

## A Numerical Study for Geomaterials Shear Strength Components Using Discrete Element Models

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### ABSTRACT

Geomaterials (ranging from clay to gravel) are usually composed of individual particles that have specific engineering properties. Those particles once packed to a certain density, exhibit a distinguished macromechanical behavior, which is a result of their micromechanical interactions at the contact levels. Soil masses are usually subjected to direct normal and indirect shear stresses; yet, they normally show shear type of failure as indicated by many researchers using experimental and numerical evidences. The shear strength concept of friction and cohesion is discussed in this paper. A Discrete Element Code (developed and owned by Caterpillar, Inc.) was used in this study to show that it is possible to drop the apparent cohesion portion and compensate for that with additional frictional resistance. Apparent cohesive bonds usually fail before mobilizing the frictional resistance and, therefore, we may not account on it to resist future stresses. The numerical simulations results for triaxial tests and excavation operations showed consistency regarding the proposed shear strength components. Triaxial simulations for fine-grained materials showed that it is possible for a numerical model to capture the stress-strain behavior if the cohesion component is dropped and, instead, additional frictional component is added to account for the dilation that many classical soil mechanics laws usually ignore. Likewise, excavation operations showed similar results using the same proposed theory. Some important observations regarding the apparent cohesion concept are discussed and shown in this paper.

**KEYWORDS:** Geomaterials, Apparent cohesion, Friction, Micromechanics and continuum mechanics, Virtual triaxial tests, Excavation and Non-cohesion theory.

### INTRODUCTION

The shear resistance components in geomaterials give the materials their shear strength; the name here (shear strength) indicates that those components should resist shear stresses. Coulomb's equation for the shear strength was used by many researchers to describe the constitutive behavior of soils. Otto Mohr later came up with his known envelope that represents a limit for shear strength

and stability. Mohr-Coulomb shear strength theory assumes that there are two components that give the soil its shear strength: internal angle of friction and apparent cohesion (Alsaleh, 2004). These are two phenomenological parameters than can be obtained for a given soil utilizing simple laboratory tests and using linear regression analysis. During soil deformation, the particles tend to roll and slide over each other, while surface friction, angular friction and particle interlocking act to prevent such kinematics from taking place (Alsaleh et al., 2004). Once those resistances are fully mobilized,

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translational and rotational motions are not restricted anymore. As a result, tremendous deformations occur and the soil mass enters an instability phase that eventually leads to failure and massive fragmentations. The ultimate resistance stress at this point is called the shear strength of the soil. The shear failure surface of a geomaterial is a non-linear thin surface that experiences tremendous shear and volumetric plastic strains (localized strains) (Alsaleh, 2004). Apparent cohesion has been defined by the geotechnical researchers as the tensile resistance that essentially builds up between two adjacent particles due to the suction at the contact surface. Such a strength component can simply collapse under very small strains. Using the above-mentioned argument, it is clear that the apparent cohesion is not sustained once the soil mass is sheared; therefore, the Mohr-Coulomb shear strength theory needs to be revisited and discussed carefully. Using an assembly of discrete particles (rigid rods), Rowe (1962) has proven and showed that this theory has some limitations, especially when it is applied to granular materials due to the fact that it does not account for soil dilatancy. Traditionally, geotechnical engineers have been classifying the soil into two types: cohesive and cohesionless soils; however, these terminologies could be misleading in many occasions. Instead, fine and coarse-grained soils should replace these terms. This is an important classification that would depend on the level of the hydraulic conductivity and the specific surface area. In fine-grained soils, the structure is able to hold water molecules for longer times, creating negative pore water pressure that produces suction at the contact area between two adjacent particles (Santamarina, 1997), resulting in apparent cohesion. In coarse sand, this phenomenon does not exist as fluid seeps out the voids quickly, leaving no chance for the negative pore water pressure to build up.

As indicated by Alsaleh *et al.* (2006), there are several microproperties that control the behavior and the strength of a discrete particle system (soils). Particle size distribution, local void ratio, particle shape and surface roughness are good examples of such properties. Micromechanical based material parameters and laws are needed to better describe the micro and macro

behavior of soil masses.

It is generally agreed that the shear strength of the normally consolidated clays, sands and gravels is highly dependent on the microproperties of the solid particles (size, local void ratio, shape and surface roughness) (Alsaleh, 2004), density of the soil mass, existence of water and the level of the effective stress (Alsaleh *et al.*, 2004; Terzaghi and Peck, 1948). Such types of geomaterials are known as  $\phi$ -soils, described only by the internal angle of friction (macrolevel). The classical definition is the summation of: (i) the repose angle, which depends on the grain microproperties, and (ii) the dilation component, which depends on the density and the level of the effective stress.

In slightly overconsolidated clayey to silty soils, the shear resistance mechanism is known to be a combination of cohesive and frictional components often represented by a Mohr – Coulomb (M–C) failure envelope. Geotechnical engineers call this type of soil a  $c$ - $\phi$  soil; the cohesion and the friction angle are used here to compute the shear strength of the soil. The authors see a danger in combining these two quantities together due to the physical fact that the cohesive bonds (if they exist) would break down prior to the mobilization of the frictional resistance components.

Considering highly overconsolidated clays, the geotechnical community has been using very high cohesion quantities and very low or zero friction angles according to the classical M–C failure criterion. It is believed here that this approach might be a serious error and its applicability to such types of soil should be questioned. The clay sheets are compacted closer to higher densities due to the high maximum past pressure and the micromechanical interactions are more efficient to give higher macro shear strength. Therefore, the authors are supporting the  $\phi$ -soil concept to be applied to such types of soils.

#### MODELING PARAMETERS

As previously mentioned, the internal angle of friction is a continuum-based parameter that accounts for the particle rotational and sliding resistance. This

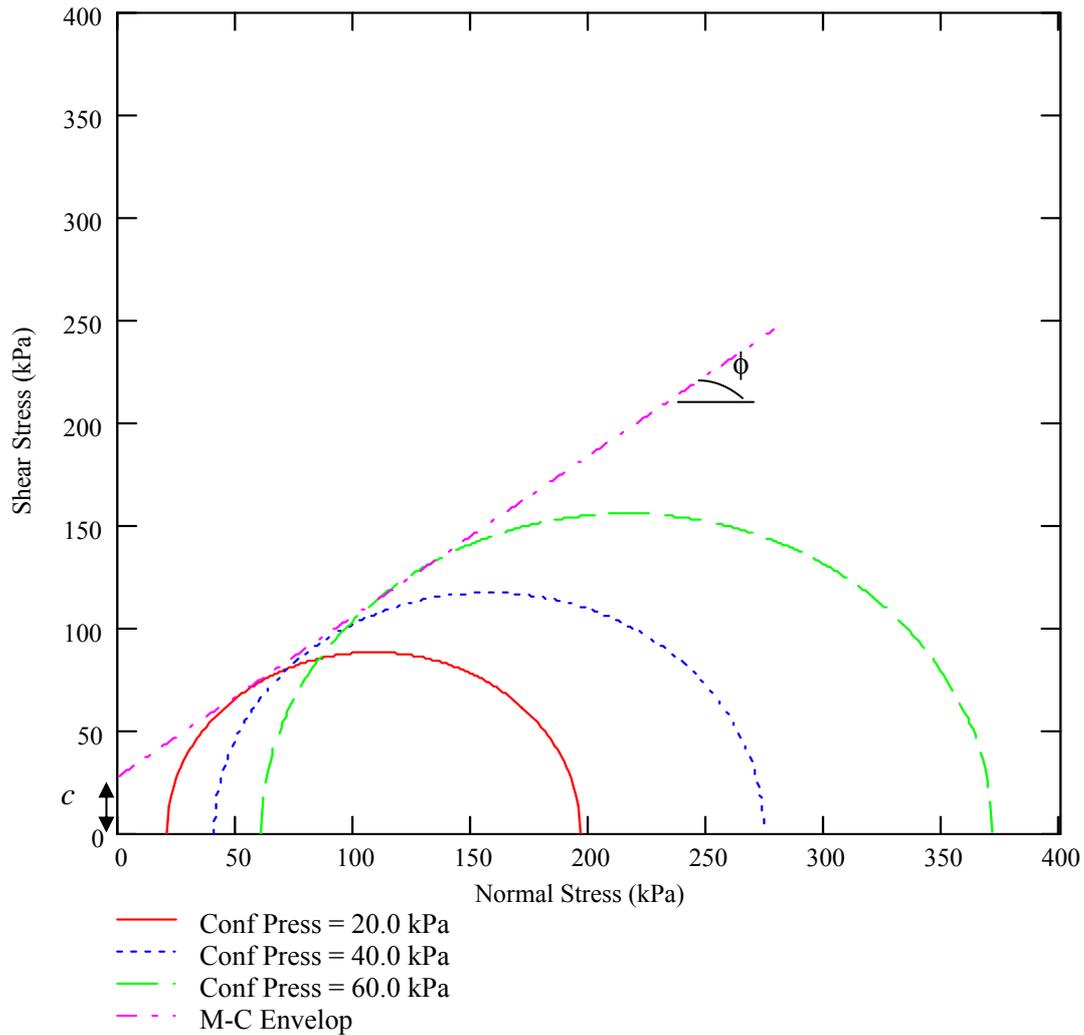
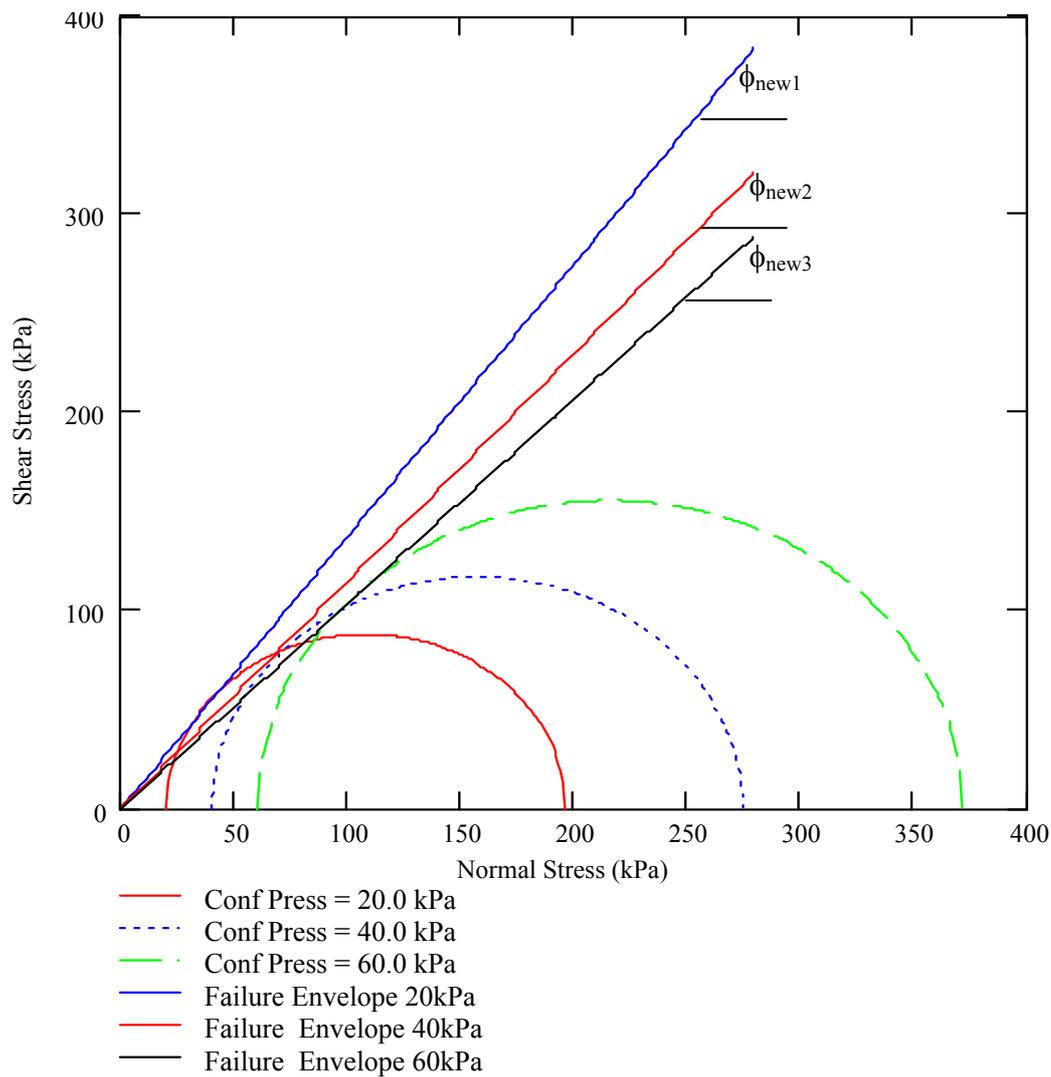


Figure 1: Classical M-C Failure Envelope Using Lab Triaxial Test on Silty Clay

parameter is meant to account for the particle-to-particle frictional resistance; however, the difficulty in measuring such microvalues enforces the continuum-based quantity. The critical state soil mechanics approach separates the peak friction angle into two components; constant volume or critical state angle of friction and dilatancy angle (Wood, 1990). This leads to the conclusion that one should consider the level of effective stress before providing a value for the peak angle of friction. If the soil does not undergo any volumetric changes, then it is called

the critical state and the angle of friction in this case is dependent only on the particle size, shape and surface roughness. Therefore, the critical state or the constant volume friction angle is a unique value for a certain soil type. On the other hand, soil particles that are confined under high stresses show lower peak friction angles than those which are confined under low stresses because dilation is reduced at high stresses. A preliminary conclusion can be made here; the unique linear M-C failure envelope (Figure 1) for



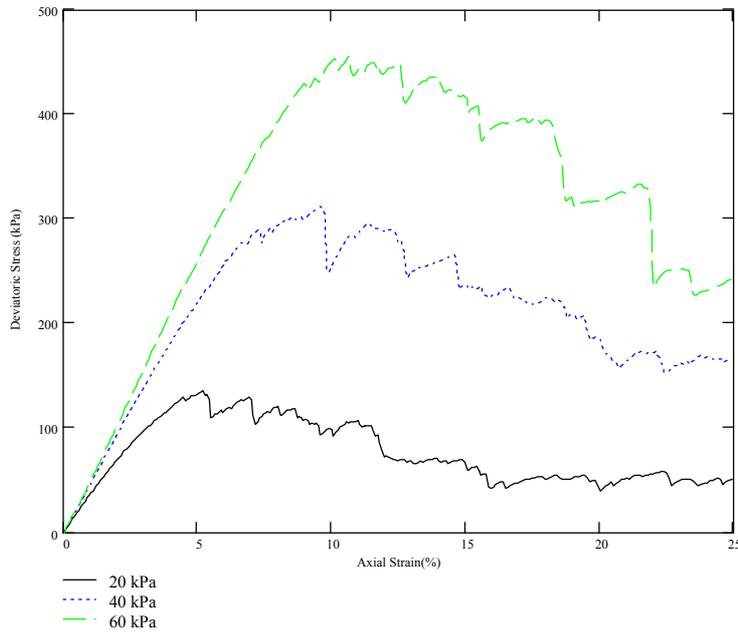
**Figure 2: Confining Stress-Dependent Failure Envelope and Friction Angles for Silty Clay Using the Same Triaxial Tests**

multiple levels of confining stress is incorrect and, instead, there exists a linear failure envelope for each level (Figure 2). As shown in Figure 1, where the classical Mohr–Coulomb theory is used to obtain the peak friction angle, it is obvious that this failure envelope could show a non-zero cohesion value. If we do not accept the concept of cohesion in soil, then the failure envelope is forced through a zero intercept on the shear stress axis and at the same time is tangent to

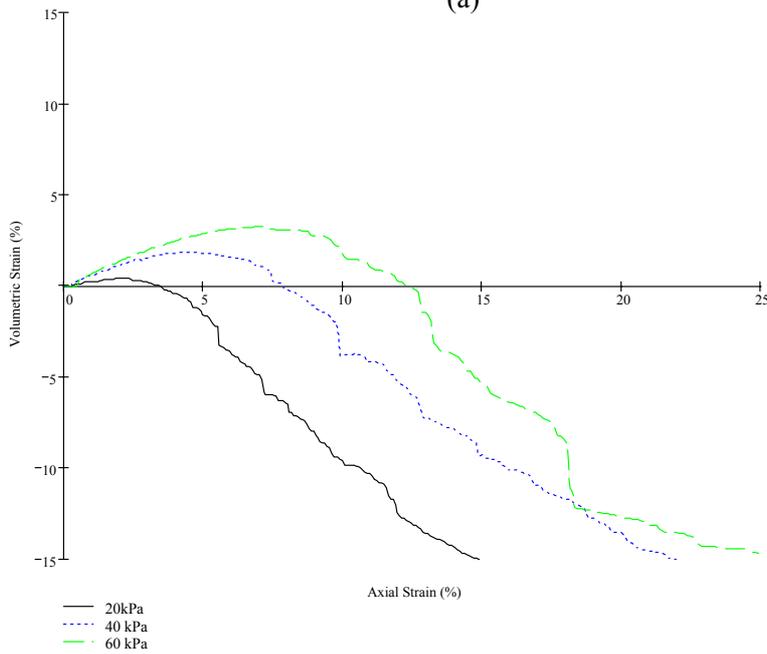
the Mohr circle that is associated with a particular confining pressure. This yields a higher peak angle of friction that would compensate for the assumed zero cohesion resistance.

As clarified above, the apparent cohesion concept was used first to fit the experimental results into a linear equation (Mohr–Coulomb shear strength theory, see Figure 1). The following equation describes the failure envelope:

$\tau = C + \sigma \tan(\phi)$  (1)  $C$  is the apparent cohesion,  
 where,  $\sigma$  is the normal stress and  
 $\tau$  is the shear stress,  $\phi$  is the internal angle of friction

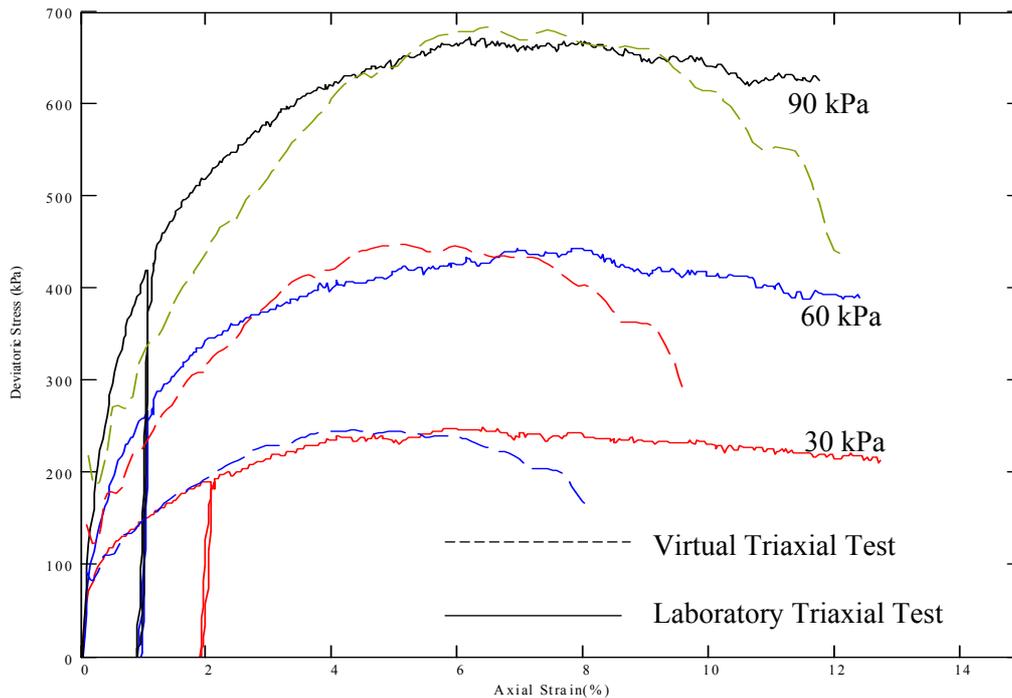


(a)



(b)

**Figure 3: Effect of the Confining Stress on (a) Deviatoric Stress and (b) Volumetric Strain Using DEM Simulations**



**Figure 4: Comparison between Predicted and Measured Deviatoric Stresses for Crushed Limestone**

The apparent cohesion is a continuum-based parameter that is trying, to some extent, to account for the small tensile forces that might exist in a partial-saturated fine-grained soil mass. The resistance that this parameter is providing within the numerical model is acceptable and has a physical meaning only if the soil mass is subjected to very small deformations, when the frictional resistance is not yet mobilized and the suction bond between two adjacent particles is not yet broken. However, in most engineering applications, the soil masses undergo large deformations that pass this limit. At that point, the soil can use only the frictional component to resist any further shearing. This fact indicates that the cohesive resistance term can be dropped from Eq. 1 and additional frictional resistance added so that the soil provides the same shear resistance. Using this argument, Eq. 1 is rewritten as:

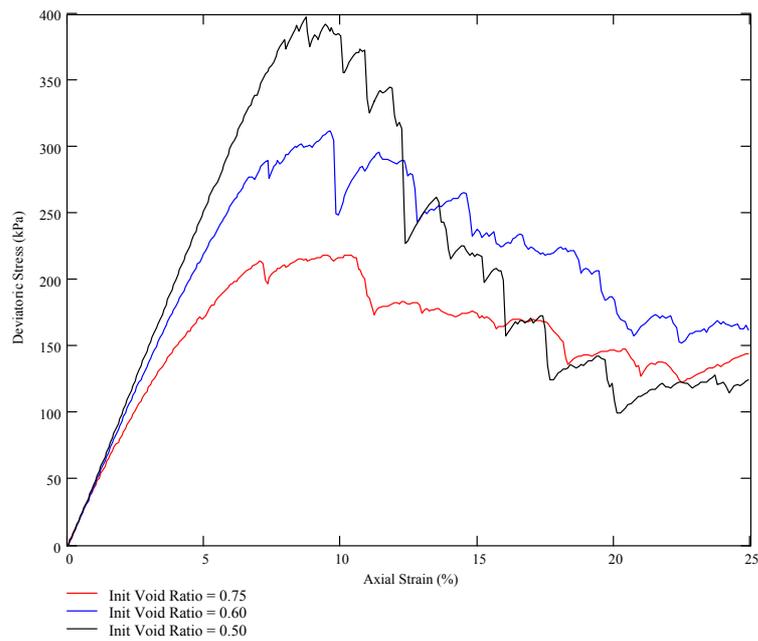
$$\tau = \sigma \tan(\phi_{new})$$

where,  $\phi_{new}$  is the internal angle of friction shown in

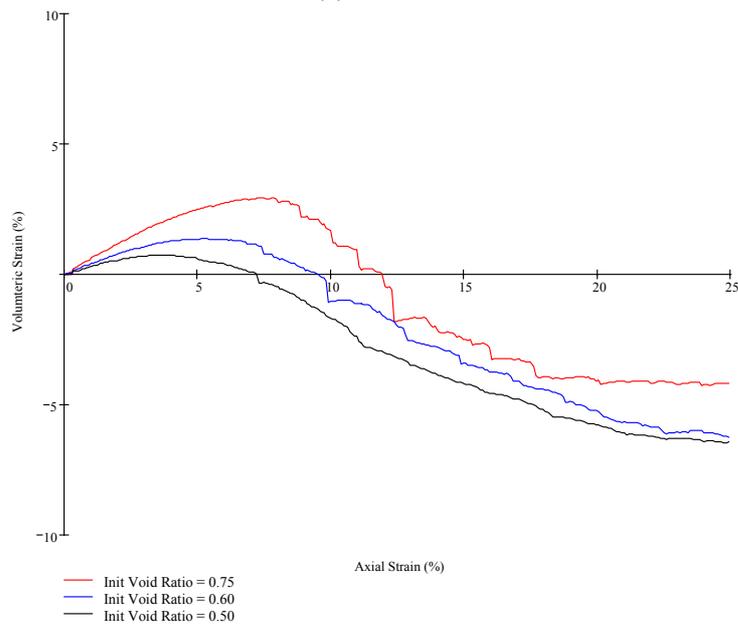
Figure 2.

Considering the example of a soil pile, it may be described by the shear strength resistance within the soil mass and by other parameters (boundary conditions, moisture content, ... etc.). The pile can stand stable or in equilibrium without any external support at a certain slope; the angle of this slope is usually called the repose angle. This angle can be measured using different techniques (Santamarina and Cho, 2001), dependent on the microfabric of the material (Alsaleh et al., 2006). The angle increases with the decreasing particle size, decreasing the coefficient of uniformity and increasing the grain angularity and surface roughness.

Given our physical understanding for the discrete materials behavior of soil, we can say that decreasing the particle or the grain size will significantly increase the frictional resistance due to the increase in the total specific area. Therefore, the finer the soil fabric, the higher the actual internal microfrictional components, leaving no need to consider the apparent cohesive resistance.

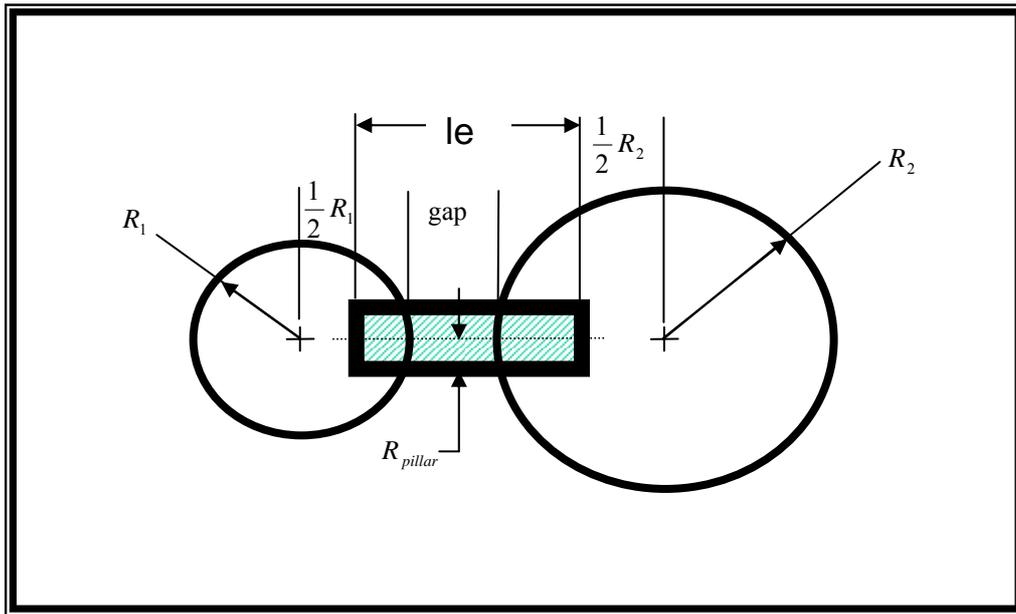


(a) Deviatoric Stress

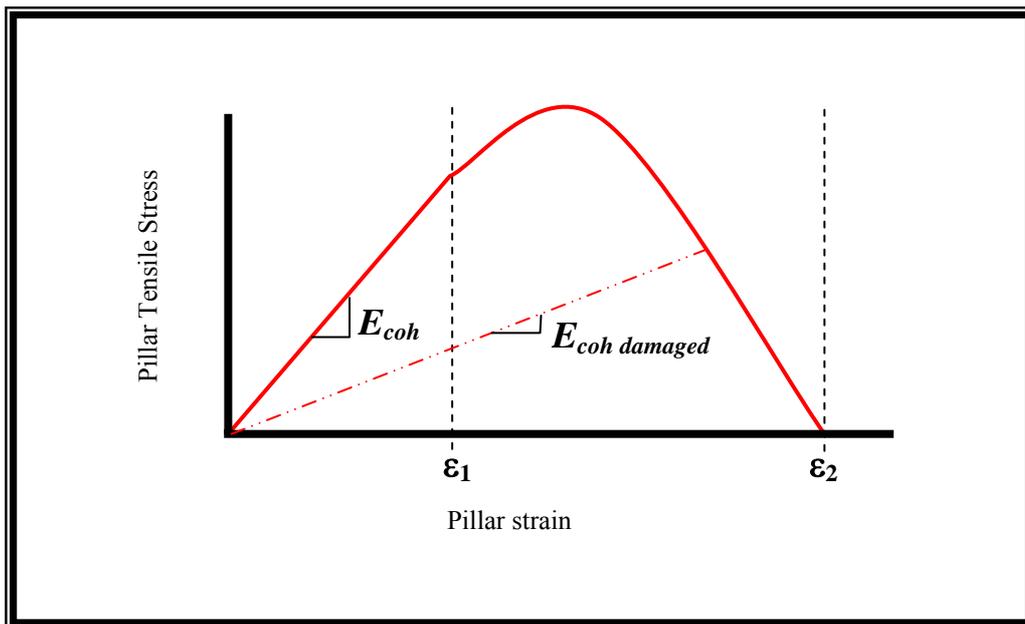


(b) Volumetric Strain

**Figure 5: The Effect of the Initial Density (Void Ratio) on the (a): Deviatoric Stress and (b): Volumetric Strain Using DEM Simulations**

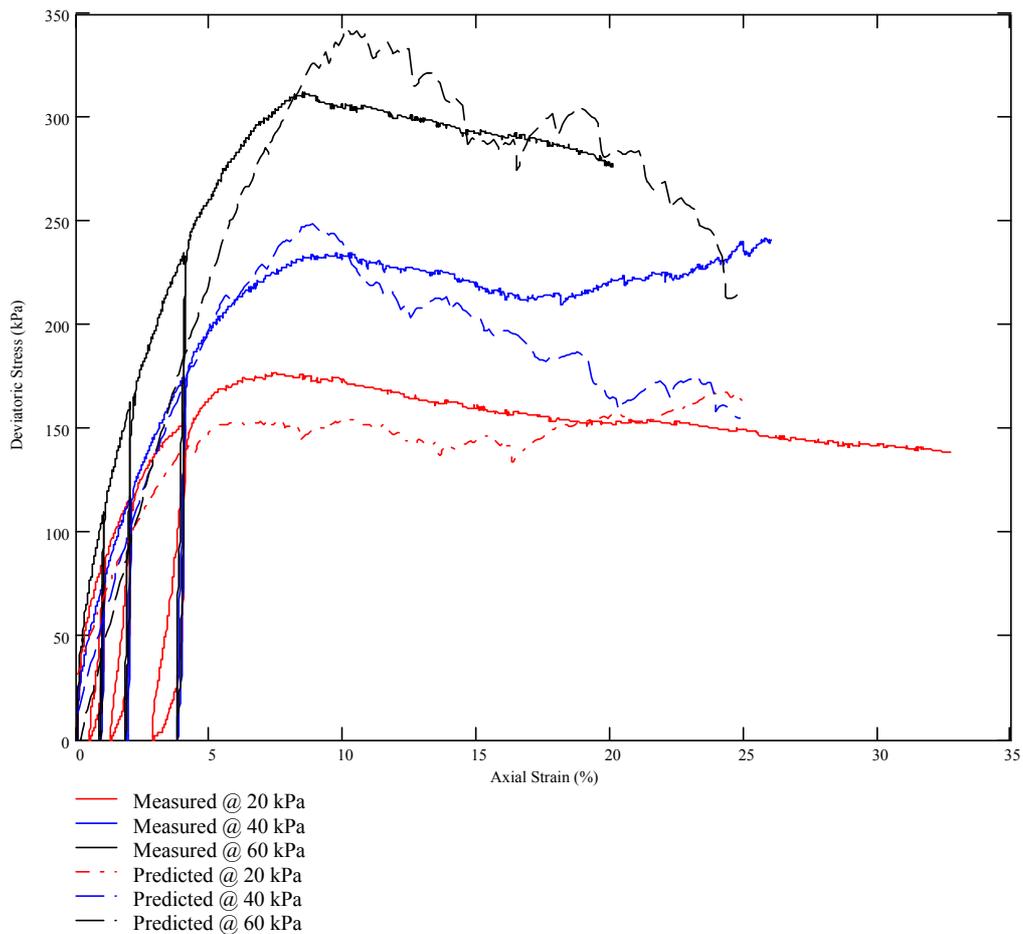


(a) Tensile Stress Pillar between Two Particles



(b) Pillar Tensile Strength

Figure 6: Apparent Cohesion Model Implemented within the Current DEM Code



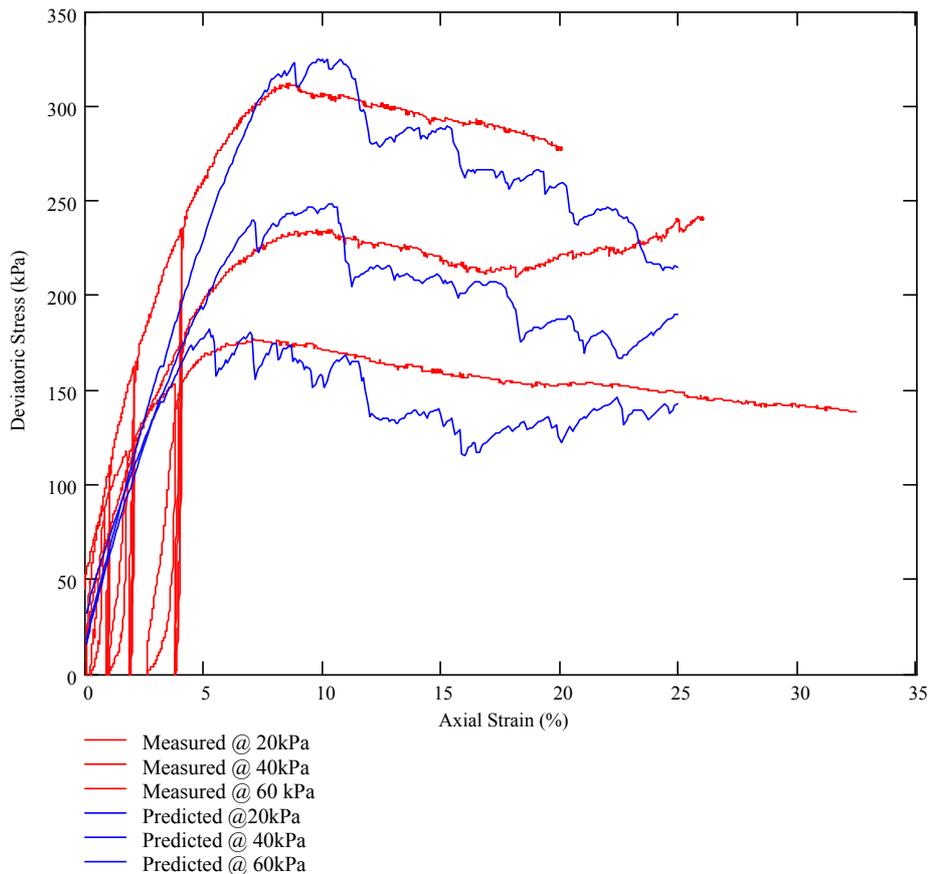
**Figure 7: Comparison between Predicted and Measured Deviatoric Stresses for Silty Clay Using Friction and Cohesion Parameters**

#### MODELING OF GEOMATERIALS

There are several numerical tools to model the behavior of geomaterials. Discrete Element (DEM), Finite Element (FEM) and Mesh Free Methods (MFM) are the most common tools to model the stress transfer and the particle flow mechanisms in soils. Each of the above-mentioned models has its own applications, advantages and limitations. In any of the above-mentioned tools, there is a need for stress-strain relations; of course such relations require material properties (shear strength parameters, stiffness modulus,... etc.).

Let us consider a Discrete Element Model (DEM)

that is mostly used to model the particle flow and stress-strain transfer for granulates. Granular material in geotechnical engineering is considered a cohesionless material, which is an acceptable assumption. Modeling a granulate assembly normally requires microfrictional parameters, particle-to-particle stiffness and damping parameters to solve for the system dynamics. Such parameters are, relatively speaking, obtainable using some simple engineering tests. However, once we try to model fine-grained cohesive soil, two issues seem to be limiting the application of the DEM. The first issue is the length scale; we are trying to model very small particles (in the order of angstroms) using a size of



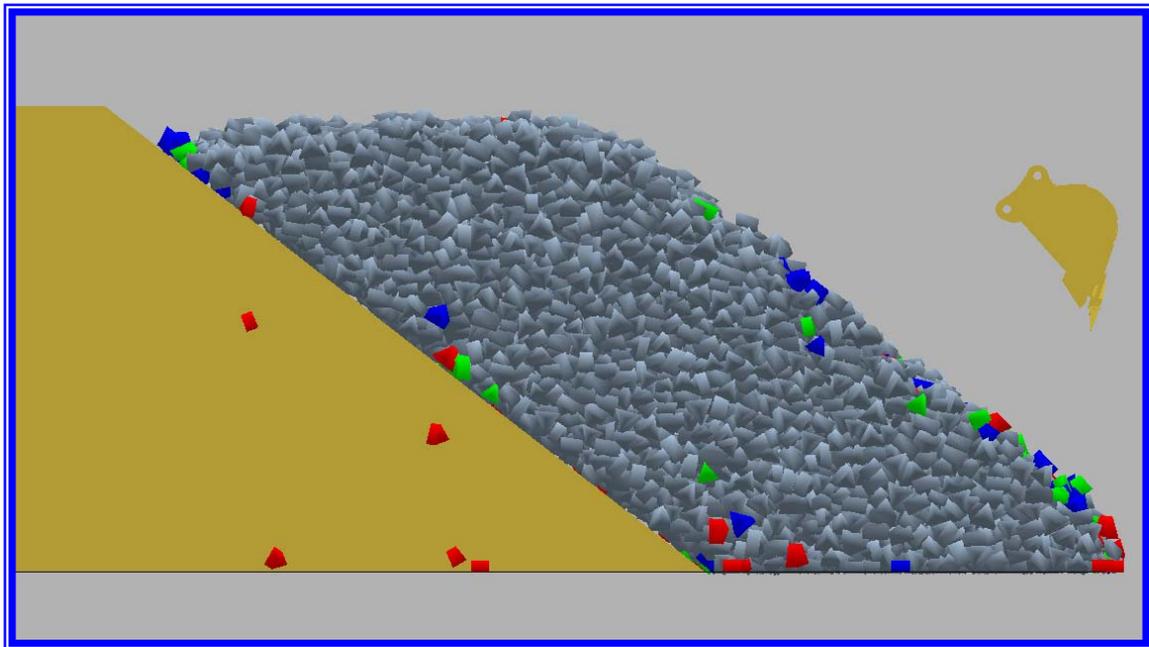
**Figure 8: Comparison between Predicted and Measured Deviatoric Stresses for Silty Clay Using Zero-cohesion theory**

multiple inches. The second issue is the cohesive characteristics that the soil mass might have as a result of the negative pore water pressure within the tiny voids. Many researchers tried to resolve the length scale issue by representing group of very small particles as large solid grains (spheres in most cases) and compensate for the high micro frictional components by some false cohesion parameters. This approach requires building some cohesive links or bonds at the particle level to keep the particles bonded to each other until a certain strain-threshold value. Once this strain-threshold is reached, failure criterion is met and the particles will travel apart from each other. This use of cohesion does not represent the true soil behavior; yet, researchers and developers tend to use such technique to model fine-

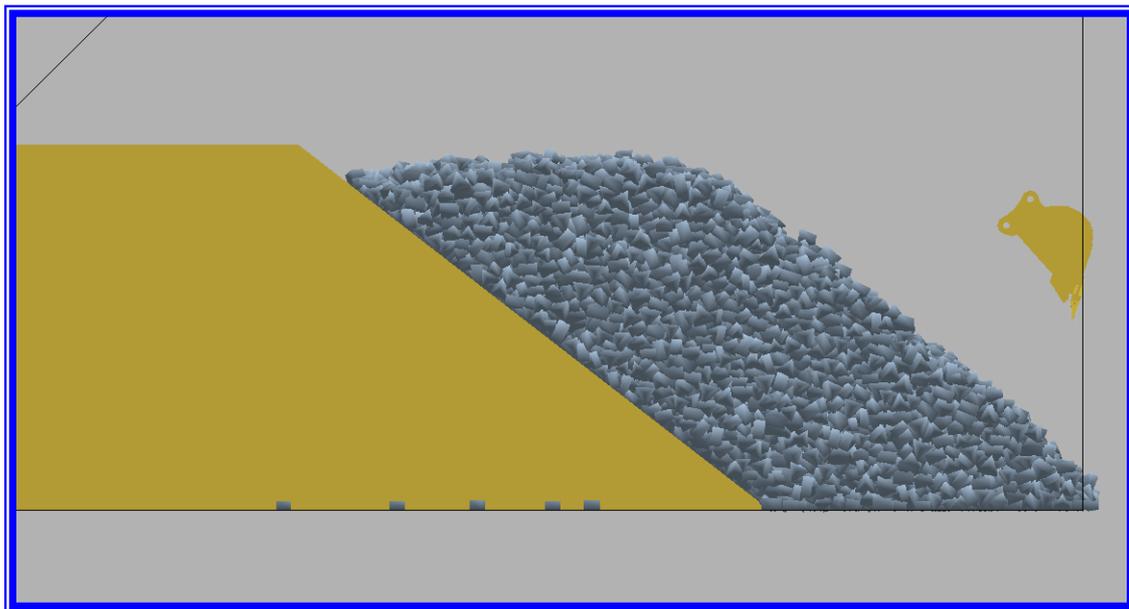
grained soil.

## RESULTS AND DISCUSSION

The Discrete Element code developed by Caterpillar, Inc. (Hofstetter, 2002) was used in this study to simulate triaxial experiments and excavation applications. The triaxial simulations have been performed to show that the current DEM code is capable of capturing the real behavior for both fine-grained and coarse-grained materials with an acceptable level of accuracy. Figure 3 shows the effect of the confining pressure on the deviatoric stress and volumetric strain predictions using DEM triaxial simulations. Dilation increases as the confining stress decreases and, as a result, the predicted peak friction angle increases. Figure 4 shows a

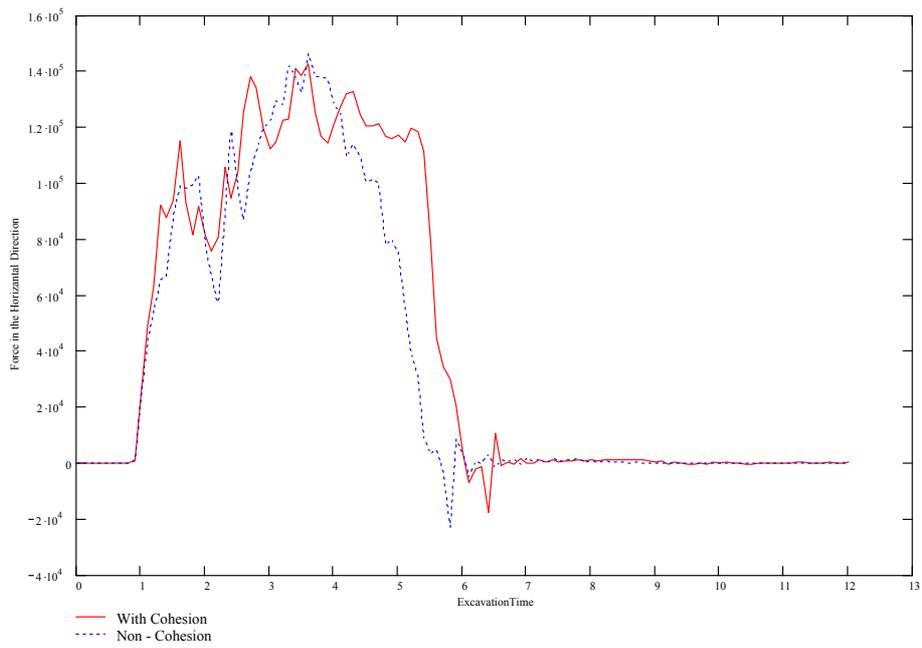


(a) With Cohesion

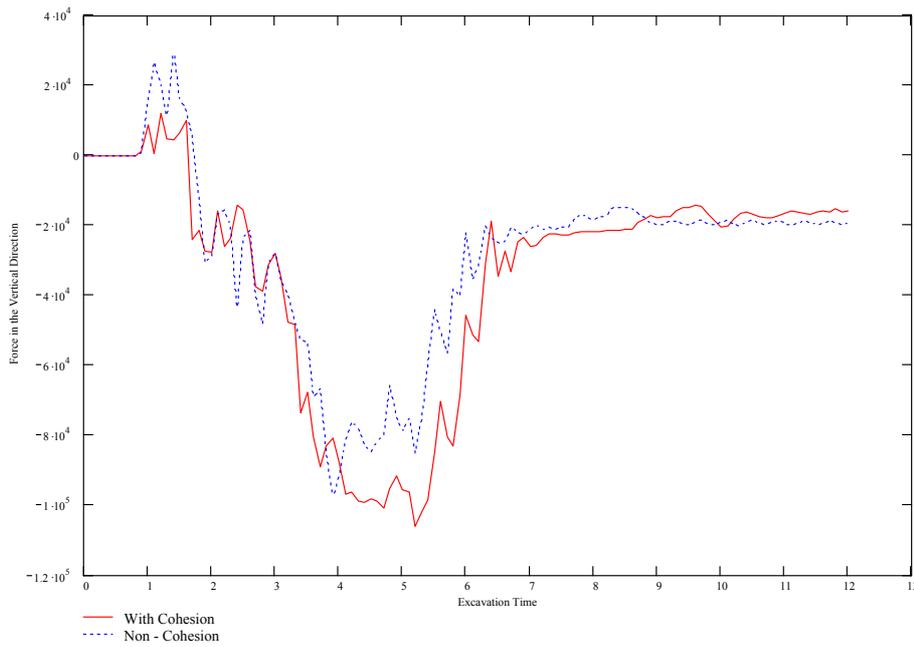


(b) Without Cohesion

Figure 9: Soil Piles Built at Steep Angles Using Caterpillar, Inc. DEM Code



(a)



(b)

**Figure 10: Comparison between Forces Acting on the Bucket during Excavation Using Both (a) Cohesive and (b) Non-Cohesive Parameters**

comparison between predicted and measured deviatoric stresses for medium-dense crushed limestone. The material parameters were obtained from simple laboratory tests and, thereafter, were mapped into the DEM code. The classical internal angle of friction was back-calculated using the virtual triaxial test results and the values agree with laboratory experimental measurements. Likewise, the DEM simulations showed that the density effect agrees with the fundamentals of soil mechanics; dense packed particles seem to dilate while loose packed particles tend to contract (Figure 5). Triaxial test results for fine-grained soil (silty clay) were used to calibrate for the DEM apparent cohesion model parameters. The apparent cohesion model implemented in this DEM code is essentially a non-linear tensile pillar that is described in Figure 6. Using the classical soil shear strength laws (Figure 1), the apparent cohesion and internal angle of friction were obtained and used to predict the stress-strain behavior for this material. Figure 7 shows comparisons between model predictions and laboratory measurements. In these predictions, the internal angle of friction was assumed to be constant following the M-C theory; this assumption of course, does not let the model respond to the effect of the confining pressure. In other words, the dilation effect is not being captured; on the other hand, if the non-cohesion theory (Figure 2) is adopted, the internal angle of friction becomes highly dependent on the level of the confining stress. This dependency agrees with the real behavior of a discrete system. The triaxial test results for the silty clay were analyzed using the non-cohesion theory and the measured internal angles of friction were used to predict the stress-strain behavior for the three different confining stresses. The comparisons in Figure 8 show that the non-cohesion theory can predict the constitutive behavior of the fine-grained soils.

In many engineering practices, the design engineer is required to replicate or build a geotechnical structure using numerical tools: soil piles, slopes, earthfill dams,... etc. Caterpillar machines deal with various types of geotechnical structures. The authors chose to use excavation applications as an example on modeling the

shear strength components for fine-grained soil piles. The modeled soil particles are usually required to be equipped with cohesive bonds in order for the pile to be stable at a given steep angle. In this case, the cohesive bonds algorithm needs to be enabled, which will introduce intensive computational overhead. Instead, using the non-cohesion theory with the additional frictional resistance ( $\phi_{new}$ , which includes the effect of dilation and other microproperties) can provide the adequate shear strength and significantly reduce the computational cost. Figure 9a shows a fine-grained soil built at a steep angle using cohesive bonds; this pile could be rebuilt using the non-cohesion theory at the same steep angle (Figure 9b). Simulating excavation, both models retain almost the same vertical and horizontal forces (see Figure 10) with much lower computational cost for the non-cohesion case.

## CONCLUSIONS

A numerical study for the shear strength components (friction and cohesion) was performed for fine and course-grained soils using Caterpillar DEM code. The classical soil shear strength laws (mainly M-C theory) assume a constant internal angle of friction and ignore the dilation effect. The authors revisited the definition for the apparent cohesion and pointed out the limitations of the concept. The apparent cohesion, if existing in partially saturated fine-grained soil, fails at very low strain levels and we may not account on it. Numerical results using a micromechanical-based DEM model showed that the cohesion part can be ignored and, instead, additional frictional components which are function of dilation and microproperties can be used to compensate for that part and the model will still retain acceptable results with much lower computational cost. The proposed alternative was applied to triaxial compression tests and excavation operations. The findings of the study are supported by the fact that the shear strength of soil is essentially caused by the particles contacts, overlapping and interlocking regardless of the particle size distribution. Then, the shear strength is function of density, surface roughness of the particles, angularity, sphericity and size.

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