

Pushover Analysis for Cold Formed Storage Rack Structures

Sreedhar Kalavagunta¹⁾, Sivakumar Naganathan²⁾ and Kamal Nasharuddin Bin Mustapha²⁾

¹⁾ Bentley Systems Limited, Singapore,

Corresponding Author. E-Mail: sreedhar.kalavagunta@bentley.com

²⁾ Department of Civil Engineering, Universiti Tenaga Nasional, Jalan IKRAM-UNITEN, 43000, Kajang, Selangor, Malaysia

ABSTRACT

This study investigated the progressive collapse of cold formed storage rack structures subjected to seismic loading, using pushover analysis. A simple storage rack cold formed steel structure was analyzed with static, non-linear procedure in accordance with FEMA 356 specifications and progressive collapse recorded such as occupancy, immediate occupancy, life safety and collapse prevention. Pushover analysis was found to be a useful analysis tool for the conventional storage racking systems giving good estimates of the overall displacement demands, base shears and plastic hinge formation.

KEYWORDS: Cold formed steel, Pushover analysis, Plastic hinge, FEMA, ATC, Capacity.

INTRODUCTION

While conventional limit-states design is typically a two-level design approach having concern in the service operational and ultimate-strength limit states for a building, performance-based design can be viewed as a multi-level design approach that additionally has explicit concern in the performance of a building at intermediate limit states related to such issues as occupancy and life-safety standards. With the emergence of the performance-based approach in design, there is a need to develop corresponding analysis tools. Nonlinear static (pushover) analysis is often an attractive choice in this regard because of its simplicity and ability in identify component and system-level deformation demands with accuracy comparable to dynamic analysis (Hasan et al., 2002; Fu and Alayed, 2006). The non-linear static procedure (NSP) or pushover analysis, as described in FEMA-273 (BSSC, 1997) and its successor FEMA-356 (ASCE,

2000) is now used by the structural engineering profession as a standard tool for estimating seismic demands for buildings. In the past, several researchers have discussed the underlying assumptions and limitations of pushover analysis (Krawinkler and Seneviratna, 1998; Elnashai, 2001) and proposed adaptive force distributions that attempt to follow the time-variant distributions of inertia forces (Bracci et al., 1997; Gupta and Kunnath, 2000). They considered more than the fundamental vibration mode (Sasaki et al., 1998; Kunnath and Gupta, 2000; Matsumori et al., 1999). Anil K. Chopra reviewed the above papers and proposed a modal pushover analysis procedure to estimate seismic demands for asymmetric-plan buildings (Chopra and Goel, 2004).

DEFINITION

Pushover Analysis: is a static, nonlinear procedure in which the magnitude of the structural loading is incrementally increased in accordance with a certain predefined pattern. In effect, the structure is pushed

sideways well into the inelastic range till total failure or collapse occurs; hence, this method is called pushover analysis (Zeynalian and Ronagh, 2001).

Purpose and Methodology: The purpose of pushover analysis is to assess the structural performance by estimating the strength and deformation capacities using static, nonlinear analysis and comparing these capacities with the demands at the corresponding performance levels. The basic procedure of this method is to perform a sequence of elastic static analysis under monotonically increasing lateral loads in each of its principle directions to stimulate the loading history of the structure during collapse. The potential of the pushover analysis has been recognized in the last decade and it has found its way into seismic guidelines [FEMA 356, 2000; ATC, 1997; SEAOC, 1995; CEN 1995].

Analysis Highlights: The pushover is expected to provide information on many response characteristics that can't be obtained from an elastic static or dynamic analysis. The following are examples of such response characteristics (Bajoria and Sangle, 2008):

- Realistic force demands on potentially brittle elements, such as axial demands on columns, moment demands on beam to column connections or shear demands on short, shear-dominated elements.
- Identification of the critical regions, where the inelastic deformations are expected to be high.
- Estimates of the deformation demands on elements that have to deform in elastically, in order to dissipate energy.
- Consequences of strength deterioration of particular elements on the overall structural stability.
- Estimates of inter-storey drifts, accounting for strength and stiffness discontinuities. In this way, damage on nonstructural elements can be controlled.
- Sequence of members yielding and failure and the

progress of the overall capacity curve of the structure.

- Identification of the strength irregularities in plan or elevation that cause changes in the dynamic characteristics in the inelastic range.
- Verification of the adequacy of the load path, considering all the elements of the system, both structural and nonstructural.

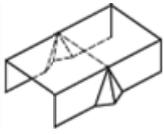
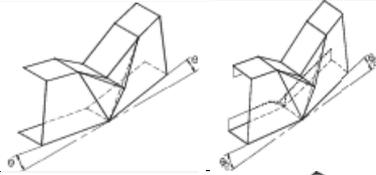
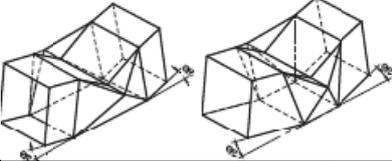
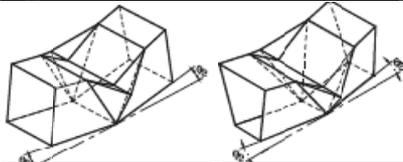
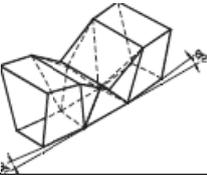
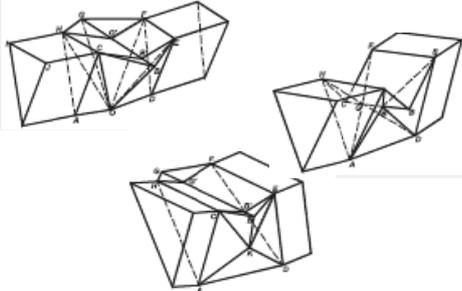
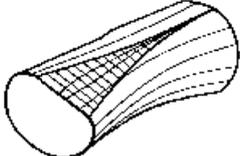
Development of Plastic Hinges: Short members of thin-walled cold formed steel sections, in compression and bending, fail by forming local plastic mechanisms (Murray and Khoo, 1981). Taking into account the localized buckling pattern, the collapse of slender members, due to the interaction between local and overall buckling modes, is always characterized by local plastic mechanism failure mode (Kotelko et al., 2000; Ungureanu and Dubina, 2004). A proper identification of the geometry of a local plastic mechanism is crucial for correct evaluation of the post buckling rigid-plastic curve (failure curve), which subsequently results in a correct estimation of the load-carrying capacity of the member, as well as of the energy absorption at collapse (Kotelko, 2004; Dubina and Ungureanu, 2000). A collection of plastic mechanisms of failure for thin-walled cold formed steel members subjected to compression and bending are tabulated [Table 1].

PUSHOVER ANALYSIS

Theory: Nonlinear static (pushover) analysis is often an attractive choice in this regard because of its simplicity and ability to identify component and system-level deformation demands with an accuracy comparable to dynamic analysis (Bentley Reference Manual; Poursha et al., 2011; Inel and Ozmen, 2006). There are two non-linear procedures using pushover methods.

- Capacity Spectrum Method (CSM).
- Displacement Coefficient Method (DCM).

Table 1. Plastic mechanisms (Ungureanu et al., 2010)

No	Beam cross-connection	Mechanism	References
1	Channel (bending in the flange plane)		Murray and Khoo (1981); Strip equilibrium method applied
2	Channel (bending in the web plane)		Kotelko (2004); Energy method applied
3	Rectangular or trapezoidal box-section		Kecman (1983); Kotelko (1996); Energy method applied
4	Rectangular or trapezoidal box-section		Kotelko (1996); Energy method applied
5	Trapezoidal box-section		Kotelko (1996); Energy method applied
6	Triangular cross-section		Kotelko and Krolak (1993); Energy method applied
7	Circular hollow section		Elchalakani et al., (2002); Elchalakani et al. (2002); Energy method applied

Capacity Spectrum Method is to develop appropriate demand and capacity spectra for the structure and to determine their intersection point. During this process, performance of each structural component is also evaluated. The structure capacity is represented by a pushover curve, often termed as capacity curve. This represents the lateral displacement as a function of the force applied to the structure. The most convenient way to plot the force displacement curve is by tracking the base shear and roof displacement (Figure 1).

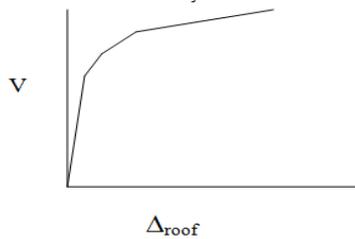


Figure 1: Roof deflection- Δ_{roof} plotted vs. base shear-V

Displacement Coefficient Method is to find target displacement which is the maximum displacement that the structure is likely to experience during the design earthquake. It provides a numerical process for estimating the displacement demand on the structure, by using a bilinear representation of capacity curve and a series of modification factors, or coefficients, to calculate a target displacement.

The target displacement is calculated as per procedure described in Section 3.3.3.3.2 of FEMA 356: 2000

$$\delta_t = C_0 C_1 C_2 C_3 S_a [T_e^2 / (4\pi^2)] g \tag{1}$$

where:

C_0 = Modification factor to relate spectral displacement to building roof displacement, as determined by Table 3-2 of FEMA 356.

C_1 = Modification factor to relate expected maximum inelastic displacements to displacements calculated for linear elastic response
 = 1.5 for $T_e < 0.1$ sec

$$= 1.0 \text{ for } T_e \geq T_s$$

$$= [1.0 + (R - 1) T_s / T_e] / R \text{ for } T_e < T_s.$$

Value of C_1 should not be less than 1.0.

T_s = Characteristic period of the response spectrum, defined as the period associated with transition from constant acceleration segment of the spectrum to the constant velocity segment of the spectrum (to be calculated from demand spectrum).

T_e = Effective fundamental time period = $T_i (K_i / K_e)^{1/2}$.

T_i = Elastic fundamental period.

K_i = Elastic lateral stiffness of the building.

K_e = Effective lateral stiffness of the building; taken as equal to the secant stiffness calculated at a base shear force equal to 60% of the effective yield strength of the structure obtained from bilinear representation of capacity curve.

R = Ratio of elastic strength demand to calculated yield strength co-efficient = $S_a / (V_y / W) C_m$.

V_y = Effective yield strength calculated using the capacity curve. For larger elements or entire structural systems composed of many components, the effective yield point represents the point at which a sufficient number of individual components or elements have yielded and the global structure begins to experience inelastic deformation.

S_a = Response spectrum acceleration, at the effective fundamental period and damping ratio of the building (to be calculated from demand spectrum).

W = Effective seismic weight .

C_m = Effective mass factor as determined by Table 3-1 of FEMA 356.

C_2 = Modification factor to represent the effect of pinched hysteretic shape, stiffness degradation and strength deterioration on maximum displacement response; taken from Table 3-3 of FEMA 356 for different framing systems and structural performance levels. Alternatively, C_2 may be taken as 1.0 for a nonlinear procedure.

C_3 = Modification factor to represent increased

displacement due to dynamic P-D effects
 = 1.0 for buildings with positive post - yield stiffness;
 = $1.0 + |\alpha|(R - 1)^{3/2}/T_c$ for buildings with negative post - yield stiffness.

α = Ratio of post - yield stiffness to effective elastic stiffness, where the non-linear force-displacement relation shall be characterized by a bilinear relation.

Frame Element Hinge Properties

Discrete hinge properties for frame elements are based on FEMA-356 criteria as per Section 5.5.2.2.2 (Figure 2).

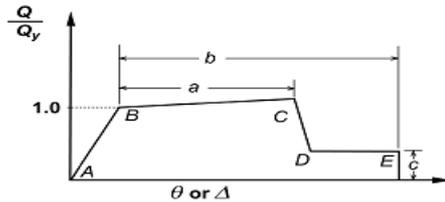


Figure 2: Generalized force-deformation relationship for components

- Point A is the origin.
- Point B represents yielding. No deformation occurs in the hinge up to point B, regardless of the deformation value specified for point B. The displacement (rotation or axial elongation as the case may be) will be subtracted from the displacements at points C, D and E. Only plastic deformation beyond point B will be exhibited by the hinge.
- Point C represents the ultimate capacity of the plastic hinge. At this point hinge strength degradation begins (hinge starts shedding load) until it reaches point D.
- Point D represents the residual strength of the plastic hinge. Beyond point D, the component responds with substantial strength to point E.
- Point E represents total failure. At deformation

greater than point E, the plastic hinge will drop load to zero.

The parameters Q and Q_{CE} (Q_y) are generalized component load and generalized component expected strength, respectively. For beams and columns, θ is the total elastic and plastic rotation of the beam or column, θ_y is the rotation at yield. For braces, Δ is the total elastic and plastic displacement and Δ_y is the yield displacement.

θ_y shall be calculated based on equations (2) and (3).

For beams:

$$\theta_y = Z \cdot F_{ye} L_b / (6 \cdot EI_b) \tag{2}$$

For columns:

$$\theta_y = Z \cdot F_{ye} L_c / (6 \cdot EI_c) (1 - P/P_{ye}) \tag{3}$$

Q and Q_{CE} are the generalized component load and generalized component expected strength, respectively. For flexural actions of beams and columns, Q_{CE} refers to the plastic moment capacity, which shall be calculated using equations (4) and (5):

For beams:

$$Q_{CE} = M_{CE} = Z \cdot F_{ye} \tag{4}$$

For columns:

$$Q_{CE} = M_{CE} = 1.18 \cdot Z \cdot F_{ye} (1 - P/P_{ye}) \tag{5}$$

where:

- E = Modulus of elasticity.
- F_{ye} = Expected yield strength of the material.
- I = Moment of inertia.
- L_b = Beam length.
- L_c = Column length.
- M_{CE} = Expected flexural strength of a member or joint, kip-in.
- P = Axial force in the member.
- P_{ye} = Expected axial yield force of the member = A_gF_{ye}.
- Q = Generalized component load.
- Q_{CE} = Generalized component expected strength = Effective expected strength, which is defined as the statistical mean value of yield strengths, Q_y, for a

population of similar components, and includes the consideration of strain hardening and plastic section development.

Validations: Pushover analysis is implemented in most of the general purpose finite element analysis software like STAAD.Pro, SAP2000, ETABS, IDARC, NISA-CIVIL,... etc. STAAD.Pro results are validated with hand calculations based on the above theory, and the proposed pushover analysis for three-dimensional storage rack structure used STAAD.Pro as per FEMA

356: 2000 and ATC 40 codes.

For validation, a transverse load P is applied to a cantilever member and increased until the member fails. Member length is 24 inch of wide flange of W16X77 standard steel section with an expected yield strength of 36 ksi. The analysis results are saved at an interval of 0.1 inch deflection of the cantilever tip and capacity curve created with base shear Vs displacements (Figure 3).

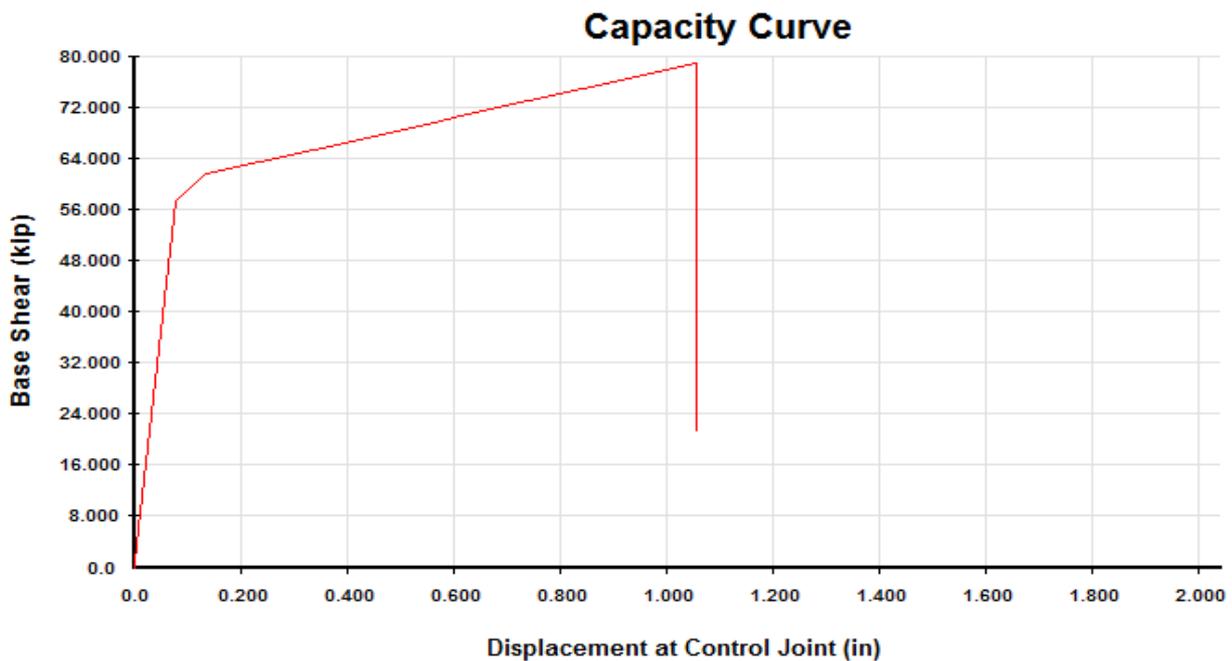


Figure 3: Capacity curve - displacement at control joint vs. base shear

Table 2. Software results vs. manual calculation

Load step	Force (free end) kips	Deflection (free end) in		Percent difference
		STAAD.Pro results	Hand calculations	
3 rd step linear	57.837	0.078	0.078	0%
14 th step non-linear	79.32	1.057	1.067	0.94%

At load step 3

Equation of elastic deflection at cantilever tip:

$$\delta_z = PL^3/3EI + PL/GAv$$

P = 57.837 kip, l = 24 in, E = 29000 ksi, I = 138.0

in^4 , $G = 11154 \text{ ksi}$, $A_v = 10.4323 \text{ in}^2$.

Thus, elastic deformation:

$$\delta_{z \text{ elastic}} = [57.837 \times 24^3 / (3 \times 29000 \times 138)] +$$

$$[48.784 \times 24 / (11154 \times 10.4323)] = 0.1097 + 0.012 = 0.122 \text{ inch.}$$

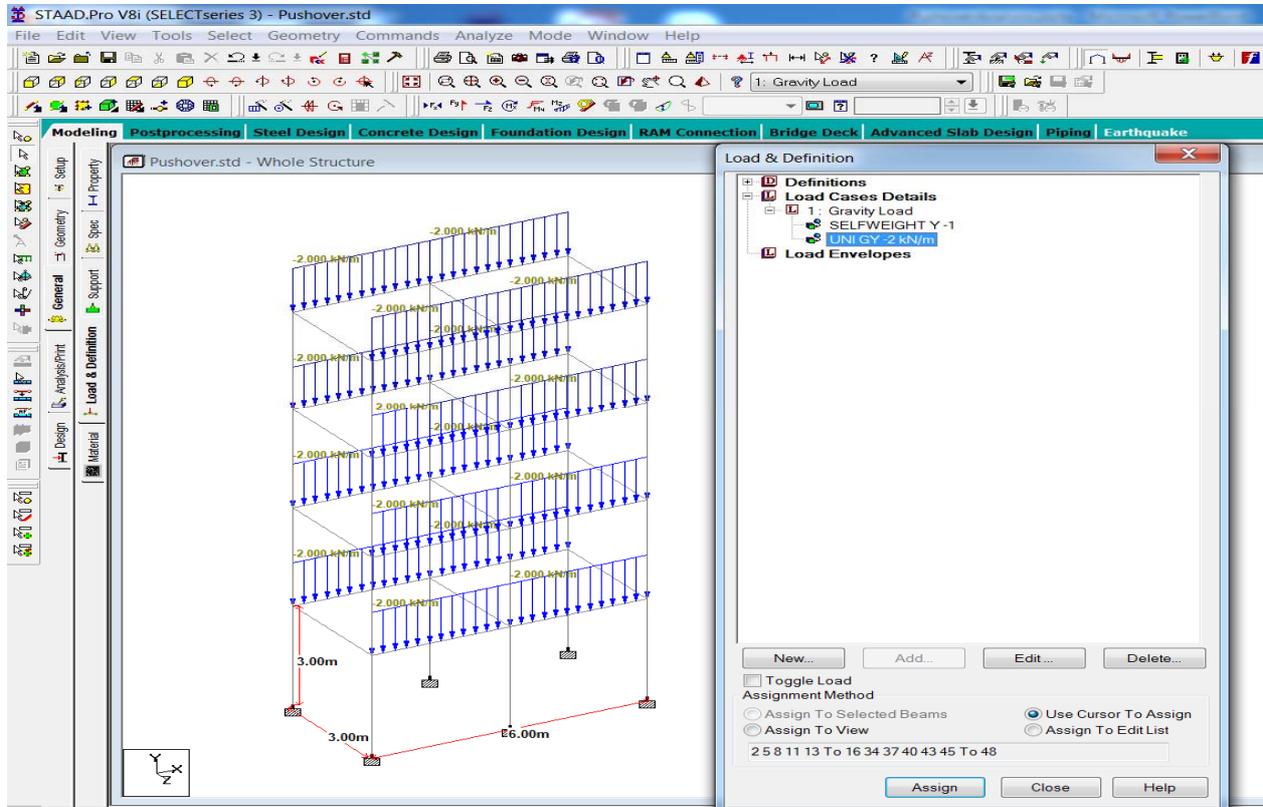


Figure 4: Typical storage rack cold formed steel structure with loading

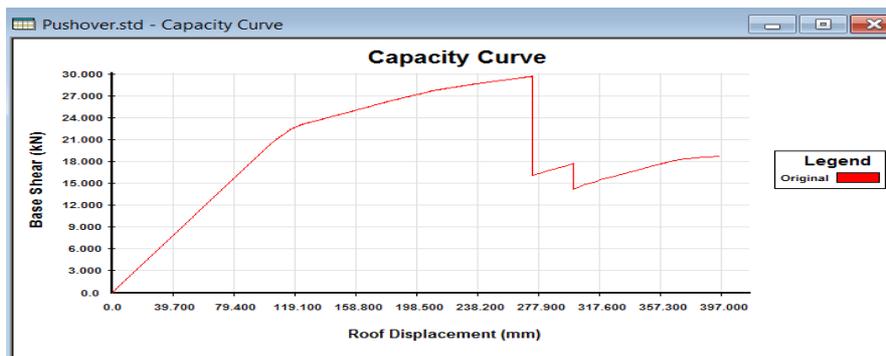


Figure 5: Capacity curve, base shear vs. roof displacement

At load step 14

Equation of elastic deflection at cantilever tip:

$$\delta_z = PL^3/3EI + PL/GA_v$$

$$P = 79.32 \text{ kip, } l = 24 \text{ inch, } E = 29000 \text{ ksi, } I = 138.0 \text{ in}^4,$$

$$G = 11154 \text{ ksi, } A_v = 10.4323 \text{ in}^2.$$

Thus, elastic deformation at cantilever tip:

$$34 / (11154 \times 10.4323) = 0.091 + 0.016 = 0.107 \text{ inch.}$$

$$\delta_{z \text{ elastic}} = [79.32 \times 30^3 / (3 \times 29000 \times 138)] + [79.32 \times$$

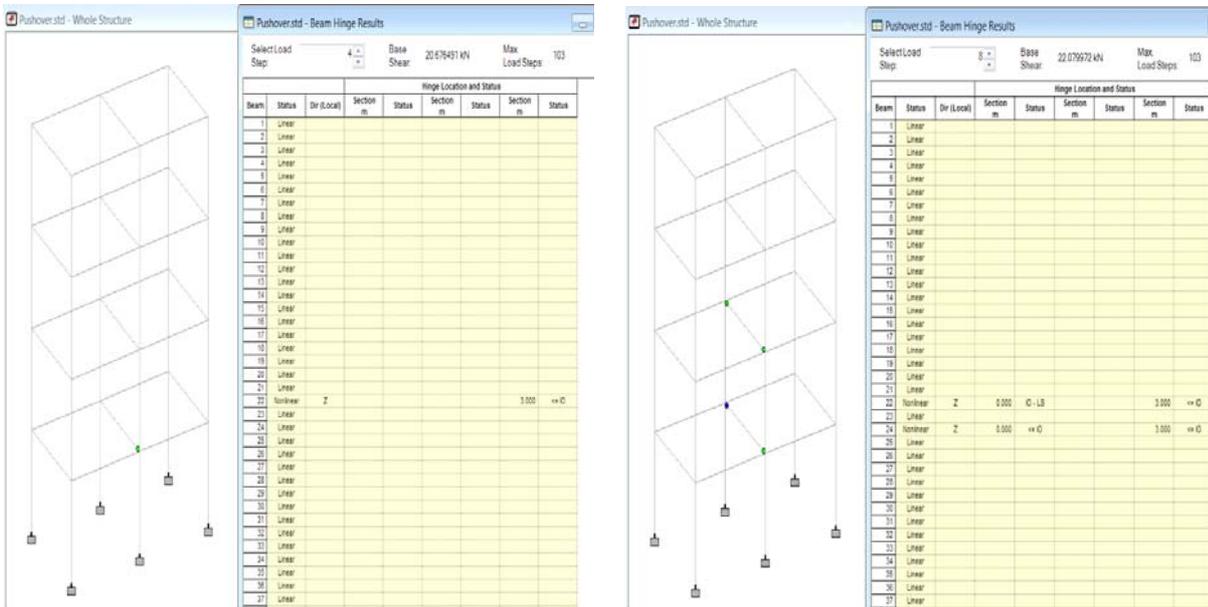


Figure 6: Plastic hinge -IO at base shear of 20.67 kN and LS at 22.07 kN

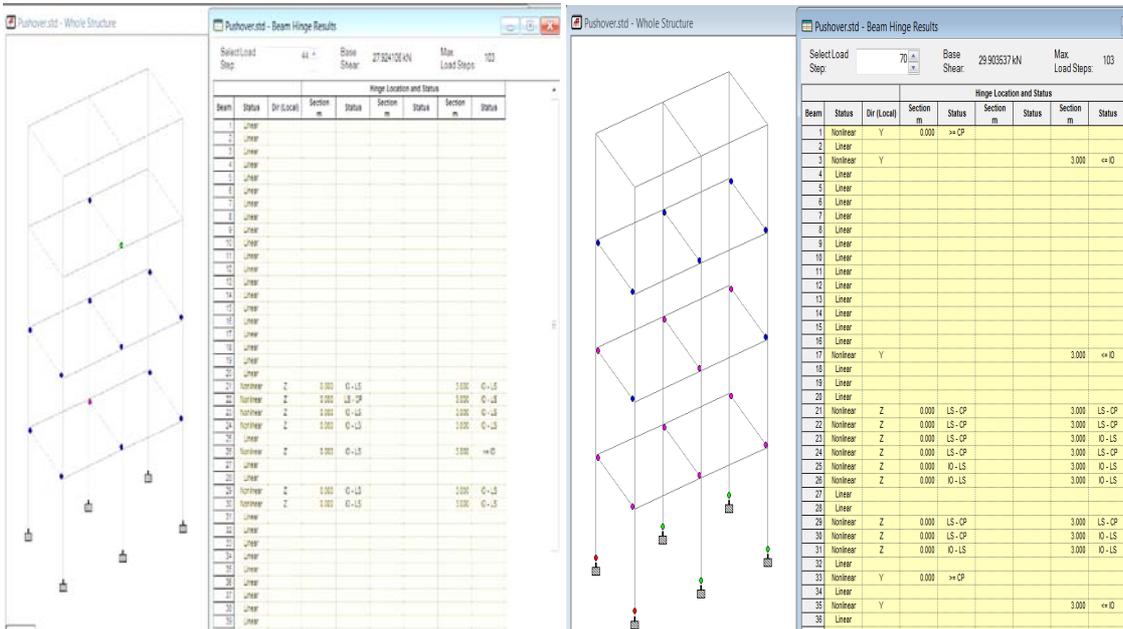


Figure 7: Plastic hinge formation of LS-CP at base shear of 27.924, total collapse of structure (CP) at 29.9 kN

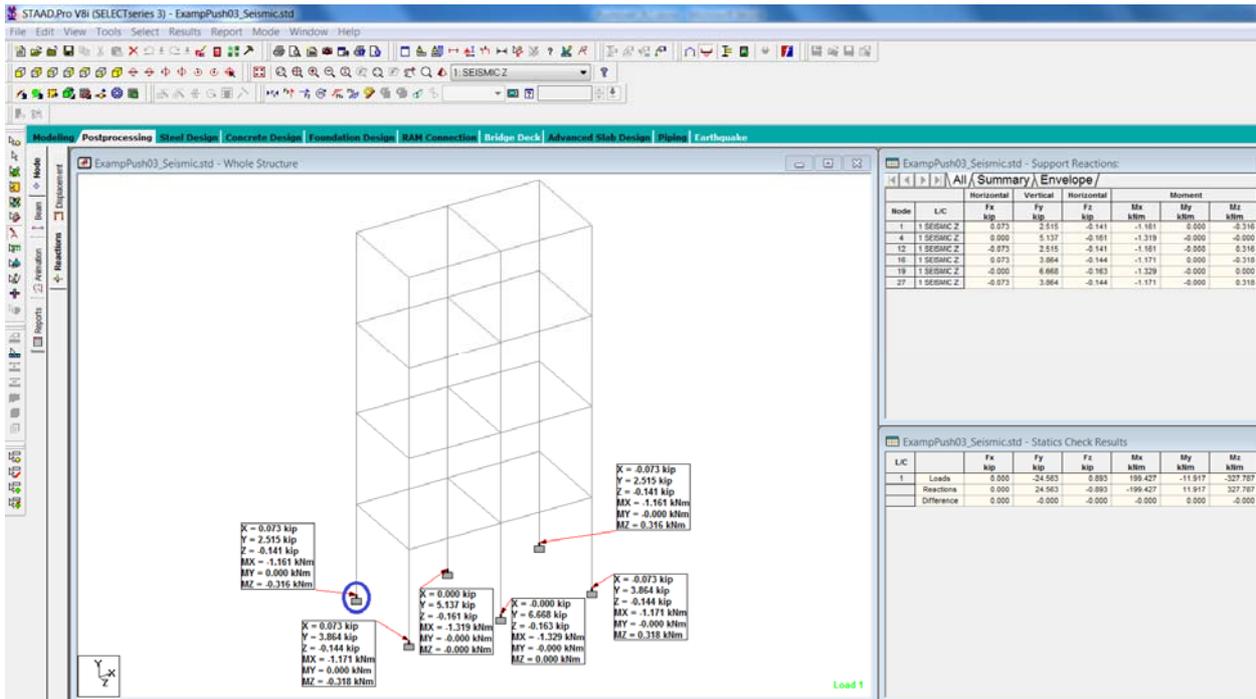


Figure 8: Analysis of four storey storage steel rack as per UBC 1997

Table 3: Pushover analysis vs. static seismic method UBC

Support reactions at node 1	Pushover analysis	UBC 1997	Difference
FX (kip)	0.073	0.073	0%
FY(kip)	2.515	2.473	1.7%
FZ(kip)	0.141	0.143	1.4%
MX (kNm)	1.161	1.178	1.5%
MY (kNm)	0.000	0.00	0%
MZ (kNm)	0.316	0.316	0%

Plastic rotation = 0.04 radian.

Since STAAD.Pro assumes small displacements ($\sin \theta = \theta$), plastic deformation at cantilever tip

$\delta_z \text{ plastic} = 1 \times \theta = 24 \times 0.04 = 0.96$ inch.

Total deflection = $\delta_z \text{ elastic} + \delta_z \text{ plastic} = 0.107 + 0.96 =$

1.067 inch.

Comparison: STAAD.Pro software results are close to manual calculations (Table 2).

DESCRIPTION OF STRUCTURE

A typical four storey storage steel rack of 3 meters width, 6 meters length and 3 meter height of each floor is taken as example. Lysaght cold formed channel section C230X76X1.5 is used for beams and C350X125X3 used for columns. Self-weight of the structure and dead load of 2 kN/m are considered as permanent gravity loads (Figure 4). Input is considered

as moment frame. Expected yield stress is 36 kip/in², displacement incremental value is 0.1 inch, base shear incremental value is 5 kip, vertical distribution of base shear is due to method 1 as per section 3.3.3.2.3 of FEMA 356, number of pushover loads is set as 1000 with defined base shear of 50 kip. Spectrum parameters: critical damping (%) of 1st spectrum is 5, mapped spectral acceleration at short period is 0.25, at one-second period is 0.1, site category as per section 1.6.1.4.1 is class 1.

In the energy-based pushover approach, the capacity curve (Figure 5) associated with each modal pushover is determined based on the work conducted in the analysis. The work is computed incrementally, typically for each step in the pushover analysis as per FEMA 356 and ATC 40.

In the Z direction, plastic hinge formation of the building mechanisms has been obtained at different displacement levels. Plastic hinge formation starts with beam ends and later proceeds to base columns of lower stories, then propagates to upper stories and continues with the yielding of interior intermediate beams (Figures 6 and 7). But, since yielding occurs at designing events B (yielding), IO (Immediate occupancy), LS (Life safety-Figure 6) and last hinge CP (collapse prevention-Figure 7), respectively, the amount of damage in this direction will be limited. The first hinge formation of base shear and displacement is at 20.67 kN and 4.1 inch.

However, the support reactions of the four storey storage steel rack is compared with static seismic method Uniform Building Code (UBC), 1997 (Figure 8) at first hinge formation base shear of 20.67 kN, and the results are close to pushover analysis (Table 3).

REFERENCES

American Society of Civil Engineers. 2000. Prestandard and commentary for the seismic rehabilitation of buildings, FEMA-356, Federal Emergency

CONCLUSIONS

The paper has presented a simple computer-based method for pushover analysis of steel storage rack subjected to equivalent static earthquake loading. A typical example illustrated that pushover analysis provides valuable information for the performance-based seismic analysis of steel storage rack moment-frame structure. The capacity curve, base shear vs. displacement graph, clearly indicates a collapse load of 29.9 kN at a displacement of 10.81 inch. The progressive collapse of storage rack initial plastic hinge (Immediate Occupancy (IO)- building is safe to occupy but possibly not useful until repair) is formed at a load of 20.67 kN, next stage of plastic hinge (Life Safety (LS) - structure may be safe in the event, but the structure may collapse partially or totally) is formed at a load of 22.07 kN, next stage of plastic hinge (Collapse Prevention (CP) - system is on the verge of experiencing partial or total collapse) is formed at a load of 29.9 kN.

However, conventional pushover analysis exhibits also shortcomings and limitations that confine its range of application and raise doubts about its effectiveness to accurately estimate structural seismic demand, as demonstrated by a number of researchers. Moreover, being a static method, pushover analysis reproduces material straining only, neglecting other sources of energy dissipation that are associated with dynamic response, such as kinetic energy and viscous damping, as well as duration effects. This clearly indicates that further research work is required before pushover can be considered as a valid alternative to nonlinear time history analysis.

Management Agency, Washington, DC.

Bajoria, K.M. and Sangle, K.K. 2008. Capacity-based design of cold formed storage rack structures under seismic load for rigid and semi-rigid connections. *The 14th World Conference on Earthquake Engineering*,

- October 12-17, Beijing, China.
- Bentley Reference Manual on STAAD.Pro, V8i.
- Bracci, J.M., Kunnath, S.K. and Reinhorn, A.M. 1997. Seismic performance and retrofit evaluation for reinforced concrete structures. *Journal of Structural Engineering (ASCE)*, 123: 3-10.
- Building Seismic Safety Council. 1997. NEHRP guidelines for the seismic rehabilitation of buildings, FEMA-273, Federal Emergency Management Agency, Washington, DC.
- Chopra, A.K. and Goel, R.K. 2004. A modal pushover analysis procedure to estimate seismic demands for unsymmetric-plan buildings. *Earthquake Engineering and Structural Dynamics*, 33: 903-927.
- Dubina, D. and Ungureanu, V. 2000. Elastic-plastic interactive buckling of thin-walled steel compression members. *Proceedings of the 15th international conference on cold formed steel structures*, St. Louis, USA, 223-237.
- Elchalakani, M., Grzebieta, R.H. and Zhao, X.L. 2002. Plastic collapse analysis of slender circular tubes subjected to large deformation pure bending. *Advances in Structural Engineering—An International Journal*, 5 (4): 241-257.
- Elchalakani, M., Zhao, X.L. and Grzebieta, R.H. 2002. Plastic mechanism analysis of circular tubes under pure bending. *International Journal of Mechanical Science*, 44 (6): 1117-1143.
- Elnashai, A.S. 2001. Advanced inelastic static (pushover) analysis for earthquake applications. *Structural Engineering and Mechanics*, 12 (1): 51-69.
- Fu, C.C. and Alayed, H. 2006. Seismic analysis of bridges using pushover analysis approach, *4th Jordanian Civil Engineering Conference*, April 10-13, Amman and the Dead Sea, Jordan.
- Gupta, B. and Kunnath, S.K. 2000. Adaptive spectra-based pushover procedure for seismic evaluation of structures. *Earthquake Spectra*, 16: 367-392.
- Hasan, R., Xu, L. and Grierson, D.E. 2002. Pushover analysis for performance-based seismic design. *Computers and Structures*, 80: 2483-2493.
- Inel, M. and Ozmen, H.B. 2006. Effects of plastic hinge properties in nonlinear analysis of reinforced concrete buildings, *Engineering Structures*, 28: 1494-1502.
- Kecman, D. 1983. Bending collapse of rectangular and square section tubes. *International Journal of Mechanical Sciences*, 25 (9-10): 623-636.
- Kotelko, M. 1996. Ultimate load and post-failure behavior of box-section beams under pure bending. *Engineering Transactions*, 44 (2): 229-251.
- Kotelko, M. 2004. Load-capacity estimation and collapse analysis of thin-walled beams and columns—recent advances. *Thin-walled Structures*, 42 (2): 153-175.
- Kotelko, M. and Krolak, M. 1993. Collapse behavior of triangular cross-section girders subject to pure bending. *Thin-walled Structures*, 15 (2): 127-141.
- Kotelko, M., Lim, T.H. and Rhodes, J. 2000. Post-failure of box sections beams under pure bending (an experimental study). *Thin-walled Structures*, 38: 179-194.
- Krawinkler, H. and Seneviratna, G.D.P.K. 1998. Pros and cons of a pushover analysis of seismic performance evaluation. *Engineering Structures*, 20: 452-464.
- Kunnath, S.K. and Gupta, B. 2000. Validity of deformation demand estimates using nonlinear static procedures. *Proceedings of the U.S.–Japan Workshop on Performance-based Engineering for R/C Building Structures*, Sapporo, Hokkaido, Japan.
- Matsumori, T., Otani, S., Shiohara, H. and Kabeyasawa, T. 1999. Earthquake member deformation demands in reinforced concrete frame structures. *Proceedings of the U.S.–Japan Workshop on Performance-based Earthquake Engineering Methodology for R/C Building Structures*, 79-94, Maui, Hawaii.
- Murray, N.W. and Khoo, P.S. 1981. Some basic mechanisms in the local buckling of thin-walled steel structures. *International Journal of Mechanical Sciences*, 23 (12): 703-713.
- Poursha, M., Khoshnoudianb, F. and Moghadamc, A.S. 2011. A consecutive modal pushover procedure for nonlinear static analysis of one-way unsymmetric-plan tall building structures, *Engineering Structures*, 33: 2417-2434.
- Sasaki, K.K., Freeman, S.A. and Paret, T.F. 1998.

- Multimode pushover procedure (MMP): a method to identify the effects of higher modes in a pushover analysis. *Proceedings of the 6th U.S. National Conference on Earthquake Engineering*, Seattle, Washington.
- Ungureanu, V. and Dubina, D. 2004. Recent research advances on ECBL approach. Part I: Plastic–elastic interactive buckling of cold-formed steel sections. *Thin-walled Structures*, 42 (2): 177-194.
- Ungureanu, V., Kotelko, M., Mania, R.J. and Dubina, D. 2010. Plastic mechanisms database for thin-walled cold-formed steel members in compression and bending. *Thin-Walled Structures*, 48: 818-826.
- Zeynalian, M. and Ronagh, H.R. 2011. A numerical study on seismic characteristics of knee-braced cold formed steel shear walls. *Thin-Walled Structures*, 49: 1517-1525.