Iso-Safety Design Charts for Singly Reinforced Concrete Sections to Eurocode 0 Recommended Target Safety Indices

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ABSTRACT

This work focused on the development of Iso-safety (Isolated Safety) design charts for flexural design of singly reinforced concrete sections at predefined reliability levels in accordance with EN 1992-1-1 (2004) design criteria. Constitutive models for reinforcing steel and concrete were selected in accordance with the EN 1992-1-1 design requirements and subsequently the flexural limit state function was derived. Charts were developed for the flexural design of rectangular reinforced concrete sections with respect to the position of neutral axis ($\frac{x}{d}$) for each concrete grade (f_{ck}) and steel grade (f_{yk}). Uncertainties in loading and geometrical properties were obtained and a program was developed taking into consideration Eurocode 2 design requirements. A safety index β value of 1.81 was achieved for various points on each of the generated curves using First Order Reliability Method (FORM). Reliability-based design charts, called Iso-safety charts, were produced to target safety indices; β_T of 3.3, 3.8 and 4.3 as the minimum recommended for the three failure consequence classes by EN 1990 (2002). This recommendation shows that EN 1992-1-1 design of singly reinforced concrete sections considering flexural failure with a safety index value of 1.81 provides designs that are below the recommended target safety indices. A singly reinforced concrete section was there after being designed using the charts and it was shown that for the same loading and geometrical considerations, the area of flexural reinforcement required increased by 40%, 55% and 75% over EN 1992-1-1 design for corresponding target safety indices of 3.3, 3.8 and 4.3, respectively.

KEYWORDS: Iso-safety, Design charts, Reliability, Target safety index, Eurocodes, Reinforced concrete.

INTRODUCTION

Design charts are very useful for fast determination of the percentage of reinforcements for singly and doubly reinforced concrete beams having known cross-sectional dimensions, characteristic strengths of the concrete and steel and the ultimate design moment (Bayagoob et al., 2013). Iso-safety design charts, however, allow the selection of appropriate design parameters based on a prescribed safety index against the occurrence of a limit state. According to Ditlevsen and Madsen (2005), engineering judgment is the art of being able to decide whether results obtained from a structural analysis or design model are sufficiently realistic so that the engineer dares basing his practical decisions on these results. Abubakar and Pius (2007) added that the aim of a structural design is to produce design and drawings for a safe and economical structure that fulfills its intended purpose. Anthony (1971) opined that it is not easy to account for all factors that affect the assessment of loads consistent with acceptable risk. Sorensen (2004) stated that for many years, it has been assumed in the design of structural systems that all loads

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and strengths are deterministic and that the strength of the designed element is determined in such a way that it exceeds the loading effect with a certain margin. The ratio between strength and load, which is denoted as the safety factor, is considered as a measure of the reliability of the structure. Wright (2003) pointed out that deterministic characteristics are "exact" models that will produce the same outcome each time they are run, therefore ignoring the uncertainties of the input. Yang and DeWolf (2002) added that such uncertainties must be taken into account to assess the safety and performance of the structure. Afolayan (2005) further noted that the traditional way of dealing with uncertainties in the `design process is to use conservative values of the uncertain quantities and/or safety factors in a deterministic framework. Zaki (2006) is the opinion that the deterministic safety factors do not provide adequate information to achieve optimal use of the available resources to maximize safety, while the probabilistic analysis does. The use of probabilistic analysis is therefore expected to provide more information about system behavior, influence of different uncertain variables on system performance and interaction between different system components.

The Eurocode establishes principles and requirements for safety, serviceability and durability of structures. It uses a statistical approach to determine realistic values for actions that occur in combination with each other. Partial factors for actions are given in this Code, whilst partial factors for materials are prescribed in other relevant Eurocodes (Anitha et al., 2007).

Abejide (2014) carried out a reliability analysis considering bending, shear and deflection criteria of reinforced concrete slabs and observed that safety margins proposed for singly reinforced concrete slabs in CP110 (1972), BS8110 (1985) and EN 1992-1-1 (2004) codes for design are not achieved at all. The reliability levels were also found to be non-uniform, thus the current design formulations are not as safe and reliable. Therefore, the design formulations in these codes need a review, so that they can at least meet the target for approved structural safety. Afolayan and Abubakar (2003) proposed Iso-safety design charts for the design of one-way slabs in accordance with BS8110 (1985). It was recommended that there was need to design the slabs using the Iso-safety charts to ensure a compromise between safety and economy in design.

This research paper focuses on the generation of Iso-safety design charts considering EN 1992-1-1 design requirements for singly reinforced concrete sections at predefined target safety index values using First Order Reliability Method (FORM).

FIRST ORDER RELIABILITY METHOD (FORM)

First-Order Reliability Method (FORM) is designed to provide approximations of probability integrals occurring in structural reliability. Gollwitzer et al. (1988) explained that they refer to FORM being designed for the approximate computation of general probability functions over given domains with locally smooth boundaries, but especially for probability integrals occurring in structural reliability. Reliability estimations are carried out by representing each variable by only its first two moments (mean and standard deviation) while ignoring higher moments. FORM is essentially a level II method of reliability analysis and would be used for reliability analysis throughout this study.

To be able to evaluate the reliability of a structure, it is necessary to select the variables of relevance. These variables are selected such that the performance of the structure or system depends upon them; that is, it is a function of the variables (Atim, 2006).

If **X** is a vector of all relevant basic variables for a given structure, each variable x_i, $(i = 1,\ldots,n)$ is considered a realization of a random variable x_i, such that the set of variables $\mathbf{x} = (x_1,\ldots,x_n)$ is a realization of the random vector:

$$\mathbf{X} = (X_1, X_2,\ldots,X_n) \quad (1)$$

Alternatively, the variable **x** is a point in an n-dimensional basic variable space; i.e., $\mathbf{X} = \mathbf{x}$ defines a particular "point" **x**. **X** = (X_1, X_2,….,X_n) is chosen in a manner that a limit state surface (or failure surface) can be defined in the 'n'- dimensional variable space. This failure can be represented as:

$$G(\mathbf{X}) = f(X_1, X_2, \ldots, X_n) = 0 \quad (2)$$

where G(**X**) is the function expressing the relationship between the limit state and basic variables. Then, G(**X**) is the locally sufficient smooth limit state function, which must be at least once differentiable. FORM assumes a set of basic random variables in such a way that G(**X**) > 0 indicates the safe or acceptable region, G(**X**) < 0 indicates an unsafe set of variables or the failure region and G(**X**) = 0 is the limit state itself or failure boundary.

Structural reliability is usually measured by the reliability index, β. FORM assumes that if the probability density functions of the resistance and load variables are normal and independent, they may be combined to define β as:

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad (3)$$

Let G be the safety margin, G = R – S, such that:

$$\beta = \frac{\mu_G}{\sigma_G} \quad (4)$$

where μ_R and μ_S are the mean values of the resistance and load variables and σ_R and σ_S are the corresponding standard deviations. β, which is referred to in other literature as the safety index, is related to the probability of failure (P_f) using (Gollwitzer et al., 1988):

$$P_f = \Phi(-\beta) \quad (5)$$

where Φ is the standard normal distribution function. β is a measure in standard deviation unit, by which the mean μ_G exceeds zero. Recall that the point "R – S = 0" represents the origin. The higher the value of β, the safer the structure and the lower the nominal probability of failure will be.

METHODOLOGY

Singly Reinforced Concrete Sections' Design Charts

Equations (6) and (7) were used to generate the singly reinforced concrete section design charts and were developed from EN 1992-1-1 rectangular stress block.

$$\frac{\boldsymbol{A_{st}}}{\boldsymbol{bd}} = 0.4536 \frac{f_{ck}}{0.87 f_{yk}} \frac{x}{d} \quad (6)$$

$$\frac{\boldsymbol{M}}{\boldsymbol{bd^2}} = 0.87 f_{yk} \rho \left(1 - \frac{0.4x}{d}\right) \quad (7)$$

For each concrete grade (*f_{ck}*) and steel grade *(f_{yk})*, the corresponding values of A_{st}/bd (Equation 6) are calculated with respect to the position of neutral axis ($\frac{x}{d}$). This provides solutions for the x-axis of the design charts on which singly reinforced rectangular concrete sections according to EN 1992-1-1 (2004) are based. For the y-axis, ($\frac{M}{bd^2}$) (Equation 7) is applied also with respect to the neutral axis depth, concrete grade (*f_{ck}*) and steel grade *(f_{yk})*, in order to calculate the moment stress.

Limit State Function

The limit state function *g(x)*, defined as the difference between strength (A) and load (B), is obtained as:

$$G(x) = A - B \quad (8)$$

where:

$$A = \rho f_{yk} \frac{z}{d} \quad (9)$$

and

$$B = \frac{M}{bd^2} = 0.4536 f_{ck} \frac{x}{d} \frac{z}{d} \quad (10)$$

The basic variables selected for the probabilistic analysis are the percentage reinforcement, ρ, steel strength in tension, f_{yk}, position of neutral axis, $\frac{x}{d}$ and the concrete strength, f_{ck}. A typical set of variables selected is shown in Table 1.

Table 1. Statistical models of the basic design variables (Muhammed et al., 2014; Ocholi and Lukman, 2012)

S/No.	Design Variable	Unit	Distribution model	Mean	Coefficient of variation
1	Concrete compressive strength (f_{ck})	N/mm^2	Lognormal	25-50	0.17
2	Steel strength (f_{yk})	N/mm^2	Lognormal	500	0.15
3	Reinforcement ratio (ρ)	_	Normal	0.16	0.16
4	Neutral axis position (x/d)	_	Lognormal	0.07672	0.01

The procedure for this research is summarized below:

1. Selecting a constitutive model for flexure in singly reinforced sections as implied in EN 1992-1-1 (2004)**.**
2. Defining the limit state equation for the failure mode considered.
3. Using appropriate basic equations, design curves to EN 1992-1-1 (2004) were plotted and these agree with the limit state equation.
4. Stochastic characteristics of the basic variables were estimated.
5. The computation of safety index **β** value for the various points on the design curves developed in step 3 above using First Order Reliability Method (FORM).
6. Subroutines in a FORTRAN module were developed and synchronized with that of First-Order Reliability Method (Gollwitzer et al., 1988) and design charts to different target reliability indices were generated according to EN 1992-1-1 (2004). Target reliability level choice was made to correspond with the target reliability indices of Table B2 of EN1990 (2002) which is presented as Table A2 in the Appendix.

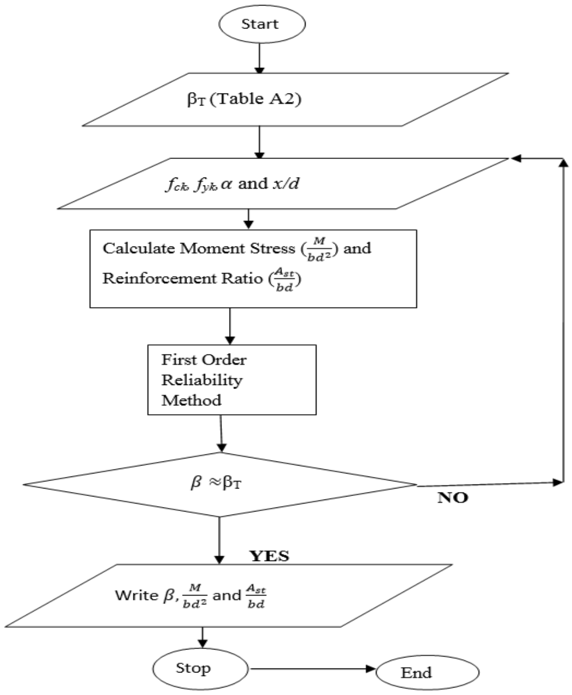

Figure (1): Program flowchart for Iso-safety charts

RESULTS AND DISCUSSION

EN 1992-1-1 Singly Reinforced Section Design Charts

All the points on each curve in Figure 2 are selected and the safety index, β, is computed for each point. The charts have 4 curves for steel grade 500 N/mm^2. This grade is the grade recognized by the Eurocode (Mosley et al., 2007). Each curve represents a particular grade of concrete. The safety index of each point on the curve was calculated *via* FORM5 (Gollwitzer et al., 1988). A program was written for this purpose and a reliability index (β_0) of 1.81 computed and found to be uniform between grades of steel and concrete, which shows the consistency of the points on the design charts, but this value of safety index does not fall within the acceptable range mostly specified in literature and the Eurocode for ULS consideration. A reinforcement ratio of 0.2% was achieved using the chart and 0.1948% (≅ 0.2%) using conventional EN 1992-1-1 design equation when a singly reinforced section was designed. The slight difference may be attributed to the scale used for the x-axis.

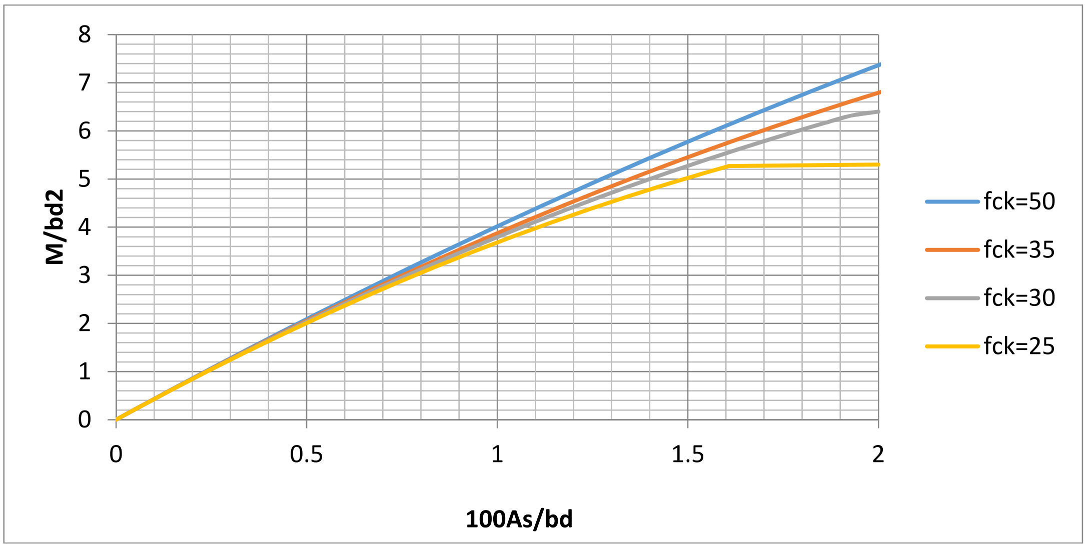

Figure (2): Design chart for singly reinforced rectangular sections (f_{yk}= 500 N/mm^2)

ISO-SAFETY DESIGN CHARTS

To develop these charts, target reliability level choice is made to correspond to values recommended for the ultimate limit state in Eurocode 0 (2002). The selected target values for the reliability index, β, are 3.3, 3.8 and 4.3 as the minimum recommended by Eurocode 0 for the 3 consequence classes. For each concrete grade (f_{ck}), steel grade (f_{yk}) and neutral axis depth (x/d), the reinforcement ratio ($\boldsymbol{\rho}$) is kept constant, while the moment stress, $M/_{bd^2}$, is allowed to vary systematically until a target value of safety index, β_T, is attained. That is, once the safety index β is computed, then the program checks the calculated value of β against the target safety index, β_T, specified at the onset. If the calculated reliability level is approximately equal to the target safety level, the design is said to be acceptable (Abubakar, 1999; Akindahunsi and Afolayan, 2009). The program writes down the values of $\frac{M}{bd^2}$ and $\frac{A_{st}}{bd}$ at the design point and stops otherwise. The design procedure is repeated until the condition is satisfied. The values of $M/_{bd^2}$ thus obtained are plotted against the computed values of $\boldsymbol{\rho}$ to generate the new reliability-based (Iso-safety) design charts for the target value of β_T. The use of the charts given below will provide explicit information on the safety index of the sections being designed. The Iso-safety charts are given in Figures (3) to (5) for the design of singly reinforced concrete sections.

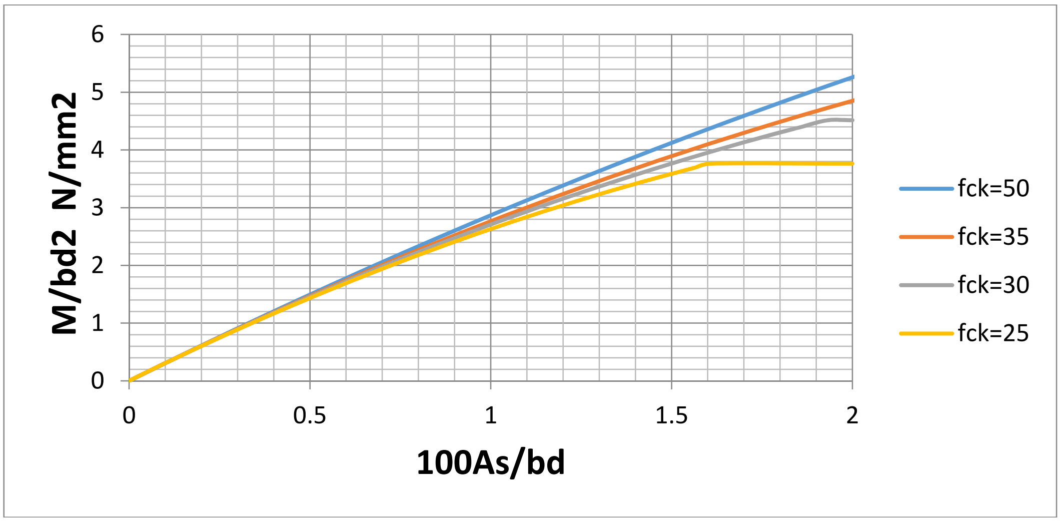

Figure (3): Iso-safety design chart for singly reinforced concrete sections, β = 3.3 and f_{yk} = 500 N/mm²

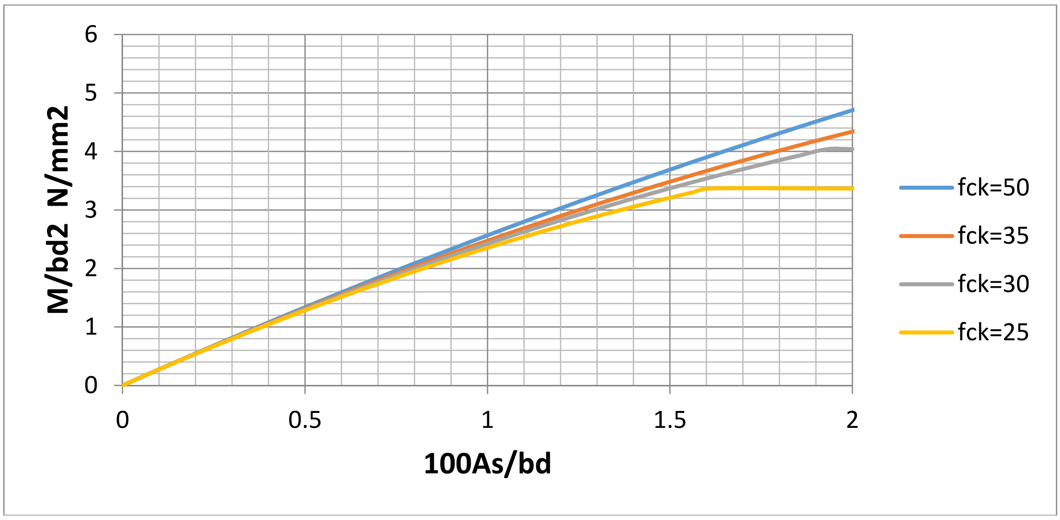

Figure (4): Iso-safety design chart for singly reinforced concrete sections, β = 3.8 and f_{yk} = 500 N/mm²

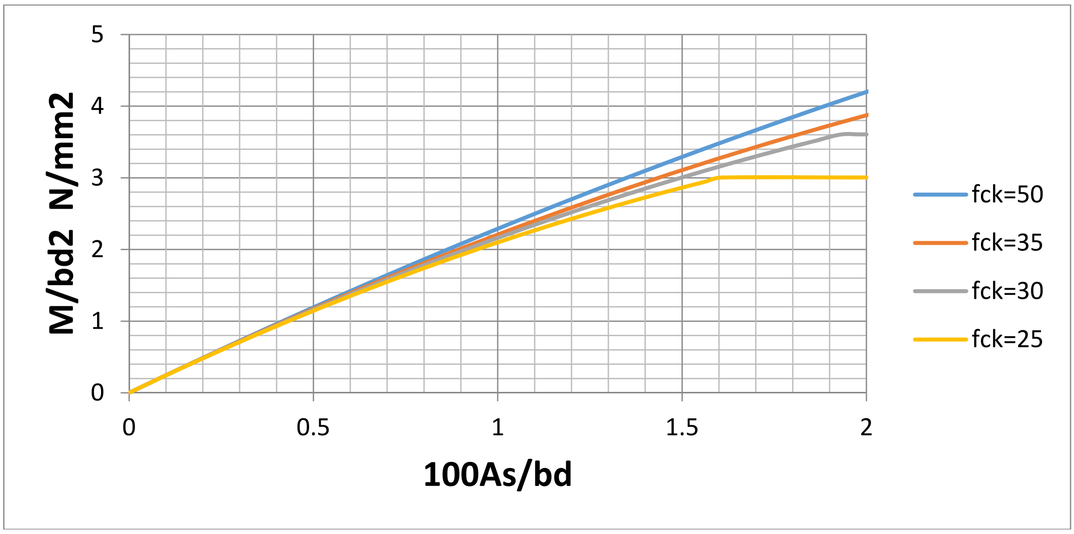

Figure (5): Iso-safety design chart for singly reinforced concrete sections, β = 4.3 and f_{yk} = 500 N/mm^2

Illustrative Example on the Use of Charts

It is required to design the cross-section of a flat slab for the ULS assuming a characteristic strength of concrete f_{ck}= 25 N/mm^2, a characteristic strength of steel f_{yk}= 500 N/mm^2, an overall depth of slab = 250 mm, a drop panel dimension = 2.5 m square, a depth of drop panel = 100mm, column head diameter = 1.2 m, a variable load = 5 kN/m^2 (Source: Mosley et al., 2007).

Figures (2), (3), (4) and (5) were respectively used to obtain the results in Table 2.

Table 2. Summary of illustrative example design solution

S/No.	Chart type	$100A_{st}/bd$	Area of Steel A_{streq}
1	EN 1992-1-1 Design Chart β = 1.81	0.20	$1640mm^2$
2	Iso-safety Design Chart β_t = 3.3	0.28	$2296mm^2$
3	Iso-safety Design Chart β_t = 3.8	0.31	$2542mm^2$
4	Iso-safety Design Chart β_t = 4.3	0.35	$2870mm^2$

From the results presented in Table (2), it was observed that EC 2 gave a cheaper area of steel requirement, but the safety index of 1.81 indicates failure of the section at ULS as it is lower than the minimum recommended by Eurocode 0 (2002). Also, using the Iso-safety design, as β_T was increased, the area of steel required increased by 40%, 55% and 75% when β_T =3.3, 3.8 and 4.3, respectively, for the example considered. This further confirms the claim of Ocholi and Lukman (2012) that a higher area of steel is required for improved safety.

CONCLUSION

This paper presents Iso-safety design charts for the design of singly reinforced concrete sections according to Eurocode 0 recommended target safety indices for steel grade 500N/mm^2. The charts allow for the selection of design target safety index and provide sections with uniform reliability. It was shown that EN 1992-1-1 provides designs below the recommended target safety indices and that using the developed Iso-safety charts for the same loading and geometrical considerations, the area of flexural reinforcement required increased by 40%, 55% and 75% over EN 1992-1-1 design for corresponding Eurocode 0 recommended target safety indices of 3.3, 3.8 and 4.3, respectively. This shows that the practice of choosing the next higher area of reinforcement in the design of reinforced concrete sections is actually a good practice.

APPENDIX

Table A1. Definition of consequence classes (EN1990. (2002))

Consequence class	Description	Examples of buildings and civil engineering works
CC3	High consequence for loss of human lives, or economic, social or environmental consequence considerations	Grandstands, public buildings where consequences of failure are high (e.g. a concert hall)
CC2	Medium consequence for loss of human lives, or economic, social or environmental consequence considerations	Residential and office buildings, public buildings where consequences of failure are medium (e.g. an office building)
CC1	Low consequence for loss of human lives, and economic, social or environmental consequences small or negligible	Agricultural buildings where people do not normally enter (e.g.storage buildings), green houses

Each of the mentioned consequence class CC (Table A1) is assigned a reliability class RC (Table A2).

Table A2. Recommended minimum values for reliability index β (ultimate limit state) (EN1990 (2002))

Reliability class	Minimum values for β	
	1-year reference period	50-year reference period
RC3	5.2	4.3
RC2	4.7	3.8
RC1	4.2	3.3

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