Jordan Journal of Civil Engineering, Volume **11**, No. **3**, **2017**

Prophecy of Plate Load Test Response from Theory of Elasticity Solution and CBR Test

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ABSTRACT

The present study deals with prophecy of plate load test response from theory of elasticity solution as well as California Bearing Ratio (CBR) test. The response of elastic half space in CBR mould model in FEM was generated and the relationship between modulus of elasticity (E) and CBR was derived. In addition, the correlation between E and CBR was proposed and using E values, coefficient of subgrade reaction (k_s) predictions were discussed. Thus, CBR test is expected to simplify the effort in determination of k_s which is used in pavement design.

KEYWORDS: Modulus of elasticity (E), California bearing ratio (CBR), Coefficient of subgrade reaction (k_s), Plate load test (PLT), 2D –plaxis.

INTRODUCTION

Soil subgrade is a stratum of natural formation which can bear wheel loads transferring from vehicles as well as from pavement materials. These loads can be ultimately received by the soil subgrade for lateral distribution onto earth mass. The main functions of subgrade pavement are principally based on parameters, such as: load bearing capacity, moisture content and swell (or) shrink behavior of the soil. These basic parameters are typically characterized by resistance to deformation under wheel load actions, which can be either a measure of strength or a measure of stiffness. Therefore, it is very significant that at least the top 50 cm thickness of subgrade soil layer should be firmly compacted under controlled conditions. When a material is said to be elastic, it is able to return to its original shape or size immediately after being stretched or squeezed by external forces or moments. Almost all materials are elastic in nature up to some degree as long as the applied load does not cause permanent deformation. Thus, the flexibility of any object or structure depends on its modulus of elasticity (E) as well as on its geometric shape. It depends on many factors, such as: loading process, soil particle organization and water content. These factors, among others, are involved in the determination of the value of E of the soil (Briaud, 2000).

Elastic half space is an identical, mutually independent, discrete and linearly elastic structure with springs. The simplified assumptions on which this approach is based are called approximations (Winkler, 1867). In fact, in this model, the subsoil is replaced by fictitious springs, the stiffness of which is equal to the coefficient of subgrade reaction (k_s). It is to be acquainted with the behavior of elastic half space, which

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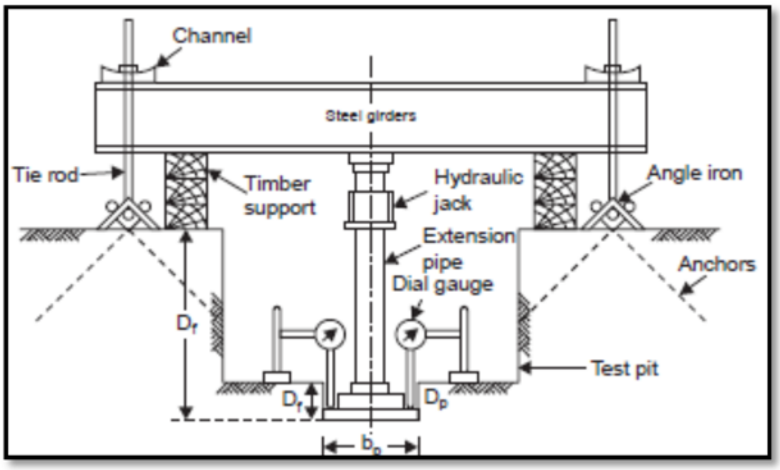
Received on 26/1/2015.
Accepted for Publication on 22/2/2015.

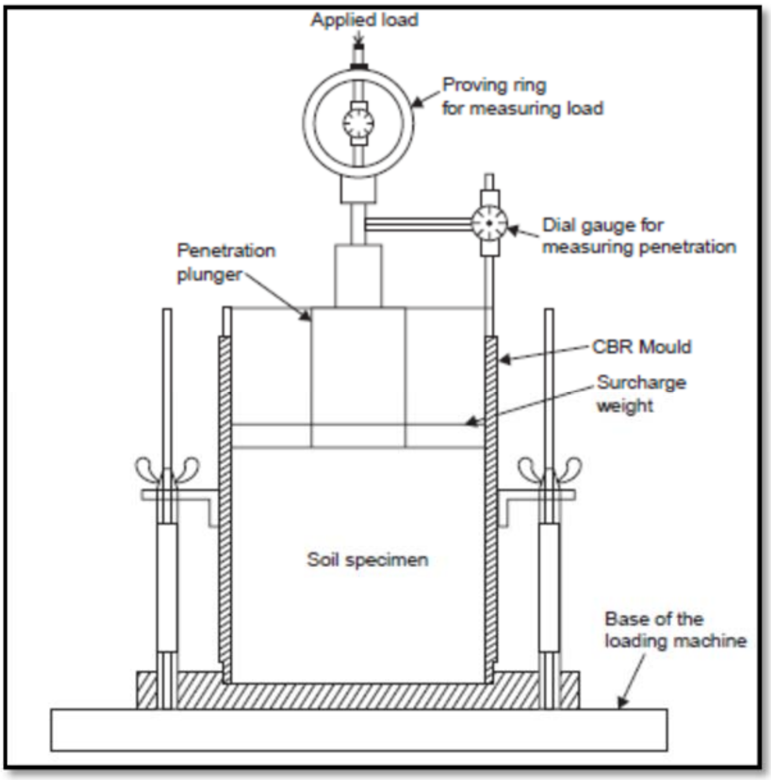
can be obtained from field (*in situ*) plate load test. This behavior is commonly used to predict deformations, failure characteristics of the subgrade and k_s, which is used in foundation design, soil-structure interaction studies and design of highway pavements (Putri, 2012). However, one of its basic limitations lies in that this model can't transmit any shear stresses, which are derived from lack of spring couplings with the assumption of linear stress-strain behavior. In general, k_s is identified from the characteristics of foundation support and has a dimension of force per length cubed. In the near past, most of the researchers have attempted to elaborate the Winkler's model in a more realistic manner by assuming interactions among the spring elements that represent soil continuum (Filonenko-Borodich, 1940; Heteneyi, 1950; Pasternak, 1954). The effective factors and determination approaches of k_s were investigated by different researchers (Biot, 1937; Terzaghi, 1955; Vesic, 1961; Horvath, 1983 a, b). It is widely used in structural analysis of foundation members, continuous footings, mats and various types of piling activities.

California Bearing Ratio (CBR) test can be used in the laboratory on a prepared specimen in a mould or an *in situ* subgrade. It is essentially an arbitrary strength test and hence cannot be used to evaluate the shear parameters of soil, such as cohesion, angle of internal friction and shearing resistance. Unless the test procedure is strictly followed, dependable results cannot be obtained and the presence of coarse grained particles would cause poor reproducibility of results. Moisture conditions in which a material is to be used vary according to the climatic conditions of a region. Thus, CBR test is used to simulate the worst likely conditions in service (Emery, 1987). A major criticism of CBR is that its poor reproducibility results from the test procedure adopted worldwide. The moisture content of a sample may show significant variance when remolding, thereby hampering reproducibility (Kleyn, 1955). From the aforementioned review of literature, an attempt has been made to investigate the response of plate load test from theory of elastic solution and laboratory CBR test results.

It is well known that when a load is imposed on a soil specimen, settlements will transpire. The main purpose of conducting CBR test is to determine the load – penetration response of soil under confined circumstances and hence, this response can be assumed as close as to the plate load test. The k_s values can now be obtained from laboratory CBR test results. The correlations of CBR *versus* E and CBR *versus* k_s are developed to fulfill the breach, which is mainly needed to integrate engineering behavior of soil subgrade. Fig. 1 (a and b) shows the schematic view of plate load test and laboratory CBR test set-up.



(a) Plate load test (PLT)



(b) Laboratory CBR test set-up

Figure 1 (a and b): Schematic views of PLT and CBR test set-up (Courtesy: C. Venkatramaiah)

METHODOLOGY

The soil used in the study was obtained from Sanga Reddy in Medak District, Hyderabad, India. On this soil, basic tests were conducted in the laboratory for its characterization. When the basic properties of soil are concerned, it is in red color and has no gravel. The IS classification of soil is clayey sand. CBR test was conducted in the laboratory as per IS: 2720 (Part 16). The procedure for conducting the test is explained later. The same mould is modeled by assuming rigid footing concept in 2D-PLAXIS. The properties obtained for clayey sand (SC) oil subgrade, such as: unit weight (γ), cohesion (c), angle of internal friction (ϕ), modulus of elasticity (E) and Poisson's ratio (μ) are input parameters to the model. Poisson's ratio of elastic soil subgrade is assumed in the range from 0.2 to 0.5 for the purpose of variation in analysis. Finite Element Method (FEM) analysis was carried out with a range of values E, chosen from 1.0E+06 to 1.0E+07 (Pa) for establishing the load – settlement response of the soil. From this load – settlement response, the CBR values are derived corresponding to 2.5 mm settlement or penetration of the CBR plunger (50 mm in diameter). FEM analysis was used to evaluate the response of plunger penetrating vertically in an elastic half space. It is observed that the analysis gives a wide range of settlements in the middle of the CBR plunger. In order to confer the laboratory CBR values with the FEM model, analysis results should be corrected to correspond to rigid analysis using the analytical results produced by Tsytovich (Harr, 1966). The center deflection of the circular loaded area of CBR plunger should be multiplied by the influence factor, K = 0.785 as per Tsytovich, in order to get the corresponding value of rigid plunger.

RESULTS AND DISCUSSION

CBR Value

The pressure - penetration relationship curve for soil considered in the study is shown in Fig. 2. It is also comparable with plate load test response due to the application of incremental pressure onto the soil in a rigid mould.

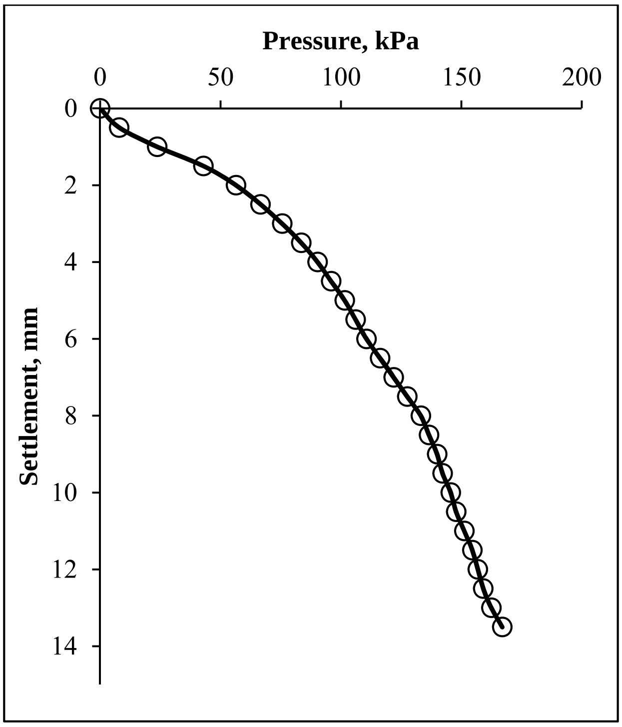


Figure (2): Load – penetration curve for soil subgrade

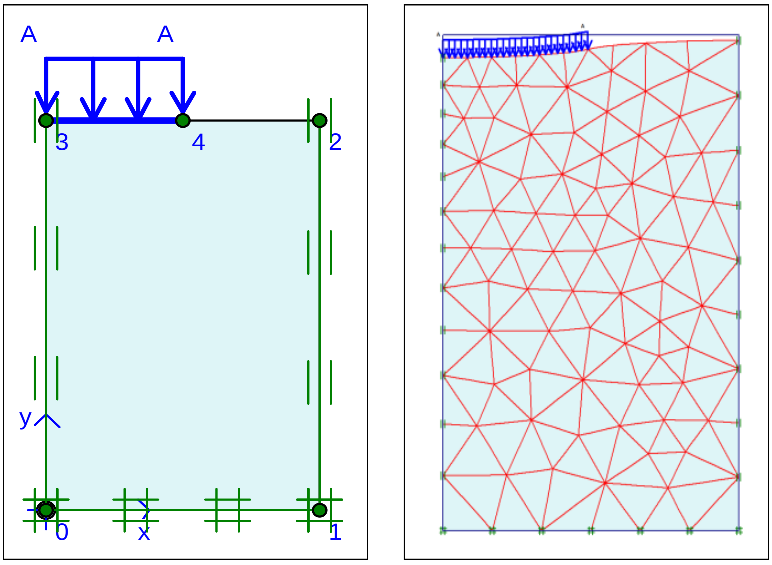
CBR apparatus consists of a mould with a base plate and a collar, a loading frame with a cylindrical plunger, dial gauges for measuring the penetration and other accessories, like spacer disc, rammer and surcharge weights as shown in Fig. 1(a). The soil samples were compacted at Maximum Dry Density (MDD) and Optimum Moisture Content (OMC) in the CBR mould and subjected to loading. Mould containing the specimen, with the base plate in position but the top face exposed, shall be placed on the lower plate of the testing machine. Surcharge weights, sufficient to produce an intensity of loading equal to the weight of base material and pavement shall be placed on the specimen. To prevent upheaval of soil into the hole of the surcharge weights, an annular mass of 2.5 kg shall be placed on the soil surface prior to seating the penetration plunger, after which the remainder of the surcharge weights shall be placed. The plunger shall be seated under a load of 4 kg, so that full contact is established between the surface of the specimen and the plunger. The load and deformation gauges shall then be set to zero. In other words, the initial load applied to the plunger shall be considered zero when determining the load - penetration relation. Load shall be applied to the plunger onto soil at a rate of 1.25 mm per minute. Reading of the load shall be taken at penetrations of 0.5, 1.0, 1.5, 2.0, 2.5, 4.0, 5.0, 7.5, 10 and 12.5 mm. Maximum load and penetration shall be recorded for a penetration of less than 12.5 mm. The plunger shall be raised and the mould detached from the loading equipment. The CBR value can be calculated using Eqn. 1.

CBR (%) = [(Load sustained by specimen at 2.5 or 5 mm penetration)/ (Load sustained by the standard crushed stone at 2.5 or 5 mm penetration)] x 100 (1)

Load sustained by the standard crushed stone at 2.5 mm penetration = 1370 kg and load sustained by the standard crushed stone at 5.0 mm penetration = 2055 kg. In general, higher values are obtained at 2.5 mm penetration and different percents of soil and cement. This higher value is considered as the CBR value of the soil sample. If higher CBR is observed at 5 mm penetration rather than at 2.5 mm penetration, then the CBR test is to be repeated. If even the repeated test faces the same problem, the value corresponding to 5 mm penetration is considered as the CBR value. In the present investigation, the CBR value of subgrade soil as per IS: 2720 – Part (16) is 1.25%.

Results from Finite Element Software - PLAXIS

In FEM, rigid footing condition was chosen to simulate load – settlement response of a rigid CBR mould. The elements used in the study are axisymmetric, 15-node triangular elements along with other volume clusters to model the geometry. FEM allows for a fully automatic mesh generation, in which the geometry is divided into a number of basic and compatible structural elements. The K_o procedure was used in the software for developing the initial stress field for the load system considered in the present investigation. Fig. 3 (a and b) shows CBR geometric model and discrete deformed mesh of the model under applied loads.



a) CBR geometric model **b) Deformed mesh of geometry**

Figure (3): CBR mould model in FEM

The results obtained for the relationships between load and settlement of subgrade soil with a range of varied E (Pa) values are presented in Fig. 4. It can be observed that the settlement is lower for higher values of E (Pa) at constant load. The same corresponding values of load are determined at 2.5 mm deflection for various values of E (Pa). However, correlation can also be developed with CBR value corresponding to 5 mm deflection of the plunger.

Relationship between CBR and Modulus of Elasticity (E)

In real practice, the plate dimensions in plate load test can vary from 300 mm to 760 mm in diameter and the shape can be square, rectangular or circular (Jones, 1997; Moayed and Janbaz, 2009). Terzaghi (1955) developed an empirical relationship between CBR and E based on plate load testing of different subgrade soils using a circular plate, 760 mm in diameter with a thickness of 16 mm. Similarly, for fine grained non-expansive soil with a soaked CBR < 1 (AASHTO, 1993; Heukelon and Klomp, 1962) and for CBR less than 5 and above 5 (NAASRA, 1950), the empirical relations were developed.

For the present investigation, the relationship between CBR and E is furnished in Fig. 5. From this figure, it can be seen that the variation is linear irrespective of the value of the coefficient. Similarly, Eqns. 2 and 3 show the relationships between E and CBR for a Poisson's ratio of 0.3 and 0.4, respectively.

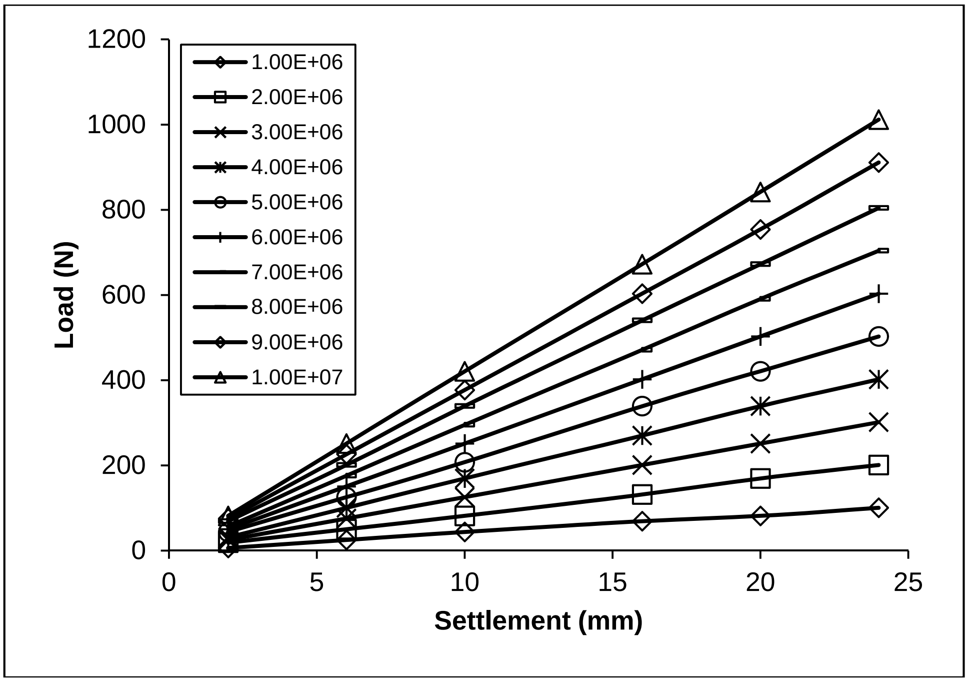


Figure (4): Load – settlement response of subgrade using FEM

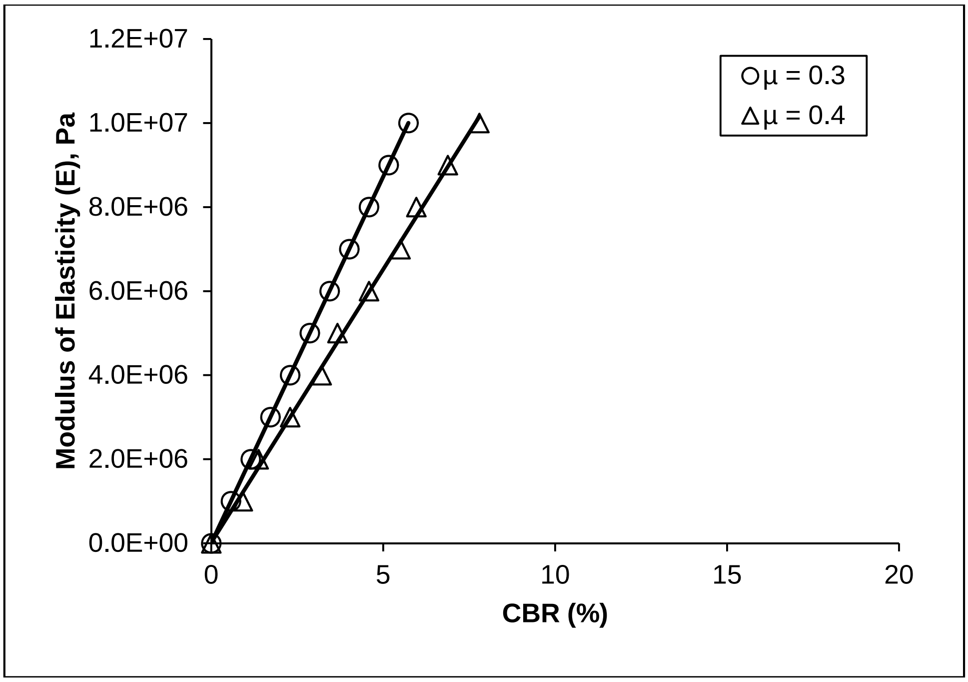


Figure (5): CBR *versus* E from FEM

$$\mathrm{E} = 2000\ \mathrm{CBR}\ (\mathrm{kPa}),\ \mu = 0.3 \tag{2}$$

$$\mathrm{E} = 1000\ \mathrm{CBR}\ (\mathrm{kPa}),\ \mu = 0.4 \tag{3}$$

Linear interpolation can be used to obtain the relationship for intermediate values of Poisson's ratio. In FEM model, confined boundary condition of CBR mould has been taken into consideration and hence E (Pa) values can be used for developing empirical relations. Corresponding to the assumed value of E (Pa), load at 2.5 mm settlement is obtained from Fig. 4. For instance, the CBR value of soil tested in laboratory is equal to 5 %, thus the value of E will be 10.000 kPa for a Poisson's ratio equal to 0.3. Similarly, for a Poisson's ratio of 0.4, the value of E will be 5000 kPa.

Prediction of Plate Load Test (PLT) Response from CBR Test Results

Plate load test (PLT) results can be used to obtain the necessary information about the soil with particular reference to design of foundation. But, test results can only reflect the behavior of soil within a depth of less than twice the width of the bearing plate. Since foundations are generally larger than test plates, settlement and shear resistance will depend on the properties of a much thicker stratum. For clayey and silty soils and for loose to medium dense sandy soils with standard penetration resistance N < 15, a 450 mm square plate or concrete blocks shall be used. In case of dense sandy or gravelly soils (15 < N < 30), three plates of sizes ranging from 300 mm to 750 mm shall be used depending upon practical considerations of reaction loading and maximum grain size. The side of the plate shall be at least four times the maximum size of soil particles present at the test location (IS: 1888 – 1997). Similarly, CBR test is used for evaluating subgrade strengths for design of flexible pavements. The ratio is used in conjunction with curves evolved through a study of the performance of flexible pavements. The CBR value of a soil can thus be considered to be an index, in which some fashion is related to its strength. The value is highly dependent on the condition of materials at rest.

Coefficient of Subgrade Reaction, k_s

The coefficient of subgrade reaction (k_s) can also be referred to as coefficient of elastic uniform compression (C_u). It is one of the important parameters obtained from plate load test and used as a primary input key parameter for pavement design models. A relationship between soil pressure and deflection is proportional as idealized in Winkler's soil model (Hetenyi, 1946; Jones, 1997). In other words, the ratio of uniform pressure imposed on the soil to the elastic part of the settlement (Kameswara Rao, 2000) is constant. Since the stress-strain curves for soils are generally non-linear, the slope of the initial straight portion of stress-strain curve can be called tangent modulus, E_i, usually taken as elastic modulus.

According to the basic theory of elastic solution, for a rigid plate on a semi- infinite elastic soil medium subjected to a concentrated load, k_s can be expressed by using the following expression (Timoshenko and Goodier, 1951; Harr, 1966; Kameswara Rao, 2000) as:

$$\mathrm{k_s} = 1.13\ \frac{\mathrm{E}}{(1-\mu^2)}\frac{1}{\sqrt{A}} \tag{4}$$

where, E = Modulus of elasticity.

μ = Poisson's ratio.

A = Area of the plate or CBR plunger.

k_s for clayey sand can be computed using Eqn. 4, presuming that k_s is to be obtained from plate load test, with a plate area and Poisson's ratio of 19.63 cm^2 and 0.4, respectively, and for clayey sand it is 1518.13 kN/m^3. The modulus of elasticity, E of clayey sand at the settlement of 2.5 mm is around 3900 kPa. Using Eqn. 4, the value of k_s for the data above was found to be 1184.14 kN/m^3. The results obtained from laboratory CBR test in Fig. 2 are embedded in Fig. 4 after modifying the abscissa as pressure in kPa and are presented in Fig. 6.

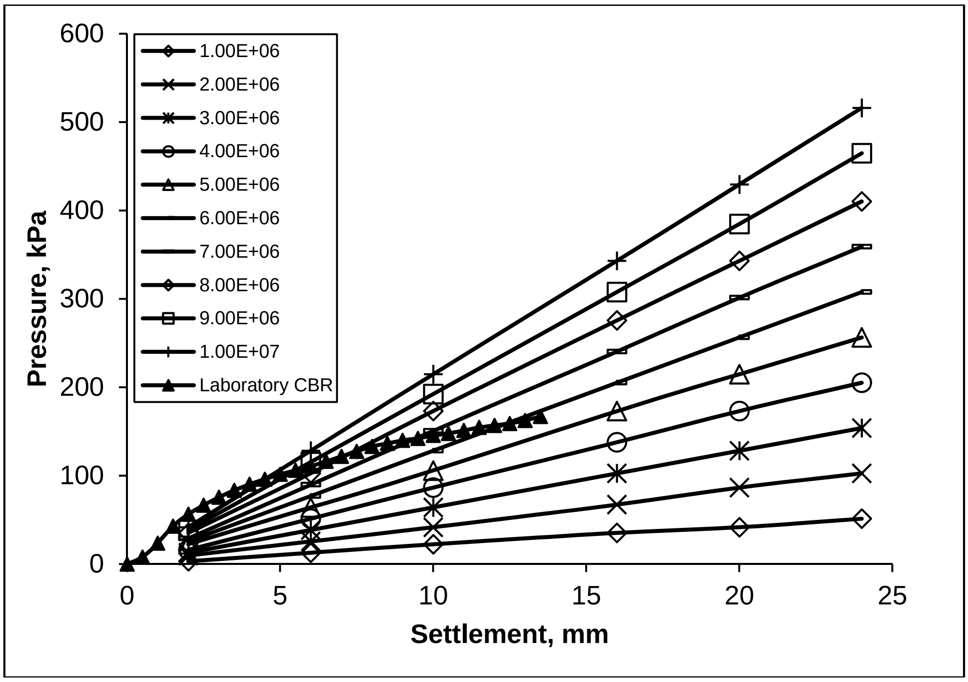


Figure (6): Pressure – settlement relationships from FEM for various E (Pa) values

Prediction of Plate Load Test (PLT) Response

Assuming that a rigid circular load is acting on a semi – infinite elastic soil medium or elastic half space,

$$(k_s)_{CBR} = 1.13\,\frac{E}{(1-\mu^2)}\frac{1}{\sqrt{A_1}} \tag{5}$$

where A_1 is the area of the CBR plunger (as per IS: 2720 (Part -16)).

For plate load test of a circular plate with a diameter of 760 mm,

$$(k_s)_{PLT} = 1.13\,\frac{E}{(1-\mu^2)}\frac{1}{\sqrt{A_2}} \tag{6}$$

From Eqns. 5 and 6, the ratio of laboratory CBR and PLT deflection can be expressed and is shown in Eqn. 7.

$$\frac{\left(\frac{p}{\Delta}\right)_{CBR}}{\left(\frac{p}{\Delta}\right)_{PLT}} = \frac{(k_s)_1}{(k_s)_2} = \sqrt{\frac{A_2}{A_1}}\;;\; \frac{(\Delta)_{PLT}}{(\Delta)_{CBR}} = \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{\pi r_2^2}{\pi r_1^2}} = \frac{r_2}{r_1} \tag{7}$$

(for a circular plate),

where A_1, Δ_{CBR} and r_1 are respectively the area of plunger penetrating into the soil, penetration or deflection and radius of the CBR plunger.

A_2, r_2 and Δ_{PLT} are respectively the area, radius and deflection of the plate.

By equating pressures, $p_{CBR} = p_{PLT}$, Eqn. 7 can be further expressed as:

$$(k_s)_{CBR} = 1.13\,\frac{E}{(1-\mu^2)}\frac{1}{\sqrt{A_1}} = \frac{(p)_{CBR}}{(\Delta)_{CBR}} \tag{8}$$

Eqn. 8 represents the correlation between PLT and laboratory CBR plunger. From CBR test results in Fig. 2, for a particular value of p, the corresponding value of $(\Delta)_{CBR}$ can be obtained. The value of pressure (p) and applied load (P) of the soil can be taken from Fig. 6.

$$\frac{(\Delta)_{PLT}}{(\Delta)_{CBR}} = \sqrt{\frac{A_2}{A_1}}$$

$$(\Delta)_{PLT} = \sqrt{\frac{\pi r_2^2}{\pi r_1^2}} \times (\Delta)_{CBR} \tag{9}$$

It can be noted that the predicted PLT can be obtained by Eqn. 9. Similarly, inverting the steps above in sequence, CBR values can also be predicted.

Fig. 7 shows the predicted results of pressure – settlement for PLT achieved from laboratory CBR test (presented in Fig. 2) for Poisson's ratio values of 0.3 and 0.4. These results are following the trend of pressure – settlement response with a linear relationship. The soil modeled in the CBR mould using FEM and the results such as load-penetration response can be used to ascertain the field behavior of soil from the bearing pressure evaluation point of view.

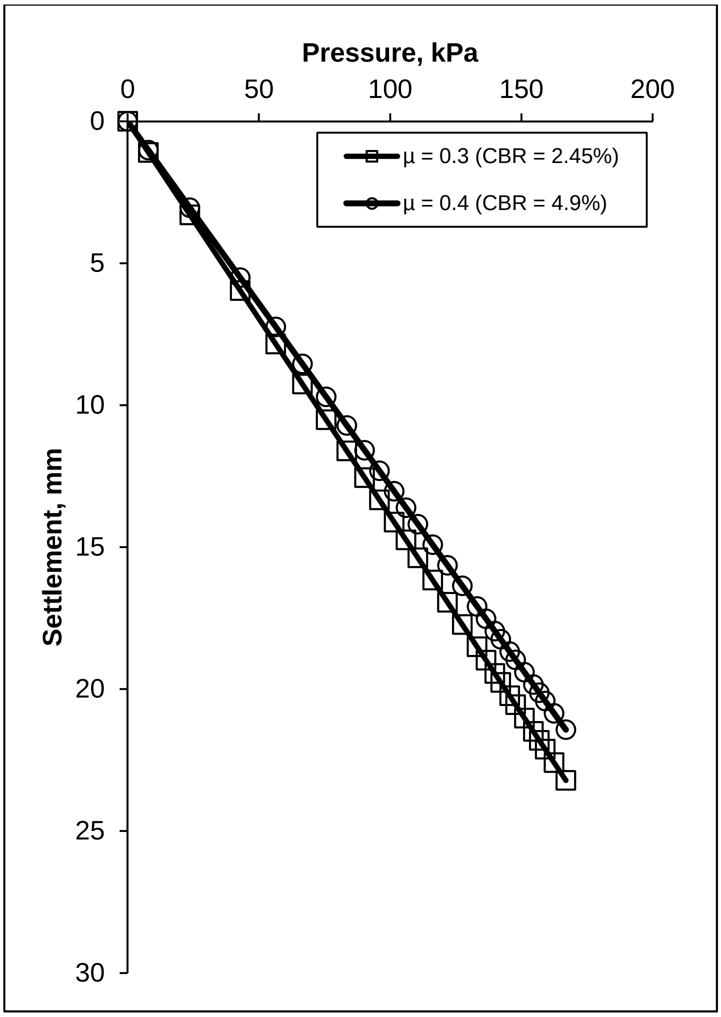


Figure (7): Predicted PLT response for a circular plate

CONCLUSIONS

The coefficient of subgrade reaction, k_s is commonly used in the design of road pavements as well as in machine foundation analysis. It is required for the design of beams on elastic foundations and for the design of flexible rafts. In general, conducting *in situ* plate load test and evaluating k_s consume a lot of time and cost. By using the universally accepted Winkler's spring methodology, it can be easy to model the behavior of the soil. Most of the recent structural design programs are developed based on finite element methods (FEM), which commonly use the Winkler's model. The response predicted is following the pressure-settlement response. The load-settlement response observed from the FEM can now be used to predict the load-settlement response of PLT for the assigned values of E, μ and deflection or pressure.

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