

Designing and Proportioning of Fiber-Reinforced Lightweight Concrete Mixtures Using Engineered Aggregate

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ABSTRACT

The goal of this paper is to construct design graphs that can be used easily by engineers to design fiber-reinforced lightweight aggregate concrete (LWAC) mixtures. Twenty LWAC mixes made from a wide range of materials were analyzed. The data were collected from nine design reports submitted by universities in the USA and Canada. Compressive strength, splitting tensile strength, unit weights with respect to percent cementitious materials (cm) by weight, percent lightweight aggregate (LWA) by volume, water-cementitious materials ratio (w/cm), percent short fibers and air content were incorporated into a set of integrated design graphs. A worked example was included at the end of the research to verify the validity of the new design approach.

KEYWORDS: Lightweight aggregate concrete, Fibers, Compressive strength, Poraver, Proportioning.

INTRODUCTION

For more than five decades, lightweight concrete (LWC) has attracted engineers. Using LWC in construction will reduce the overall weight of the structure, since much of the weight of a concrete structure is accounted for by its own dead load. Consequently, the required dead load reinforcement can be reduced, which results in cost saving. Since thermal conductivity and density of concrete are proportional (Kosmatka and Wilson, 2011), LWC can contribute to heat preservation, insulation, energy saving and high fire resistance. Furthermore, Carrilo et al. (2015) have found that lightweight low-strength shear wall concrete provides larger shear strength, drift ratios and energy dissipation than normal weight concrete. Among other advantages of using LWC are ease of construction and

being a relatively “green” building material (Suprenant and Malisch, 1999).

LWC can be used either in non-structural members as wall panels, exterior finishing materials, bricks or structural members. Structural members made from concrete with a density of (1360-1840 kg/m³) (85-115 lb/ft³) and a compressive strength greater than 17 MPa (2500 psi) have been used as floors in high-rise buildings. ACI-213 (2003) defines structural lightweight concrete (SLWC) as concrete with a density between 1120 kg/m³ and 1920 kg/m³ (70 lb/ft³ and 120 lb/ft³) and minimum compressive strength of 17 MPa (2500 psi) at the age of 28 days. LWC with a compressive strength of 0.7-7 MPa (100-1000 psi) at the age of 28 days with an oven dry density of less than or equal to 800 kg/m³ (50 lb/ft³) is called “insulating concrete”. This type of concrete is used for thermal and sound insulation. Moderate-strength LWC has a compressive strength of 7-17 MPa (1000-2500 psi) and a density of 800-1900 kg/m³ (50-120 lb/ft³). This type of

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concrete is typically used for thermal and sound insulation in walls and roofs.

With the advances in material technology, it has become possible to produce LWC effectively and efficiently with excellent engineering properties and characteristics. The key in producing LWC is lightweight aggregate (LWA) with a density $\leq 640 \text{ kg/m}^3$ (40 lb/ft^3). However, LWA with greater densities can be used. LWC manufactured with LWA is called lightweight aggregate concrete (LWAC). A combination of LWA of different particle sizes and strengths can be used. A wide range of LWA products are available in the market these days. Strength, unit weight, size and cost vary widely. For example, compressive strength can vary from 1.7 to 193 MPa (250 to 28,000 psi).

Adding randomly dispersed short fibers in low volume dosages (often less than 1% by volume) to the mixture will improve the characteristics of concrete due to their ability to provide post-cracking tension resistance to concrete (White and McGregor, 2012). Arisoy and Wu (2008) have developed high-performance fiber-reinforced lightweight concrete that shows high flexural strength, high flexural ductility and excellent toughness using polyvinyl alcohol (PVA) fibers. Despite their negligible strength in compression, fibers are a very important constituent in LWAC. The amount of fiber should be limited to 1.5% by volume. It was observed that the use of polypropylene fibers (PPF) in a percentage of $\geq 0.1\%$ by volume of concrete reduces plastic shrinkage cracking (Cui et al., 2012). In their work, Bagherzadeh et al. (2012) have found that an increase in splitting tensile strength was associated with an increase in the amount of fibers, but the length of fibers had only a slight effect on strength.

In order to achieve certain properties in concrete effectively and economically and to improve the performance of concrete, chemical admixtures can be used. These admixtures usually come in liquid form. Water-reducing and air-entraining agents are the primary admixtures. These two admixtures play a major role in LWAC. Superplastizires are high-range water

reducing agents. Water-reducing admixtures increase workability of concrete without increasing water content. Reducing water content in concrete mixes will increase the strength of the final product. Air-entraining agents increase the mount of air bubbles in concrete, resulting in a reduction in weight. However, the increase in air content will weaken the concrete. Some other chemical admixtures are: permeability reducing shrinkage reducing and viscosity modifying admixtures.

OBJECTIVES

There is a lack of standards to design and proportion fiber-reinforced LWAC using engineered aggregates. The objective of this research is to construct a set of design graphs that enable engineers to design such concrete mixes. This study is based on nine design reports. Appendix A shows twenty different LWAC mixes submitted by universities in the USA and Canada. LWAC engineering properties can be determined by specifying some common and essential parameters being used in designing and proportioning concrete mixtures. A worked example was included at the end of the research.

MATERIALS

A wide range of materials with different proportions were used in making concrete mixes. Each material has a different effect on the overall properties and behavior of the concrete. These properties are, for example, compressive strength, tensile strength, unit weight/density, ductility, flexural resistance and fracture toughness. These materials can be classified into five categories: cementitious materials, lightweight aggregate, fibers, chemical admixtures and water. Cementitious materials consist of cement and supplementary cementitious materials (SCM). The materials used in making the vast majority of the LWAC specimens are as follows:

- ◆ Cement type I/GU with a specific gravity (S.G.) of 3.15.

- ◆ A combination of one or more supplementary cementitious materials (SCM): Pozzolan VCAS™ 160 with an S.G. of 2.6; slag grade 120 with an S.G. of 2.9; metakaolin with an S.G. of 2.6; fly ash with an S.G. of 2.15; silica fume with an S.G. of 2.3; hydrated lime type S with an S.G. of 2.6. Table 1 summarizes the proportions, by weight, of cementitious materials used in the LWAC mixes.
- ◆ A combination of one or more Nycon® polyvinyl alcohol (PVA) fibers of lengths of 6, 8, 12, 19, 30 and 38 mm with an S.G. of 1.3.
- ◆ A combination of high-range water reducing agents/superplasticizers (Master Glenium® 7700, Glenium® 3030 NS, ADVA® CAST), air-entraining agents (Darex®, Daravair® AT30, Micro-Air®), shrinkage reducing agents (Eclipse®, Hydra Mix) and permeability reducing agents (Hycrete X1002, Xypex Xycrylic).
- ◆ A combination of at least four different particle sizes of engineered LWA. Four different types of LWA were used:
 - (1) 3M™ glass bubbles: these bubbles are strong, lightweight, high strength-to-density ratio and low thermal conductivity hollow glass microspheres.
 - (2) Poraver® expanded spherical glass granules. Poraver is a lightweight, alkali-resistant, insensitive to heat and cold properties' aggregate.
 - (3) Cenospheres CW300 (hollow ceramic spheres).
 - (4) Q-cell® 6014 (ultra-light hollow microspheres).

Table 2 shows some physical properties of the LWA used in this research, whereas Table 3 summarizes the aggregate proportions, by volume, used in making the LWAC mixes.

CALCULATIONS

In this study, twenty different concrete mixes with different proportions were used for analysis. The first mix was labeled LWAC1, the second mix was labeled LWAC2, ... and so on. For each mix, water/cementitious materials ratio w/cm , percent fibers by volume, percent cementitious materials by weight, percent aggregate by

volume and theoretical unit weight were calculated. w/cm is the mass of water divided by the mass of cementitious materials. Detailed fiber-reinforced LWAC mix proportions for all concrete mix specimens are shown in Table 4.

Compressive strength of cylindrical concrete specimens at the age of 28 days, splitting tensile strength (f_{ct}) and dry unit weight were collected and used in the analysis. Based on the design reports, it was observed that fiber-reinforced LWAC with engineered LWA showed a compressive strength increase of 7-8% from 14 to 28 days of age, as well as an increase of 13% from 7 to 14 days of age.

In some design reports, tensile strength was given as modulus of rupture (MOR), f_r . On average, $f_r = 1.5 f_{ct}$ (White and McGregor, 2012). According to ACI-318 (2008), Equation 1 can be used to calculate splitting tensile strength.

$$f_{ct} = 0.56 \lambda \sqrt{f'_c} \quad (1)$$

where λ equals 0.75 for all lightweight aggregates, which is our case, and f'_c is the compressive strength at the age of 28 days.

Due to lower aggregate specific gravity, LWAC doesn't slump as much as normal weight concrete (NWC). It was observed that the slump for almost all specimens ranged from 50 mm to 76 mm (2-3 in).

In order to ensure gradual gradation of particle sizes, all twenty concrete mixes used spherical aggregates of different particle sizes. A combined gradation can result in better control workability, pumpability, shrinkage and other properties of concrete. Figure 1 shows aggregate gradation curves for some LWAC mix specimens and fine aggregate grading limits as specified in ASTM C33 (2006).

The dry unit weight of the LWAC mix depends on the amount and type of LWA used. Table 1 shows that different types of LWA have different absorptions. It was observed in some mixes that the ratio of measured dry unit weight to measured wet unit weight was 0.88. On the other hand, the same ratio was 0.99 in some other

mixes. Based on the nine design reports with twenty design mixes, compressive strength ≥ 10 MPa (≥ 1470 psi), unit weight of 900-1200 kg/m³ (55-75 pcf) and splitting tensile strength ≥ 1.5 MPa (≥ 200 psi) were observed to fall within the following ranges: *w/cm* 0.3-0.5, % cementitious materials by weight 0.41-0.5, %

lightweight aggregate by volume 0.55-0.7, % fibers by volume 0.1%-1.2% with an average value of 0.5% and air content 2%-20% with an average value of 10%. Note that 2% is the lowest air content reported in the design reports, whereas 20% is the highest air content reported in the design reports.

Table 1. Proportions of cementitious materials and water by weight

Specimen	Cement	Pozzolans	Slag	Silica Fume	Fly Ash	Metakaolin	Hydrated Lime	Water
LWAC1	1	1	0.5	0	0	0	0	1.12
LWAC2	1	1	0.5	0	0	0	0	1.12
LWAC3	1	0.29	1.41	0	0	0	0.15	1.07
LWAC4	1	0	0	0	0	0	0	0.38
LWAC5	1	0.28	1.44	0	0	0	0.15	1.08
LWAC6	1	0.81	2.22	0	0	0	0	2.65
LWAC7	1	0.79	2.18	0	0	0	0	1.36
LWAC8	1	0.79	2.18	0	0	0	0	1.19
LWAC9	1	0.52	0	0.24	0	0.31	0	0.85
LWAC10	1	0.53	0	0.24	0	0.3	0	0.9
LWAC11	1	0	0	0.84	0	0.31	0	0.9
LWAC12	1	0.25	1.25	0	0	0	0	0.91
LWAC13	1	0.25	1.25	0	0	0	0	0.93
LWAC14	1	0.25	0.08	0	0.62	0	0	1.17
LWAC15	1	0	0	0	0	0	0	0.38
LWAC16	1	0.5	1.64	0	0	0	0.17	1.26
LWAC17	1	0.5	1.64	0	0	0	0.17	1.19
LWAC18	1	0.5	1.64	0	0	0	0.17	1.17
LWAC19	1	1.23	0	0	0	0	0	0.89
LWAC20	1	0.64	0.42	0	0	0	0	0.82

Table 2. Physical properties of lightweight aggregate

Lightweight Aggregate	Average S.G. (Oven Dry)	% Absorption by Mass	Compressive Strength MPa (psi)	Diameter (mm)
3M™ K1 Glass Bubbles	0.125	0	1.7 (250)	<0.177
3M™ K15 Glass Bubbles	0.15	0	2.0 (300)	<0.177
3M™ K20 Glass Bubbles	0.2	0	3.5 (500)	<0.177
3M™ S38 Glass Bubbles	0.38	0	28.0 (4,000)	<0.105
3M® iM16K Glass Bubbles	0.46	0	110.0 (16,000)	<0.105
3M® iM30K Glass Bubbles	0.6	0	193 (28,000)	<0.105
Poraver® Expanded Glass I	1.4	40	3.0 (430)	0.04-0.125
Poraver® Expanded Glass II	0.9	35	2.8 (406)	0.1-0.3
Poraver® Expanded Glass III	0.6	30	2.6 (377)	0.25-0.5
Poraver® Expanded Glass IV	0.47	25	2.0 (290)	0.5-1
Poraver® Expanded Glass V	0.39	20	1.6 (232)	1-2
Poraver® Expanded Glass VI	0.32	15	1.4 (203)	2-4
Q-Cel® 6014 Microspheres	0.14	0	1.7 (250)	0.005-0.2
Cenospheres CW300	0.9	0	23.0 (3200)	0.01-0.3

Table 3. Ratios of the lightweight aggregate constituents by volume to the total volume of aggregates

Specimen	Poraver 0.04-0.125 mm	Poraver 0.1-0.3 mm	Poraver 0.25-0.5 mm	Poraver 0.5-1 mm	Poraver 1-2 mm	Poraver 2-4 mm	3MK1	3MK15	3MK20	3MS38	Q-cel	Cenospheres
LWAC1	-	0.1	0.16	0.2	0.19	-	-	0.35	-	-	-	-
LWAC2	-	0.1	0.16	0.2	0.19	-	-	0.35	-	-	-	-
LWAC3	0.05	0.25	0.58	-	-	-	-	-	-	0.06	0.05	-
LWAC4	0.01	0.07	0.37	0.29	0.16	-	-	-	-	0.03	0.08	-
LWAC5	-	0.05	0.35	0.32	0.21	-	-	-	-	0.04	0.04	-
LWAC6	-	-	0.24	-	-	-	-	0.11	-	0.65	-	-
LWAC7	-	-	0.2	0.29	0.3	-	-	0.1	-	0.1	-	-
LWAC8	-	-	0.2	0.3	0.3	-	-	0.1	-	0.1	-	-
LWAC9	-	0.04	0.09	0.24	0.46	-	-	0.17	-	-	-	-
LWAC10	-	0.05	0.1	0.29	0.55	-	-	-	-	-	-	-
LWAC11	-	0.05	0.1	0.29	0.55	-	-	-	-	-	-	-
LWAC12	-	0.05	0.4	0.33	0.15	-	0.04	-	-	0.04	-	-
LWAC13	-	0.06	0.41	0.32	0.12	-	0.05	-	-	0.05	-	-
LWAC14 ⁱ	-	-	0.09	0.14	0.11	-	-	-	0.23	-	-	0.25
LWAC15	-	0.07	0.44	0.33	0.16	-	-	-	-	-	-	-
LWAC16	-	0.07	0.44	0.33	0.16	-	-	-	-	-	-	-
LWAC17	-	0.06	0.31	0.53	0.1	-	-	-	-	-	-	-
LWAC18	-	0.07	0.44	0.33	0.16	-	-	-	-	-	-	-
LWAC19	-	0.08	0.12	0.23	0.25	0.32	-	-	-	-	-	-
LWAC20	-	-	0.11	0.3	0.3	-	0.3	-	-	-	-	-

ⁱ plus 0.06 “3MiM16K”, and 0.12 “3MiM30K”

Table 4. Fiber-reinforced lightweight aggregate concrete mix proportions

Specimen	w/cm	% fiber by volume	% cementitious materials by weight	% aggregate by volume	Theoretical density kg/m ³ (lb/ft ³)
LWAC1	0.47	1.2	45	58	933 (58.31)
LWAC2	0.47	1.2	45	57	934 (58.37)
LWAC3	0.377	0.43	47	57	1000 (62.42)
LWAC4	0.377	0.3	48	56	1228 (76.73)
LWAC5	0.377	0.3	45	63	1071 (66.94)
LWAC6	0.66	0.1	66	0.57	937 (58.52)
LWAC7	0.34	0.1	34	65	913 (57.06)
LWAC8	0.3	0.1	42	70	901 (56.28)
LWAC9	0.41	0.1	43	69	916 (57.2)
LWAC10	0.43	1.1	44	65	1018 (63.6)
LWAC11	0.42	1.1	45	65	1023 (63.9)
LWAC12	0.36	0.27	47	62	1040 (64.92)
LWAC13	0.37	0.33	50	57	1096 (68.45)
LWAC14	0.6	0.8	38	68	890 (55.62)
LWAC15	0.38	0.32	47	60	1104 (68.95)
LWAC16	0.38	0.32	47	58	1086 (67.87)
LWAC17	0.36	0.13	49	57	1102 (68.83)
LWAC18	0.35	0.32	48	60	1087 (67.94)
LWAC19	0.4	1.0	44	67	946 (59.1)
LWAC20	0.4	1.1	47	67	914 (57.13)

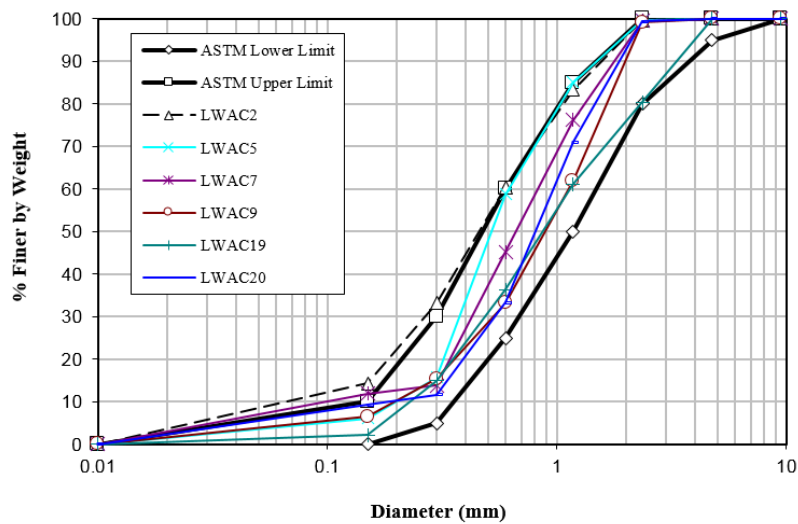


Figure (1): Aggregate gradation curves for some selected LWAC mixes

RESULTS AND DISCUSSION

Compressive Strength

From Figure 2 and Figure 3, it can be seen that the compressive strength of LWAC increases with an increase in percent of cementitious materials by weight, while the compressive strength of LWAC decreases with an increase in percent aggregate by volume.

Figure 2 shows that a single value of “% cementitious materials” can produce different compressive strength values. These values are affected by many factors. Among these factors are w/cm , air content and % LWA of the LWAC mix. Similar linear relationship was obtained by Stanton (2016) based on five data points. The above argument can be used for Figure 3, but this time “% cementitious materials by weight” should be replaced by “% aggregate by volume” and *vice versa*. In both Figures, a simple line was used to fit the data.

It was observed that there is a good correlation between % cementitious materials, by weight, and % aggregate, by volume, as shown in Figure 4. A simple line was used to fit the data points. The compressive strength of the LWAC specimens was in the range from 12 MPa (1740 psi) to 23 MPa (3300 psi).

Another crucial factor in concrete mix design is water/cementitious materials ratio (w/cm). Compressive strength and w/cm are inversely proportional. Figure 5 can be used as a starting point to select a water/cementitious materials ratio with respect to the required compressive strength. It can be seen that the data points are scattered over a wide area.

Due to the difficulty of representing all data points using a single relationship, a band (shaded area) that covers 90% of them was used. However, a simple line was also used to fit the data. This band represents the ideal range for w/cm . The same concept was used in ACI 211.2 (1998) to construct a relationship between compressive strength and cement content. To simplify the process of selecting the w/cm , the shaded area was divided using four lines parallel to the fitting line.

A single value of w/cm ratio can produce different

values of compressive strength. There are many factors that contribute to producing different values of compressive strength for a single w/cm ratio, such as: air content, % cementitious materials and % aggregate of the LWAC mix. In general, for the same w/cm , % cementitious materials by weight and % aggregate by volume, the upper region of the shaded area can be used for low air content (< 10%), while the lower region of the shaded area can be used for high air content (> 15%). For moderate air content (10-15%), the fitting line can be used.

Air content has a significant effect on the compressive strength of LWAC. Figure 6 shows a 40% reduction in strength when air content increases from 2% to 20%, while w/cm , % cementitious materials by weight and % aggregate by volume remain constant with a tolerance of $\pm 1\%$, $\pm 2\%$ and $\pm 2\%$, respectively. A second-order polynomial was used to fit the data. Air content can be calculated using Equation 2.

$$\text{Air content, \%} = \frac{T-D}{T} \times 100 \quad (2)$$

where T is the theoretical unit weight and D is the measured unit weight.

Unit Weight (Density)

From Figure 7 and Figure 8, it can be seen clearly that the theoretical unit weight of LWAC increases with an increase in percent of cementitious materials by weight, while the unit weight of LWAC decreases with an increase in percent aggregate by volume. Theoretical unit weight of LWA was based on saturated dry surface. The unit weight can be predicted with respect to compressive strength using Figure 9. Unit weight and compressive strength are proportional; i.e., unit weight of LWAC increases with an increase in compressive strength. A simple line was used to fit the data points. Most LWAC compressive strength values are associated with more than one unit weight. There are many factors that contribute to producing different values of unit weight for a single compressive strength. Among these factors are: air content, absorption of LWA, % LWA and % cm .

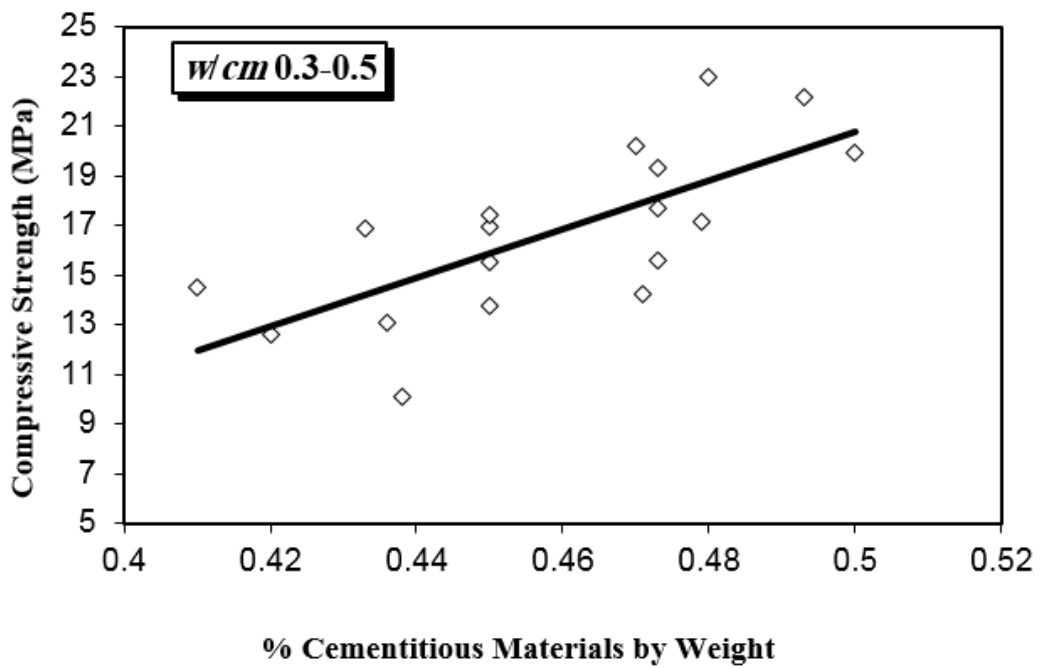


Figure (2): Relationship between compressive strength and % cementitious materials by weight using LWA

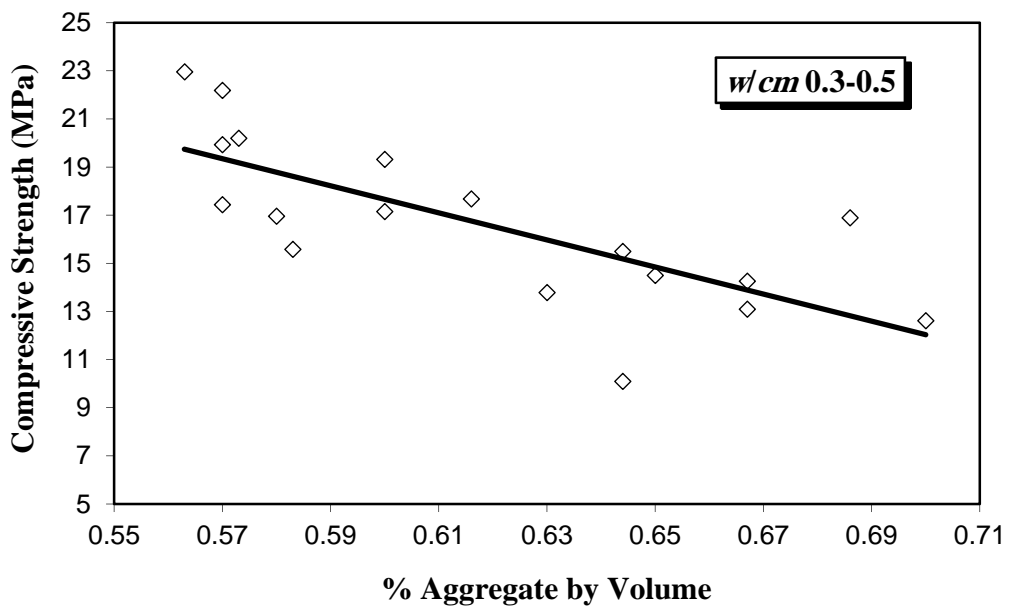


Figure (3): Relationship between compressive strength and % aggregate by volume using LWA

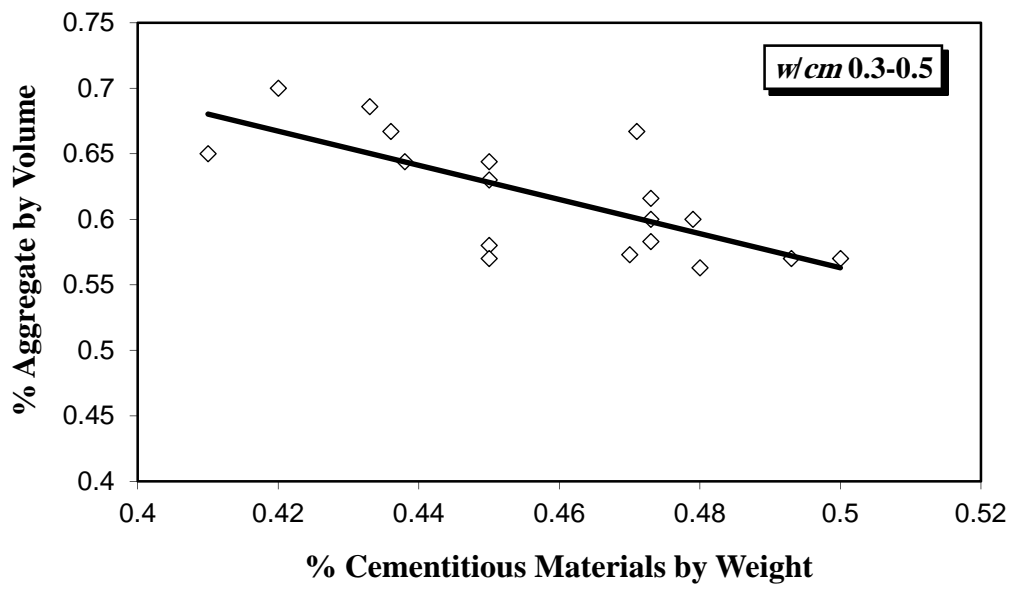


Figure (4): Relationship between % aggregate, by volume, and % cementitious materials, by weight, in LWAC

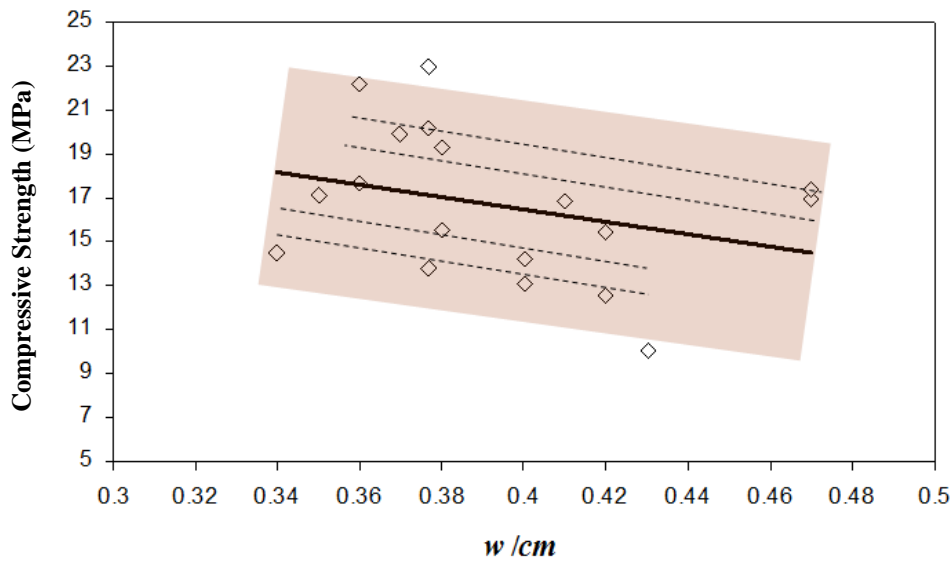


Figure (5): Relationship between compressive strength and w/cm using LWA

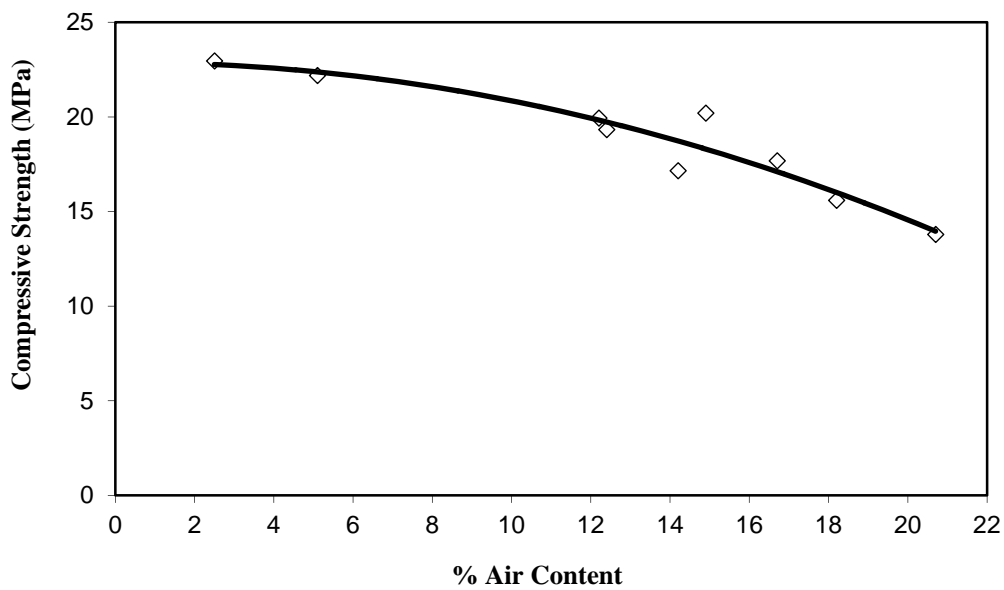


Figure (6): Relationship between compressive strength and air content for the same w/cm , % cementitious materials by weight and % aggregate by volume in LWAC

Splitting Tensile Strength

Figure 10 shows the relationship between splitting tensile strength and compressive strength. Equation 1 is recommended by ACI-318. Tensile strength increases with an increase in compressive strength. Based on the results experimentally obtained from the design reports, Equation 1 can be slightly adjusted to account for the effect of short fiber in the LWAC by using Equations 3 and 4.

$$f_{ct} = 0.54 \lambda F_f \sqrt{f'_c} \quad (3)$$

$$F_f = 1.15 - 0.07V_f \quad (0.1 \leq V_f \leq 1.2) \quad (4)$$

where F_f is the “fiber factor” and V_f is % fibers by volume. The average value of % fibers by volume for all LWAC specimens was 0.53%. Thus, the value of 0.5% will be considered the optimum fiber content.

Verification

The following example, taken from a different design report, Appendix B, has been included to verify the validity of this new procedure.

Example: Fiber-reinforced LWAC is required with a compressive strength of 14.5 MPa (2100 psi). Calculate the proportions needed for the specified compressive strength, the expected unit weight and the splitting tensile strength.

Solution: Table 5 shows the model/expected values based on Figures 2, 3, 4, 5 and 8 and the suggested equation for splitting tensile strength. It also shows the experimental results.

It can be seen that there is a negligible difference in w/cm , % cementitious materials by weight and measured unit weight between the model and the experimental results, while there is a slight difference in theoretical unit weight and % aggregate by volume (< 8.5%).

There is a noticeable difference in splitting tensile strength. However, the “author” believes that the experimental value should be a little bit higher.

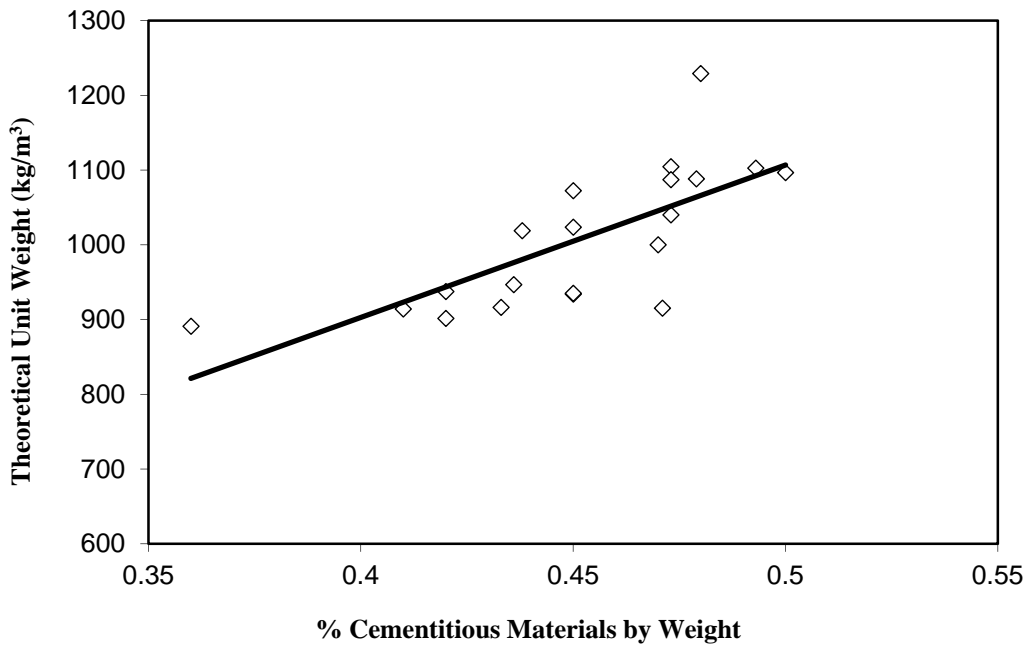


Figure (7): Theoretical unit weight vs. % cementitious materials by weight in LWAC

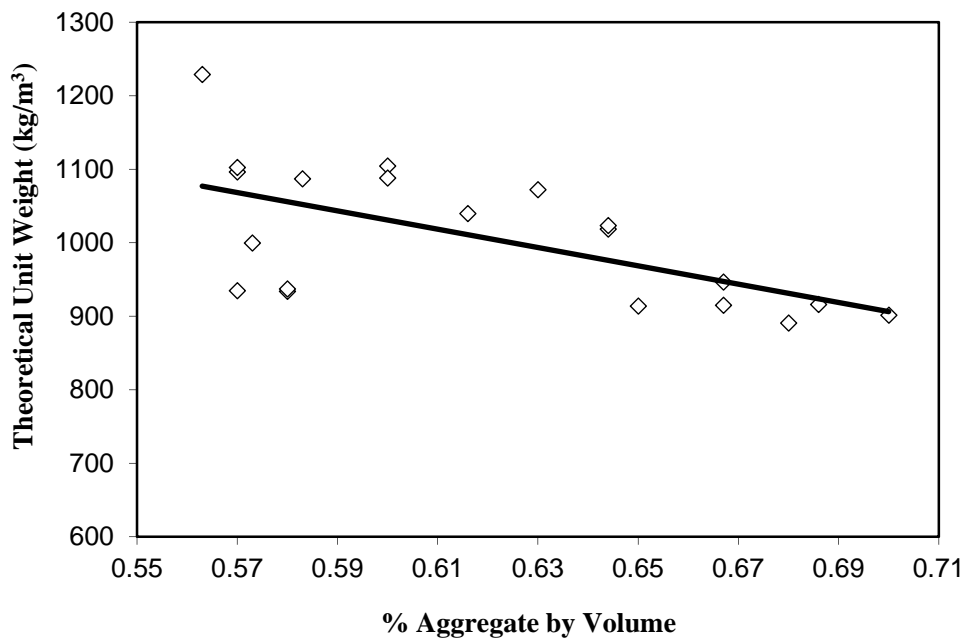


Figure (8): Theoretical unit weight vs. % aggregate by volume in LWAC

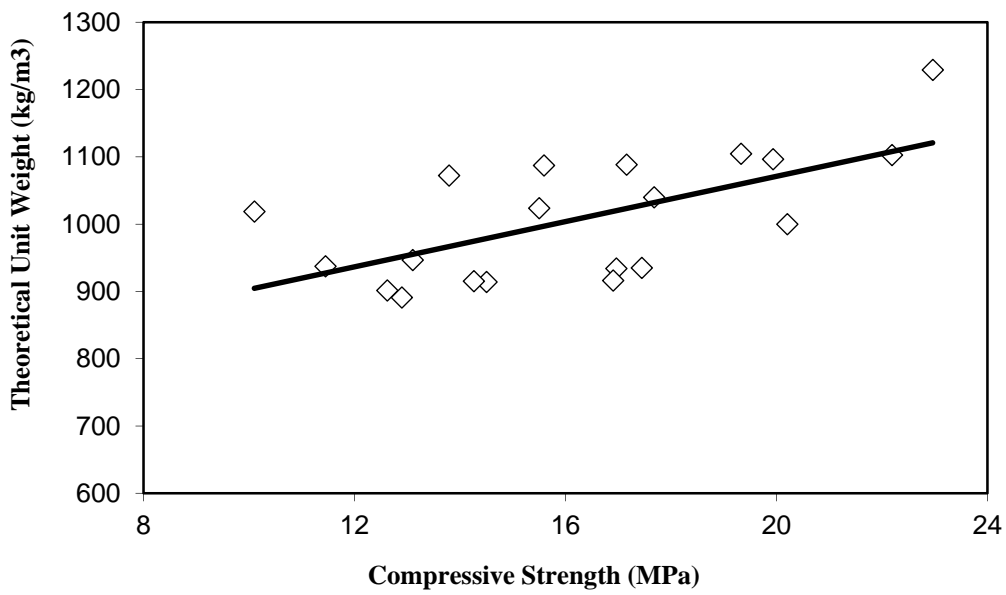


Figure (9): Theoretical unit weight vs. compressive strength in LWAC

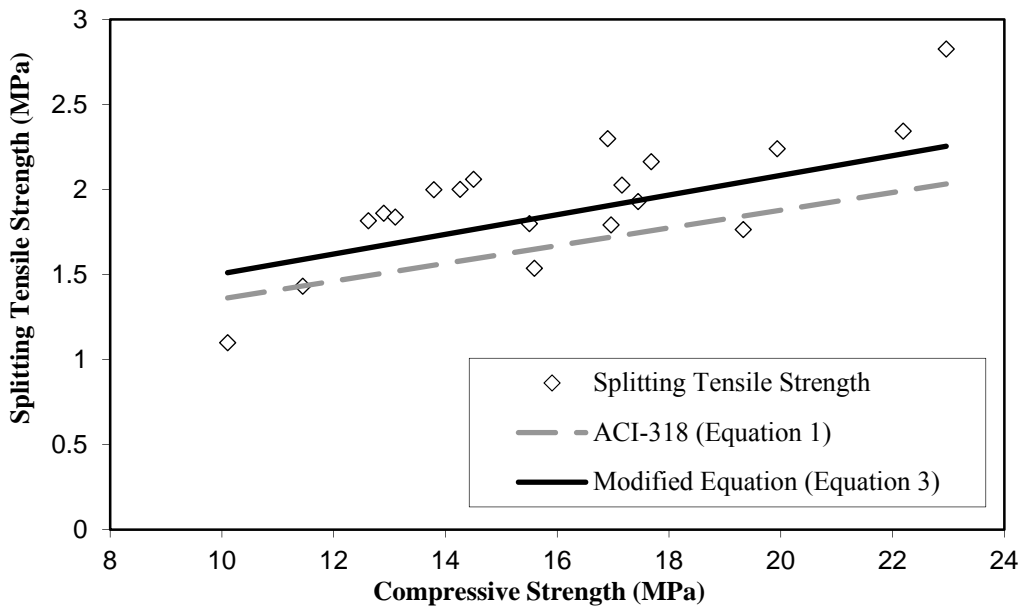


Figure (10): Relationship between splitting tensile strength and compressive strength

It should be noted that Figure 7 can be used instead of Figure 8 with no difference in the results, in this

example, as shown in Figure 11.

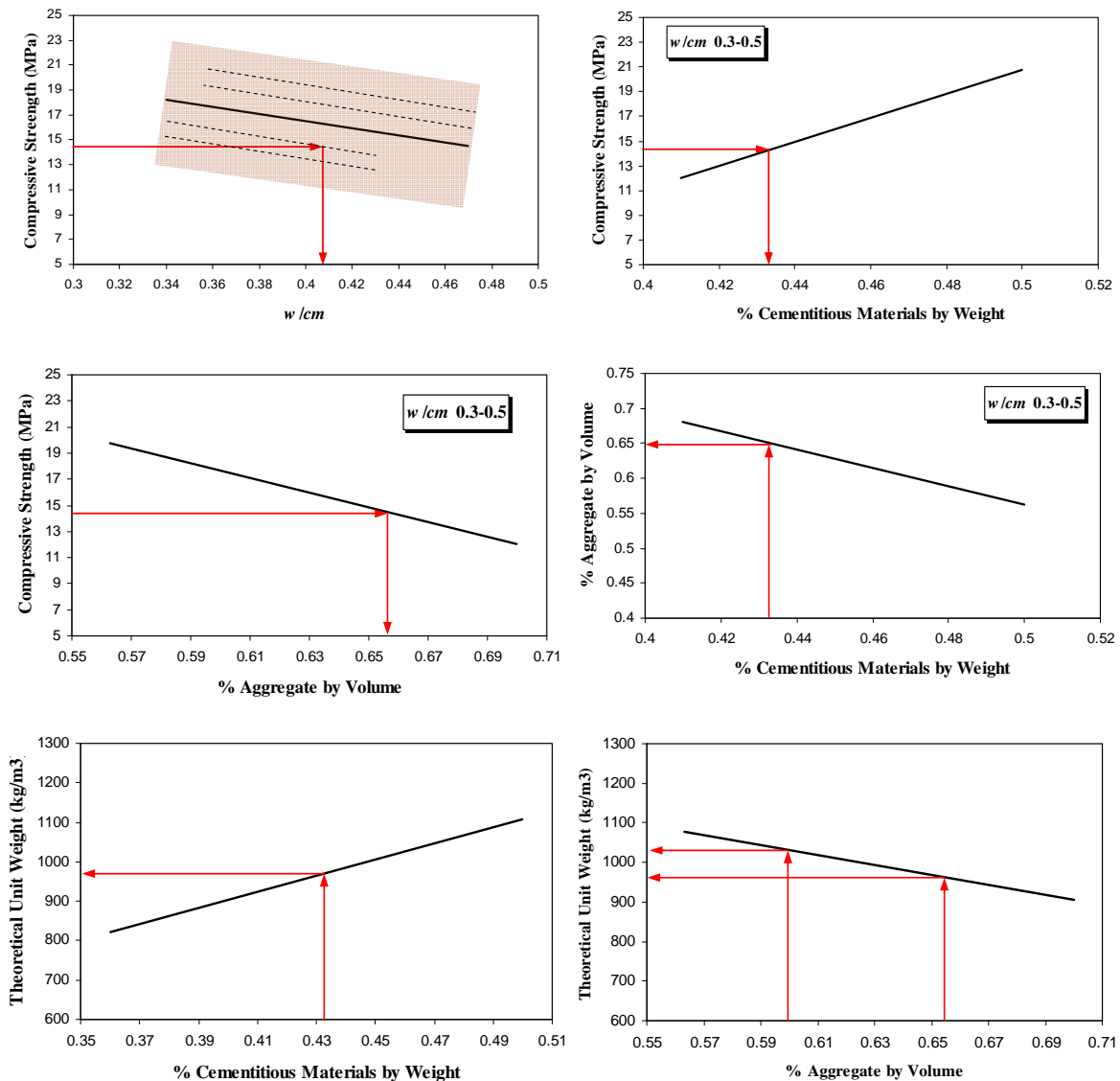


Figure (11): Procedure to determine the required parameters in the worked example

CONCLUSIONS

This research aims at the development of design graphs for fiber-reinforced LWAC. Engineered lightweight aggregate was used. Compressive strength

ranging between 10 MPa and 23 MPa (1470 psi and 3300 psi), unit weight ranging between 800 kg/m³ and 1200 kg/m³ (50 lb/ft³ and 75 lb/ft³) and splitting tensile strength ranging between 1.4 MPa and 2.1 MPa (200 psi and 300 psi) can be predicted by using the proposed

design graphs. The following proportioning parameters: w/cm , % cementitious materials by weight, % lightweight aggregate by volume, % fibers by volume

should be limited to 0.3-0.5, 0.41-0.5%, 0.55-0.7%, 0.1-1.2%, respectively. This new approach has shown an excellent agreement with the experimental results.

Table 5. Theoretical vs. experimental results

Proportions	Model Results	Experimental Results
w/cm	0.41	0.4
% cementitious materials by weight	0.435	0.44
% aggregate by volume	0.65	0.6
theoretical unit weight	980 kg/m ³ (61 lb/ft ³)	1073 kg/m ³ (67 lb/ft ³)
measured unit weight (air content included)	833 kg/m ³ (52 lb/ft ³)	865 kg/m ³ (54 lb/ft ³)
splitting tensile strength	1.7 MPa (240 psi)	1.2 MPa (175 psi)
% fibers by volume	0.5%	0.2%
air content	~15%	20%

SUMMARY

- 1- Fiber-reinforced lightweight concrete can be made out of:
 - ◆ Cementitious materials such as: cement, pozzolans, slag, silica fume, fly ash,... etc.
 - ◆ Engineered lightweight aggregate of different particle sizes and types.
 - ◆ Randomly dispersed chopped fibers.
 - ◆ Chemical admixtures, especially water-reducing and air-entraining agents.
- 2- Fiber-reinforced lightweight concrete, with desired properties, can be made with the aid of design graphs. These properties are:
 - ◆ Compressive strength at the age of 28 days.
 - ◆ Unit weight (density).
 - ◆ Splitting tensile strength.
- 3- The parameters or factors that affect properties of fiber-reinforced lightweight concrete are:
 - ◆ Water/cementitious materials ratio.
 - ◆ Percent cementitious materials by weight.
 - ◆ Percent aggregate by volume.
 - ◆ Air content.
 - ◆ Percent fiber by volume.

APPENDIX A

The data used for analysis, in this paper, were collected from the following design reports:

- [1] California Polytechnic State University, San Luis Obispo (2015), "Jumanji", National Concrete Canoe Competition Design Paper, California Polytechnic State University, San Luis Obispo, CA.
- [2] California Polytechnic State University, San Luis Obispo (2012), "Andromeda", National Concrete Canoe Competition Design Paper, California Polytechnic State University, San Luis Obispo, CA.
- [3] California Polytechnic State University, San Luis Obispo (2013), "Sentinel", National Concrete Canoe Competition Design Paper, California Polytechnic State University, San Luis Obispo, CA.
- [4] McGill University (2014), "Anakalypse", National Concrete Canoe Competition Design Paper, McGill University, Montreal, QC.
- [5] Michigan Technology University (2013), "Mesektet", National Concrete Canoe Competition Design Paper, Michigan Technology University, Houghton, MI.
- [6] University of California, Los Angeles (2015), "Arcturus", National Concrete Canoe Competition Design Paper, University of California, Los Angeles, CA.

- [7] University of Florida (2015), “Foreverglades”, National Concrete Canoe Competition Design Paper, University of Florida, Gainesville, FL.
- [8] University of Nevada, Reno (2014) Canoe, “Alluvium”, National Concrete Canoe Competition Design Paper, University of Nevada, Reno, NV.
- [9] Utah State University (2013), “Canoebis”, National Concrete Canoe Competition Design Paper, Utah State University, Logan, UT.

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APPENDIX B

The data used for verification, in this paper, were collected from the following design report:

- [1] California Polytechnic State University, Pomona (2014), “Gidget”, National Concrete Canoe Competition Design Paper, California Polytechnic State University, Pomona, CA.

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