_i L_i A_i}{6} \begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix} \quad (4)$$

where ρ_i , A_i and L_i are the density, area of the cross-section and length of the i^{th} element in the sub-structure, respectively.

Accordingly, Equation (1) can be rewritten in a matrix form as:

$$[\mathbf{A}] * \{\mathbf{B}\} = \{\mathbf{G}\} \quad (5)$$

where $[\mathbf{A}]$ is a matrix of size $(3 \times n) \times L_s$; n is the total number of sample time points; L_s is the total number of elements and damping coefficients in the sub-structure and can be expressed as:

$$[\mathbf{A}] = [\mathbf{Q}^1 \dot{\mathbf{x}}_s(t) \quad \mathbf{Q}^2 \dot{\mathbf{x}}_s(t) \quad \dots \quad \mathbf{Q}^{nes} \dot{\mathbf{x}}_s(t) \quad \mathbf{Q}^1 \ddot{\mathbf{x}}_s(t) \quad \mathbf{Q}^2 \ddot{\mathbf{x}}_s(t) \quad \dots \quad \mathbf{Q}^{nes} \ddot{\mathbf{x}}_s(t) \quad \mathbf{M}_s \dot{\mathbf{x}}_s(t)] \quad (6)$$

where \mathbf{Q} is the 6x6 matrix in Equation (2) excluding (EA/L) for each element in the sub-structure and nes is the total number of elements in the sub-structure.

$\{\mathbf{B}\}$ vector in Equation (5) is a vector of size $L_s \times 1$ and

can be shown to be:

$$\{\mathbf{B}\} = \left\{ \begin{array}{c} \mathbf{k}_1 \\ \mathbf{k}_2 \\ \dots \\ \mathbf{k}_{nes} \\ \beta \mathbf{k}_1 \\ \beta \mathbf{k}_2 \\ \dots \\ \beta \mathbf{k}_{nes} \\ \alpha \end{array} \right\} \quad (7)$$

$\{\mathbf{G}\}$ vector in Equation (5) is a vector of size $(3 \times n) \times 1$ and can be shown to be:

$$\{\mathbf{G}\} = \left\{ \begin{array}{c} f_1(t) - \mathbf{M}_s \ddot{\mathbf{x}}_s(t) \\ f_2(t) - \mathbf{M}_s \ddot{\mathbf{x}}_s(t) \\ \dots \\ f_{TD}(t) - \mathbf{M}_s \ddot{\mathbf{x}}_s(t) \end{array} \right\} \quad (8)$$

where f is the unknown dynamic force needed to be identified and TD is the total number of DOF in the sub-structure.

The stiffness of each element (EA/L) in the three-dimensional trusses could be assumed known and can be provided from the ‘‘As Built’’ drawings. Since it is sometimes difficult to obtain this information from ‘‘As Built’’ drawings, especially for old structures, it is assumed that the stiffness of each element is unknown and will be identified with the unknown dynamic force.

A least-squares-based procedure proposed by Wang and Haldar (1994) is used in this paper for the solution of the unknown dynamic force $f(t)$ by starting an iteration process assuming the unknown dynamic force to be zero at all n time sample points. This assumption will assure a nonsingular solution of Equation (5), without compromising the convergence or the accuracy

of the method. It is observed through the numerical examples shown below that the method is not sensitive to this initial assumption, or the type and form of excitation.

Using the least-squares-based procedure proposed by Wang and Haldar (1994), the solutions of unknown system parameters $\{\mathbf{B}\}$ and unknown dynamic force $f(t)$ are evaluated using the following expression:

$$\{\mathbf{B}\} = \left[\mathbf{A}^T \right] \left[\mathbf{A} \right]^{-1} \left[\mathbf{A}^T \right] \left[\mathbf{G} \right]. \quad (9)$$

The algorithm will iterate until a convergence in the unknown dynamic force with a predetermined tolerance set to be infinitesimal; i.e. 10^{-8} . Accordingly, the unknown dynamic force is determined with a reasonable accuracy.

The basic steps for the iterative algorithm can be summarized as follows:

- Step 1:** Formulate the local stiffness matrix for each element from Equation (2).
- Step 2:** Assemble the global consistent mass matrix (\mathbf{M}_s) for all the elements in the sub-structure from the local mass matrix (\mathbf{M}_i); i.e. Equation (4).
- Step 3:** Formulate the $[\mathbf{A}]$ matrix from Equation (6) which is composed of global consistent mass matrix \mathbf{M}_s , the local stiffness matrices of each element and the velocity and displacement response of the system as at each DOF in the sub-structure.
- Step 4:** Formulate the $\{\mathbf{G}\}$ vector from Equation (8).
- Step 5:** Assume the dynamic excitation force vector $f(t)$ to be zero at all time points.
- Step 6:** Obtain the first estimation of $\{\mathbf{B}\}$ vector by solving Equation (9) using the least-squares concept.
- Step 7:** Substitute $\{\mathbf{B}\}$ vector estimated from Step 6 into Equation (1) to obtain the unknown dynamic excitation force vector $f(t)$ at all time points.
- Step 8:** Iterate until a convergence in the unknown

dynamic force with a predetermined tolerance set to be infinitesimal; i.e. 10^{-8} .

NUMERICAL EXAMPLES

A three-dimensional steel truss (shown in Figure 1) is used to validate the effectiveness of the method in identifying the unknown dynamic forces. The length of the truss is 6.0 m at the base and 3.0 m at the top, the width is 6.0 m and the height is 8.0 m as shown in Figure 2. Steel tubes are used for all members; the outer and inner diameters are 11.4 cm and 10.2 cm, respectively. The nominal wall thickness of the tubes is 0.60 cm and the area (A) of each member is 19.16 cm^2 . The truss is made of 20 nodes and 52 elements. Each node consists of 3 DOF; translation in x , y and z . The total number of DOF for the whole structure is 48 considering nodes 9, 10, 19 and 20 are pin supports.

Two cases representing two types of dynamic forces are adopted in this example:

Case 1: A harmonic force $\mathbf{f}(t) = 10 \sin(20\pi t)$ is applied on node 1 of the three-dimensional truss. Figure 3 shows the details of the harmonic force.

Case 2: An impact force of 10 kN at 0 sec and 0 kN at 0.05 sec as shown in Figure 4 is applied on node 1 of the three-dimensional truss.

Based on the basic modeling and formulation of the least-squares method mentioned above, a sub-structure is needed for identifying the unknown dynamic force. The selection of the sub-structure is started by determining the node where the unknown dynamic force is applied which is node 1 in this example, followed by determining the nodes and elements attached to node 1, which are: nodes 1, 2, 3, 4 and 11 and elements 1, 6, 14 and 35. Figure 5 shows the sub-structure needed. It consists of 5 nodes and 4 elements. Accordingly, five accelerometers are needed to be placed at nodes 1, 2, 3, 4 and 11 to measure the x , y and z dynamic translation response; i.e. the total number of DOF required for this sub-structure is 15.

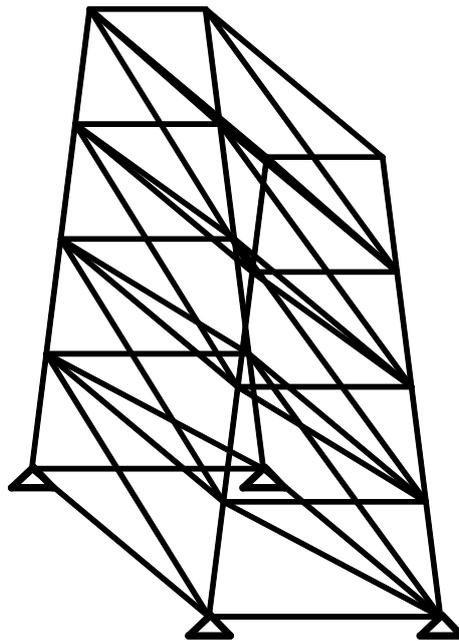


Figure 1: Three-dimensional steel truss used in the numerical examples

Case 1: Identifying Unknown Harmonic Force

The 3D steel truss is modeled using finite element software package SAP 2000. The harmonic force is applied on node 1. The theoretical dynamic responses and the acceleration, velocity and displacement of all 48 DOF were obtained. These responses resemble the recorded data of a real structure under the effect of dynamic loadings. As soon as the theoretical response has been evaluated, the information on the harmonic force is completely ignored, and the nodal response of the sub-structure; i.e. 15 DOF is only used in the algorithm. The location of the input harmonic force is assumed to be known in the identification and the stiffness of each element (EA/L) in the sub-structure is assumed to be unknown since it is sometimes difficult to obtain this information from “As Built” drawings as mentioned previously. The response used in the algorithm is with a short length of measurement. In this case, the response is from 0.03 sec to 0.50 sec at a time interval of 0.01 sec, yielding 48 time points only. The

response is assumed to be noise-free. However, from an experimental point of view, noise in the response measurements cannot be avoided. To address the issue of noise in the dynamic response, a numerically generated noise with an intensity of 8% of the root mean square (RMS) values of the response observed at all DOF is added to the theoretical response. Accordingly, the unknown harmonic force is identified by using both noise-free and noise-including dynamic response.

Figure 6 shows the results of the harmonic force identification for noise-free and noise-including cases compared with the exact force. It is obvious that the algorithm and the sub-structure identified the unknown harmonic force very effectively in both cases. The maximum error in force identification in the noise-free case was less than 1%, and this percentage was more for the noise-including dynamic response but less than 3%.

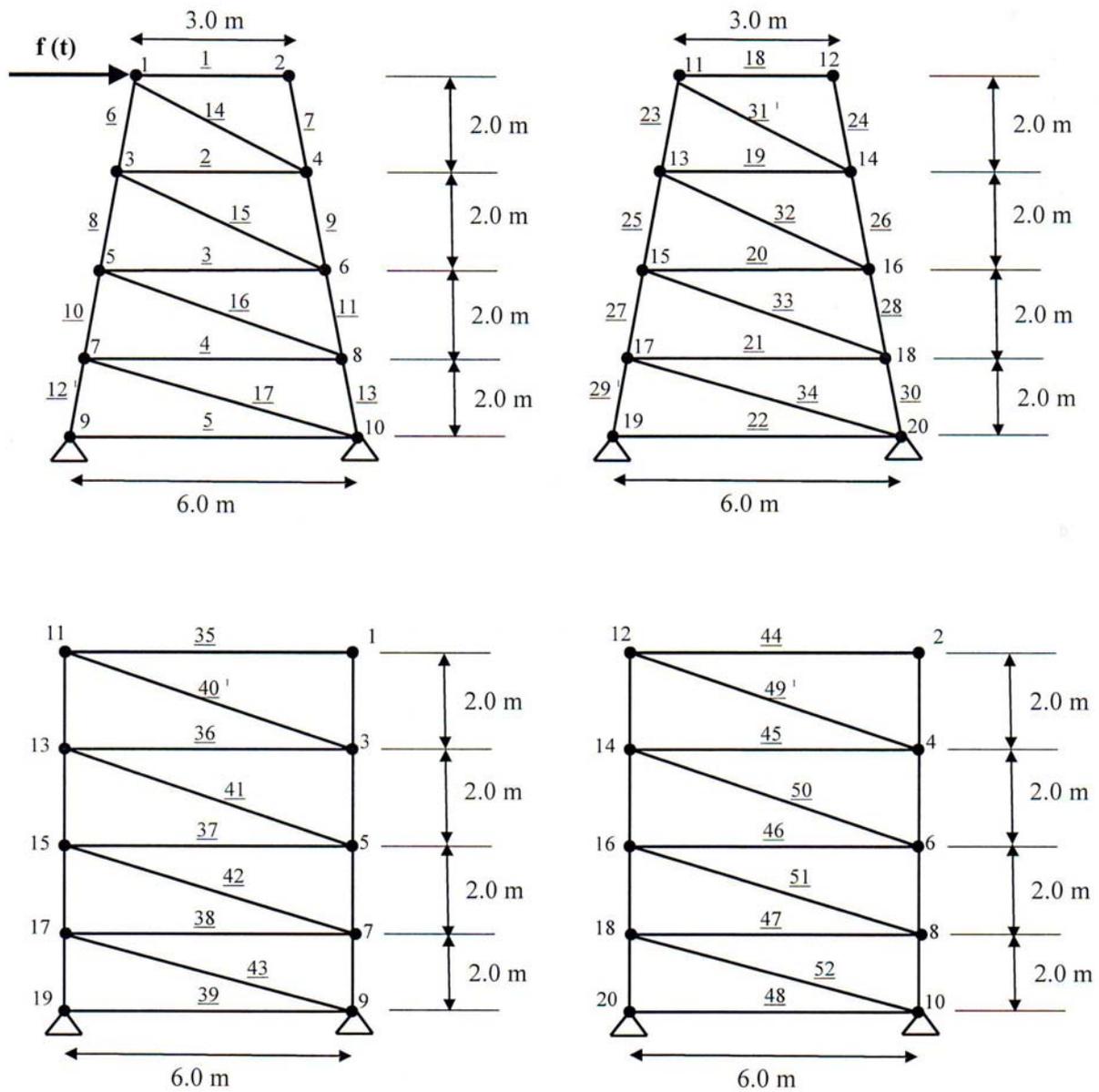


Figure 2: Two-dimensional elevations and side views of the three-dimensional truss

Case 2: Identifying Unknown Impact Force

The 3D steel truss is modeled again using finite element software package SAP 2000. The impact force is applied on node 1. The theoretical dynamic response including the acceleration, velocity and displacement of all the 48 DOF were obtained. After the theoretical

response is evaluated, the information on the impact force is completely ignored and the nodal response of the sub-structure; i.e. 15 DOF is only used in the algorithm. The response used in the algorithm is with a short length of measurement. In this case, the response is from 0.01 sec to 0.05 sec at a time interval of 0.001

sec, yielding 41 time points only. The unknown impact force is identified by using both noise-free and noise-including dynamic response. To address the issue of noise in the dynamic response, a numerically generated

noise with an intensity of 10% of the root mean square (RMS) values of the response observed at all DOF is added to the theoretical response.

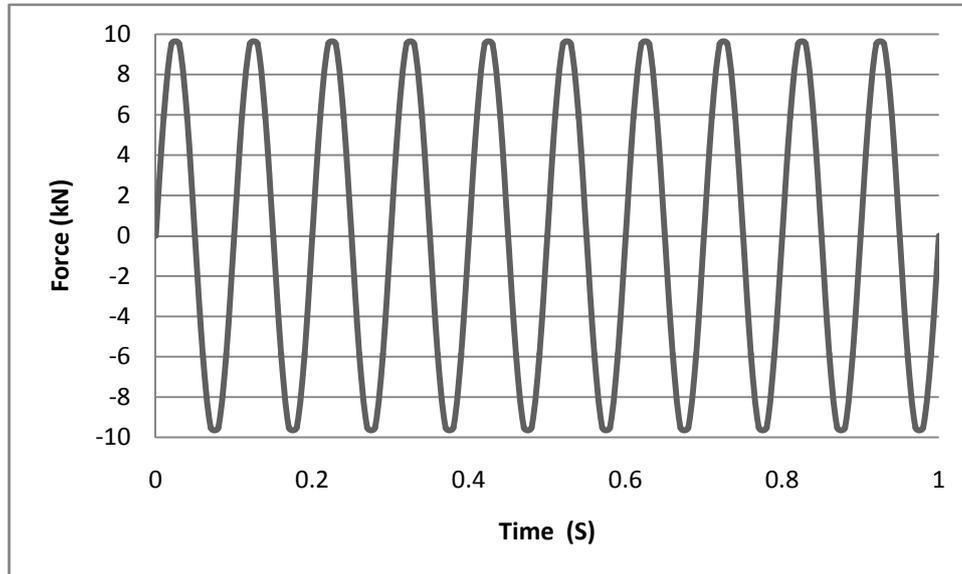


Figure 3: Harmonic force applied on the 3D truss

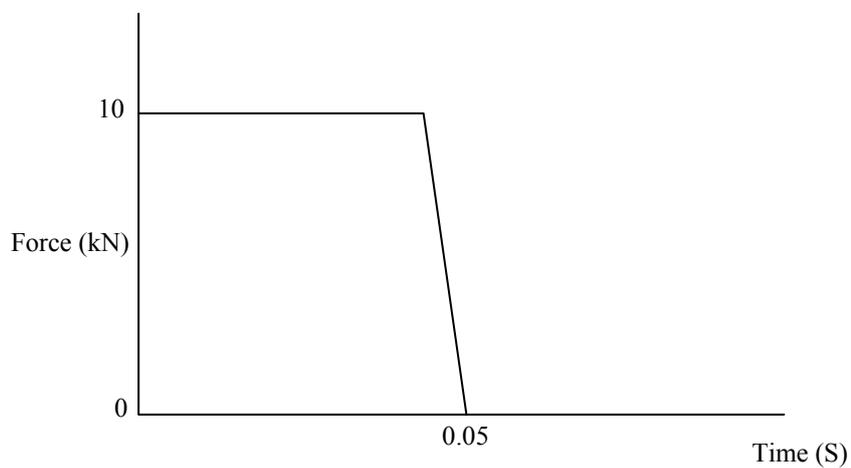


Figure 4: Impact force applied on the 3D truss

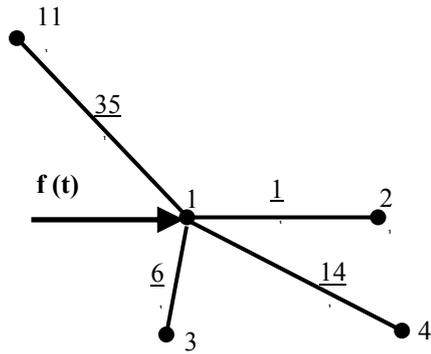
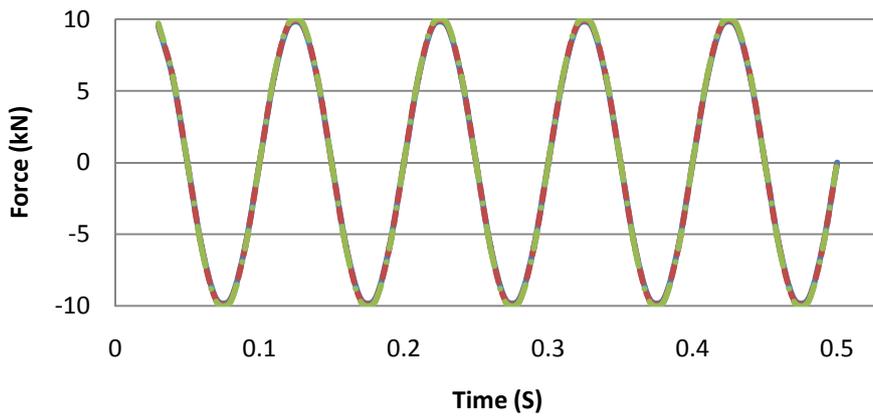
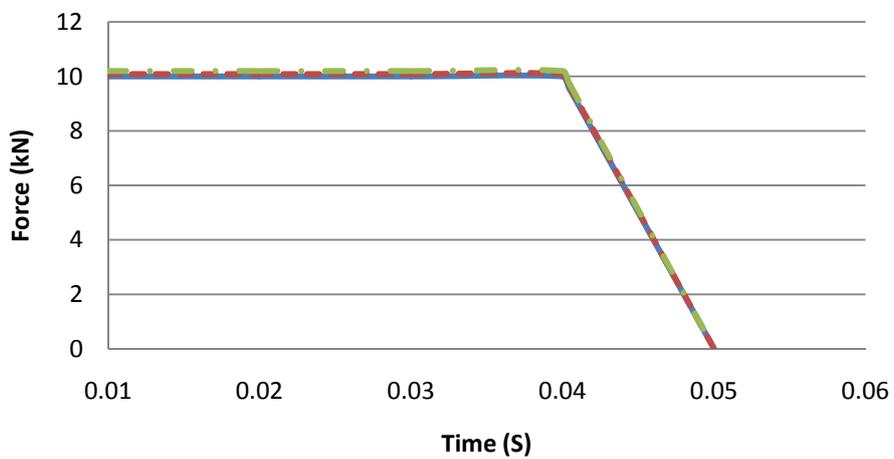


Figure 5: Sub-structure needed for identifying the unknown dynamic force



— Exact - - - Identified Noise Free - · - Identified Noise Included

Figure 6: Force identification at Node 1 for Case 1



— Exact - - - Identified Noise Free - · - Identified Noise Included

Figure 7: Force identification at Node 1 for Case 2

Figure 7 shows the results of impact force identification for noise-free and noise-including cases compared with the exact force. It is obvious that the algorithm and the sub-structure identified the unknown impact force very well in both cases. The maximum error in force identification in the noise-free case was less than 0.8%, and this percentage was more for the noise-including dynamic response but less than 4%.

CONCLUSIVE REMARKS AND DISCUSSION

For the two cases studied in this paper, the method identified the unknown dynamic forces that are applied on three-dimensional steel trusses very well. The main advantage of this method over many other methods available in the literature was the fact that it uses an optimum number of accelerometers in the identification process. It is not feasible or practical to place a large number of accelerometers on all nodes of 3D trusses and record the accelerations for all DOF to use those dynamic response records in identifying unknown dynamic forces. As was shown in the numerical examples, the steel truss has 20 nodes, 52 elements and 48 DOF, but only a sub-structure with 5 nodes, 4 elements and 15 DOF was enough to identify the unknown dynamic forces accurately.

Another advantage of this approach was the short length of measurement used in the force identification. In case 1; i.e. harmonic force, 48 time points were only used, as well as a time interval from 0.03 sec to 0.50 sec at 0.01 sec. For case 2; i.e. impact force, 41 time points only were used, as well as a time interval from 0.01 sec to 0.050 sec at 0.001 sec. Although the algorithm used a small number of time points, the results of force identification are considered to be accurate. It has been observed by the author that increasing the number of sample points does not have major impact on the accuracy of force identification. This can be considered as an advantage, since most of time domain methods available in the literature are very sensitive to the number of sample points used in identification.

On the other hand, the two cases studied in this paper showed that the method is capable of identifying the unknown dynamic force accurately regardless the type of the unknown dynamic load applied on the three-dimensional steel trusses. The algorithm identified the harmonic and impact forces with a maximum error in identification less than 1% and 0.8%, respectively. Even more, the algorithm identified the unknown dynamic forces very well by using noise-including response. The algorithm identified the harmonic and impact forces with a maximum error in identification less than 3% and 4% for noise-including response with an intensity of 8% and 10% of the root mean square (RMS) values of the response observed, respectively. These results can be considered another substantial advantage, since most of time domain methods available in the literature are very dependent on the type of forces applied on the structures and very sensitive to the noise-including response.

It is worth mentioning that these results were obtained based on the assumption that the stiffness (EA/L) of each member in the sub-structure used is unknown, since it is sometimes difficult to obtain this information from "As Built" drawings, especially for old structures as previously mentioned. It seems that this assumption does not have major impact on the accuracy of force identification.

Accordingly, this method can be considered as an effective method in identifying unknown dynamic forces that can be used in system identification approaches in time and frequency domains to identify the locations of the structural elements that suffered structural damage and to determine the amount and importance of the defects on the overall structural behavior.

CONCLUSIONS

A time domain approach based on the structural response is presented for identifying dynamic excitation forces applied on three-dimensional steel trusses. A sub-structure finite element model with a

short length of measurement from only four or five accelerometers was required for the iterative least-squares algorithm to identify the unknown dynamic force applied on the structure.

The results showed that the method identified the unknown dynamic force applied on three-dimensional trusses accurately for both harmonic and impulsive

forces. Also, the results showed that the approach can identify unknown excitations within very limited iterations with high accuracy. The approach also showed its robustness in the case that even noise-polluted dynamic response measurements had been utilized.

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