Reliability-Based Determination of the Coefficients of Lateral Earth Pressure on Retaining Walls Subjected to Seismic Loading

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ABSTRACT

The active and passive lateral earth pressures against retaining walls under seismic loading are investigated, taking into consideration the variations of internal friction angle ϕ and seismic effect represented by the seismic angle θ. This study basically involves the computation of the active and passive lateral earth pressure coefficients, k_{ae} and k_{pe}, and their coefficients of variation, $\text{cov}(k_{ae})$ and $\text{cov}(k_{pe})$, using a first order Taylor's series expansion. The concept of extreme value distributions (maxima and minima) is utilized for evaluating the reliability of these earth pressure coefficients. Design charts are developed for a wide range of granular soils. Examples illustrating the design method based on the usage of the reliability of these coefficients are also presented. Values of load factor utilized in the conventional deterministic approach are correlated with reliability levels; the load factor values vary depending on the importance of the structure and the variations of the design input values as contrasted to the constant value of load factor used in the conventional deterministic approach.

KEYWORDS: Retaining walls, Active and passive lateral earth pressures, Seismic coefficient, Reliability, Probability, Granular soils.

INTRODUCTION

The most important component in the design of rigid retaining walls, which are widely used in engineering practice, is the determination of the coefficients of active and passive lateral earth pressures. There are several types of rigid retaining walls in common use; namely, cantilever walls, counterfort or buttressed walls and gravity walls. The magnitude of the earth pressure exerted by the soil on the retaining wall mainly depends on the properties of the soil, wall friction and conditions of loads and their variations (Bowles 1996). Due to the large variations or uncertainties that may be encountered in soil properties and loading conditions, probabilistic techniques with different reliability levels can be used in addition to the conventional deterministic approach in which the concept of load factor (i.e., factor of safety or strength reduction factors) is utilized. Christian et al. (1994), Tang et al. (1999), Malkawi et al. (2000), Duncan (2000) and Chalermyanont and Bensen (2004) have successfully described examples of using reliability in geotechnical engineering that may involve either stability or settlement problems. Babu and Basha (2008) emphasized that the alternative to the conventional approach of using safety factor is to use probabilistic analysis, where uncertainty in the design parameters is considered in a mathematical framework, because the main advantage of the probabilistic approach is a direct linkage between uncertainty in the design parameters and probability of failure or reliability. However, according to Duncan (2000),

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reliability (or probability of failure) should not be viewed as a replacement for factor of safety, but as a supplement. Reliability analyses provide a logical framework for choosing factors of safety that are appropriate for the degree of uncertainty and consequences of failure. In other words, the factor of safety alone is not a sufficient measure for risk assessment, and it is hard to evaluate how much safer a retaining wall becomes as the factor of safety increases (Whitman, 2000); it is, therefore, suggested to use the factor of safety and reliability together as complementary measures of acceptable design (Duncan, 2000). Al-Zoubi (2010) used the Coulomb theory for lateral earth pressures against rigid retaining walls considering the soil friction angle ϕ and wall friction angle δ and their variations to evaluate the coefficients of active and passive earth pressures and to correlate the load factor utilized in the conventional deterministic approach with different levels of reliability. It should be pointed out that reliability-based designs and probabilistic approaches in civil engineering have been the subject of numerous other studies over the past decades (e.g., Benjamin and Cornell, 1970; Hoeg and Murarka, 1974; Rhomberg and Street, 1981; Chowdhury and Xu, 1995; Mirza, 1996; Low and Tang, 1997; Fenton and Griffiths, 2002; Castillo, 2004; Christian, 2004; Fenton et al., 2005:19; Low, 2005; Zevgolis and Bourdeau, 2010; Beer et al., 2013).

The general purpose of this study is to suggest a design method based on a predetermined reliability (or probability of failure) for selecting the coefficients of active and passive lateral earth pressures and their variations under seismic conditions. The load factors that can be used in the conventional deterministic approach are also obtained for different levels of reliability taking into account the mean and variance of the independent input variables as well as the distribution function of the dependent variable. A first order second moment Taylor's series expansion is used to evaluate the variations of the active and passive lateral earth pressure coefficients, and the concept of extreme value distributions (maxima and minima) is utilized to evaluate the reliability of these coefficients (Haldar and Mahadevan, 2000). The design method based on the reliability of these coefficients is illustrated by two examples. A strong (non-linear) relationship between the load factor utilized in the conventional deterministic approach and the reliability level is established. Hence, the selection of the load factor to be used in the design of retaining walls by the deterministic approach depends on the adopted level of reliability.

Mathematical Framework

The coefficients of the active and passive lateral earth pressures are evaluated using the Mononobe-Okabe dynamic earth pressure approach modified to be similar to the Coulomb equations (Bowles, 1996). Figures 1 and 2 show the forces acting on the soil failure wedge behind a rigid retaining wall with a granular backfill for the active and passive earth pressures, respectively, including seismic forces. The active or passive thrust (P_{ae} or P_{pe}) on a unit length of the wall with granular soil as a backfill can, according to (Bowles, 1996; Das, 2006), be given as follows:

$$\begin{Bmatrix} P_{ae} \\ P_{pe} \end{Bmatrix} = \frac{1}{2}\gamma H^2 (1-k_v) \begin{Bmatrix} k_{ae} \\ k_{pe} \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} k_{ae} \\ k_{pe} \end{Bmatrix} = \frac{\sin^2(\alpha \pm \theta \mp \phi)}{\cos\theta \ \sin^2(\alpha)\sin(\alpha \pm \theta \pm \delta)\left(1 \pm \sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi \mp \beta - \theta)}{\sin(\alpha \pm \theta \pm \delta)\sin(\alpha - \beta)}}\right)^2} \tag{2}$$

and

$$\theta = \tan^{-1}\left(\frac{k_h}{1-k_v}\right) \tag{3}$$

where k_h is the horizontal earthquake acceleration component divided by the acceleration of gravity, g;

k_v is the vertical earthquake acceleration component divided by the acceleration of gravity g; ϕ is the internal friction angle; θ is the seismic angle; δ is the friction angle between the wall and the soil; β is the angle of inclination of the backfill behind the wall; and finally α is the angle of inclination of the back face of the wall.

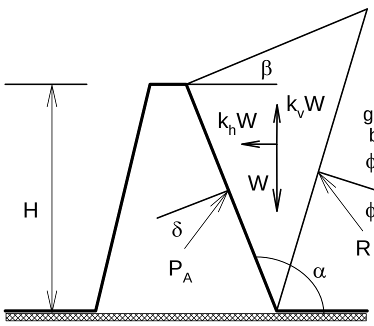

Figure (1): Forces acting on a rigid retaining wall in the active case (wall moves away from soil)

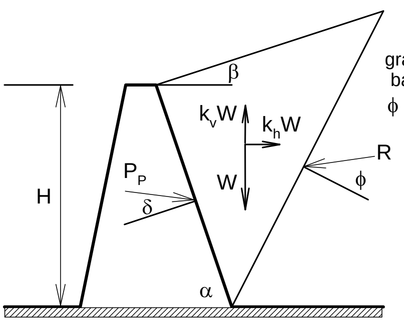

Figure (2): Forces acting on a rigid retaining wall in the passive case (wall moves towards soil)

For simplicity, the study presented herein has been performed for $\alpha = 90^o$ and $\beta = 0^o$. Therefore, the coefficient of active earth pressure can be given as follows:

$$k_{ae} = \frac{\cos^2(\phi-\theta)}{\cos\theta\ \cos(\theta+\delta)\left(1+\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-\theta)}{\cos(\theta+\delta)}}\right)^2} . \quad (4)$$

Similarly, the coefficient of passive earth pressure may be given as follows:

$$k_{pe} = \frac{\cos^2(\phi-\theta)}{\cos\theta\ \cos(\theta+\delta)\left(1-\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-\theta)}{\cos(\theta+\delta)}}\right)^2} . \quad (5)$$

The variations of the coefficients of the active and passive lateral earth pressures with the soil friction angle ϕ and seismic angle θ are demonstrated in Figs. 3 and 4. Figure 3(a) shows that the k_{ae} value decreases with the friction angle ϕ for a given seismic angle θ; whereas Fig. 3(b) shows that the k_{ae} value increases with the seismic angle θ for a given friction angle ϕ. Figure 4(a) shows that the k_{pe} value increases drastically with the friction angle ϕ for a given seismic angle θ; whereas Fig. 4(b) shows that the k_{pe} value decreases almost linearly with the seismic angle θ for a given soil friction angle ϕ.

Design Equations and Charts for Evaluating the Variations of k_{ae} and k_{pe}

The variations of the active or passive lateral earth pressure coefficients, k_{ae} or k_{pe}, (the dependent variables) are evaluated by using a first order second moment Taylor's series expansion about the mean (Haldar and Mahadevan, 2000) along with the variations of the design input values of friction angle ϕ and seismic angle θ (the independent variables). An explanation of the technique can be illustrated as follows:

Consider the following equation for the dependent variable Y as a function of the independent variables x_i:

$$Y = f(x_1, x_2, x_3, ..., x_n) \quad (6)$$

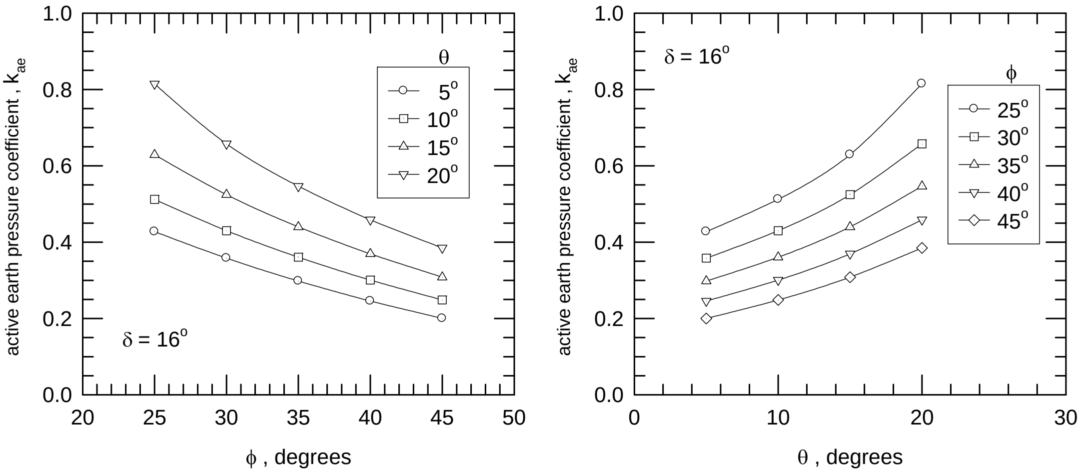

Figure (3): **(a) Variations of k_{ae} with soil friction angle ϕ for various θ-values (b) Variations of k_{ae} with seismic angle θ for various ϕ-values.**

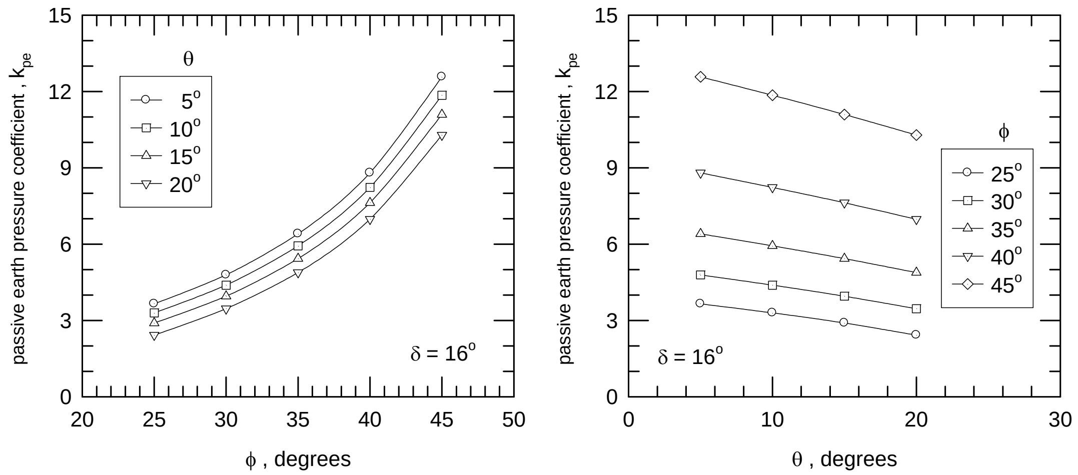

Figure (4): **(a) Variations of k_{pe} with soil friction angle ϕ for various θ-values (b) Variations of k_{pe} with seismic angle θ for various ϕ-values**

The Taylor series expansion about the mean yields the follows expressions:

$$\overline{Y} = f\left(\overline{x}_1, \overline{x}_2, \overline{x}_3, \ldots\ldots.., \overline{x}_n\right) \qquad (7)$$

and

$$\sigma_y^2 = \sum \left(\frac{\partial Y}{\partial x_i}\right)^2 \sigma_{x_i}^2 \, . \qquad (8)$$

Bars are used over the variables to indicate their mean values, where σ^2 is the variance.

This method of estimating the mean and variance of the random variable has proven to be effective (within 10%) for actual values, especially when the independent random variables have relatively small coefficients of variation ($\text{cov} < 30\,\%$) and well behaved functions near the mean (Basma, 1991).

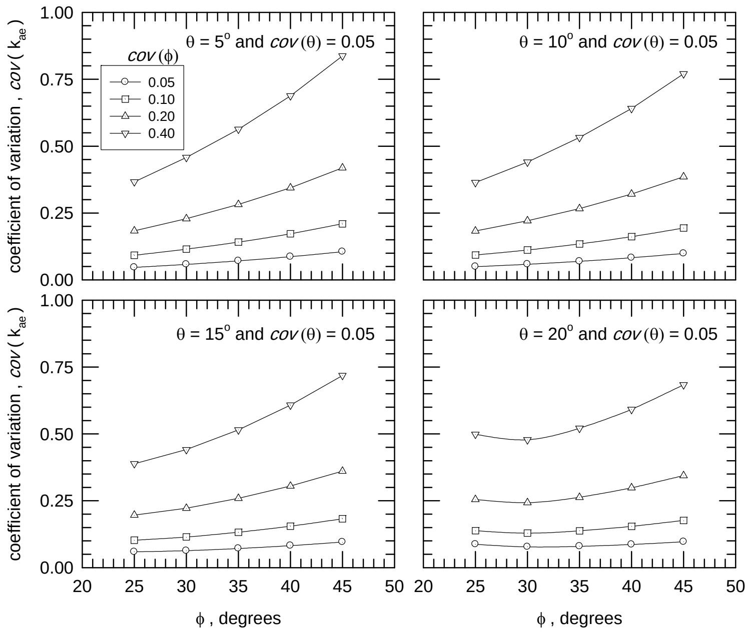

Figure (5): The coefficient of variation of k_{ae} with soil friction angle ϕ for various values of the coefficient of variation of ϕ and seismic angle θ: $\text{cov}(\theta) = 0.05$

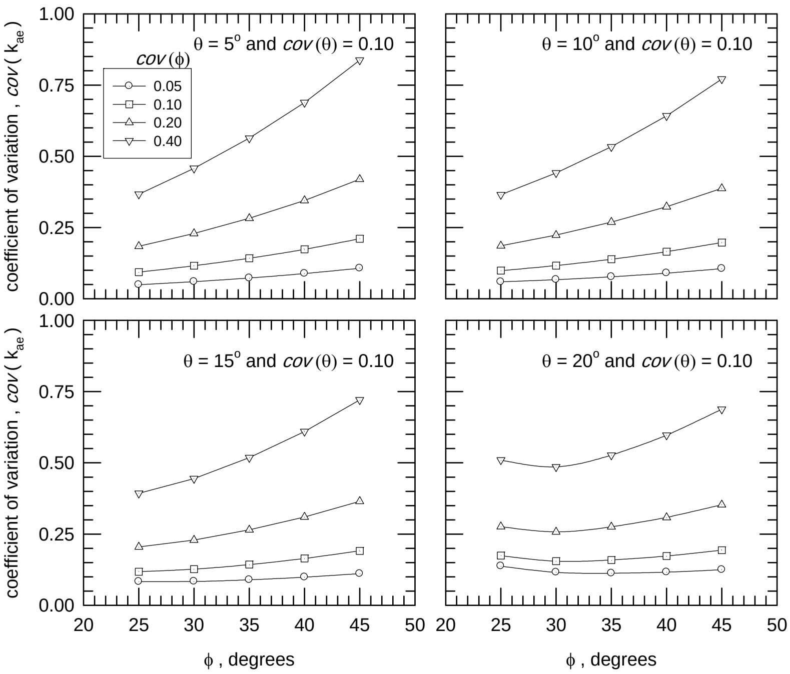

Figure (6): The coefficient of variation of k_{ae} with soil friction angle ϕ for various values of the coefficient of variation of ϕ and seismic angle θ: $\text{cov}(\theta) = 0.10$

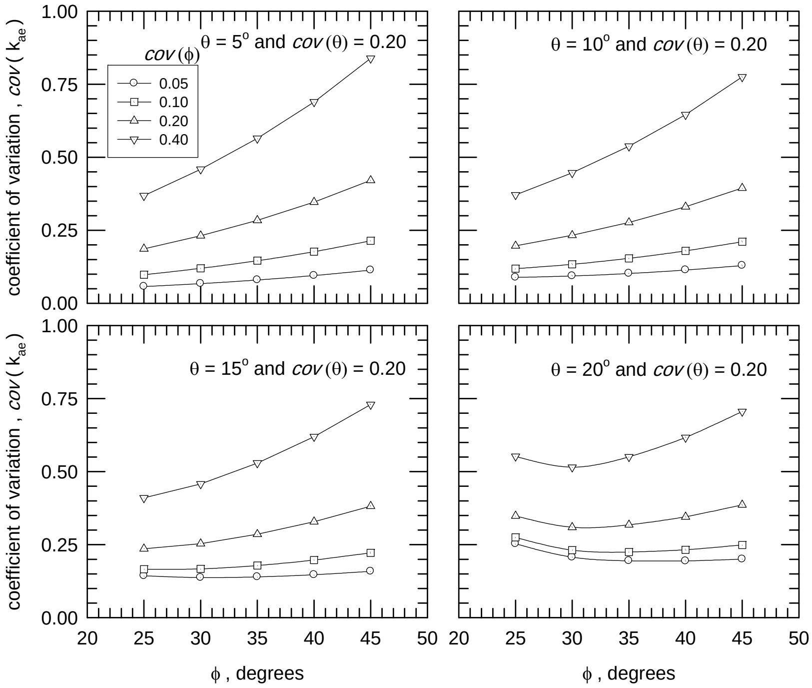

Figure (7): The coefficient of variation of k_{ae} with soil friction angle ϕ for various values of the coefficient of variation of ϕ and seismic angle θ : $\text{cov}(\theta) = 0.20$

Table 1. The coefficients of active and passive lateral earth pressures with seismic loading (k_{ae} and k_{pe})

Theta	**Friction angle , ϕ**							
	25°		**30°**		**35°**		**40°**	
	k_{ae}	k_{pe}	k_{ae}	k_{pe}	k_{ae}	k_{pe}	k_{ae}	k_{pe}
5°	0.4275	3.6540	**0.3580**	**4.7910**	0.2980	6.4044	0.2457	8.8014
10°	0.5121	3.2979	0.4300	4.3881	0.3605	5.9326	0.3005	8.2290
15°	0.6291	2.9010	0.5243	3.9526	0.4400	5.4310	0.3692	7.6267
20°	0.8147	2.4220	0.6575	3.4617	0.5465	4.8822	0.4583	6.9783

The variance of the active (or passive) earth pressure coefficient can be obtained in term of the variations of the mean and variance of ϕ and θ using the following expression:

$$\sigma_k^2 = \left(\frac{\partial k}{\partial \phi}\right)^2 \sigma_\phi^2 + \left(\frac{\partial k}{\partial \theta}\right)^2 \sigma_\theta^2 . \qquad (9)$$

Hence, the coefficient of variation of the active or passive earth pressure coefficient may be obtained by the following expression:

$$\text{cov}(k) = \frac{\sigma_k}{\bar{k}} . \qquad (10)$$

A computer program was developed for evaluating the variance and coefficient of variation of the active and passive lateral earth pressure coefficients using Eqs. 9 and 10 taking into account Eqs. 4 and 5 and their derivatives with respect to ϕ and θ to give design solutions for the conditions mentioned earlier (i.e., $\alpha = 90$, $\beta = 0$, $\delta = 16^o$).

Relatively wide ranges for the internal friction angle of granular soil (25^o to 45^o) and for the seismic angle (5^o to 20^o) were considered. The coefficient of variation $\text{cov}(k)$ for both angles was considered to vary from 5% to 20%.

The results of the computer program for the active and passive earth pressure coefficients for typical ϕ and θ values are listed in Table 1; whereas their coefficients of variation are provided in Table 2 for a specific condition where $\theta = 5^o$ and $\text{cov}(\theta) = 0.10$, while ϕ varies from 25 to 45 degrees and $\text{cov}(\phi)$ varies from 5% to 20%. However, all the results of the coefficients of variation are represented graphically as shown in Figs. 5 to 7 for the case of active earth pressure and in Figs. 8 to 10 for the case of passive earth pressure. These graphs can be used to evaluate the coefficients of variation of the active and passive lateral earth pressure coefficients as will be demonstrated later in this paper.

Table 2. Coefficient of variation of the active earth pressure coefficient $\text{cov}(k_{ae})$ and Coefficient of Variation of the passive earth pressure coefficient $\text{cov}(k_{pe})$

seismic angle = $\theta = 5^o$ Coefficient of Variation of (θ) = 0.10								
$\text{cov}(\phi)$	**Soil friction angle = ϕ ^{o}**							
	25		**30**		**35**		**40**	
	k_{ae}	k_{pe}	k_{ae}	k_{pe}	k_{ae}	k_{pe}	k_{ae}	k_{pe}
0.05	0.0490	0.0666	0.0599	0.0842	0.0726	0.1061	0.0883	0.1342
0.10	0.0932	0.1321	**0.1157**	**0.1678**	0.1421	0.2119	0.1732	0.2682
0.20	0.1841	0.2638	0.2293	0.3353	0.2822	0.4236	0.3447	0.5363
0.40	0.3669	0.5273	0.4576	0.6705	0.5635	0.8471	0.6884	1.0726

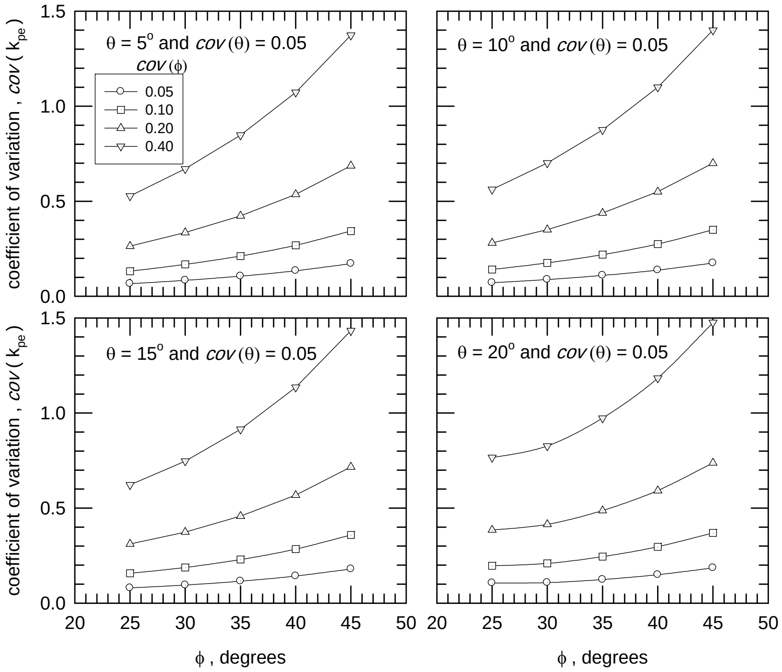

Figure (8): The coefficient of variation of k_{pe} with soil friction angle ϕ for various values of the coefficient of variation of ϕ and seismic angle θ : $\text{cov}(\theta) = 0.05$

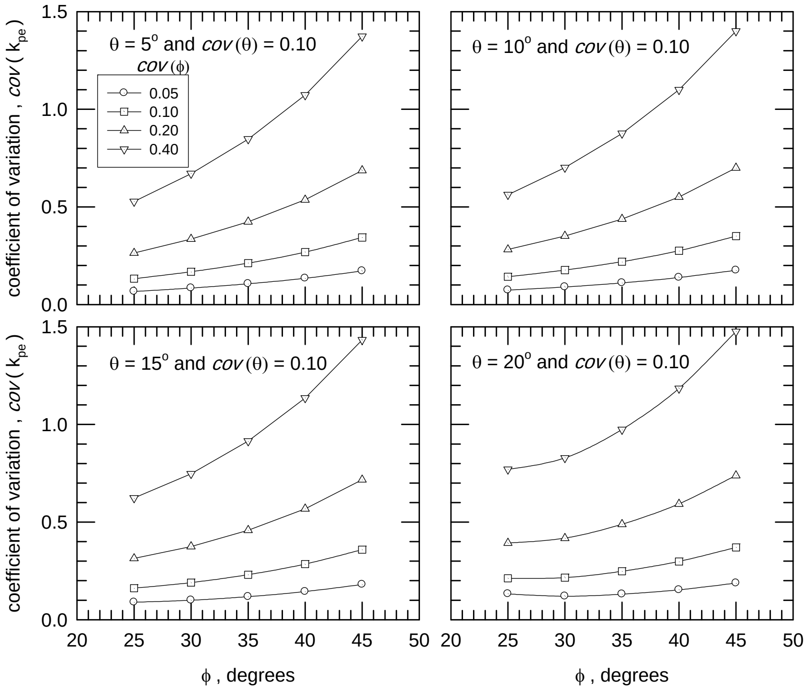

Figure (9): The coefficient of variation of k_{pe} with soil friction angle ϕ for various values of the coefficient of variation of ϕ and seismic angle θ: $\mathrm{cov}(\theta) = 0.10$

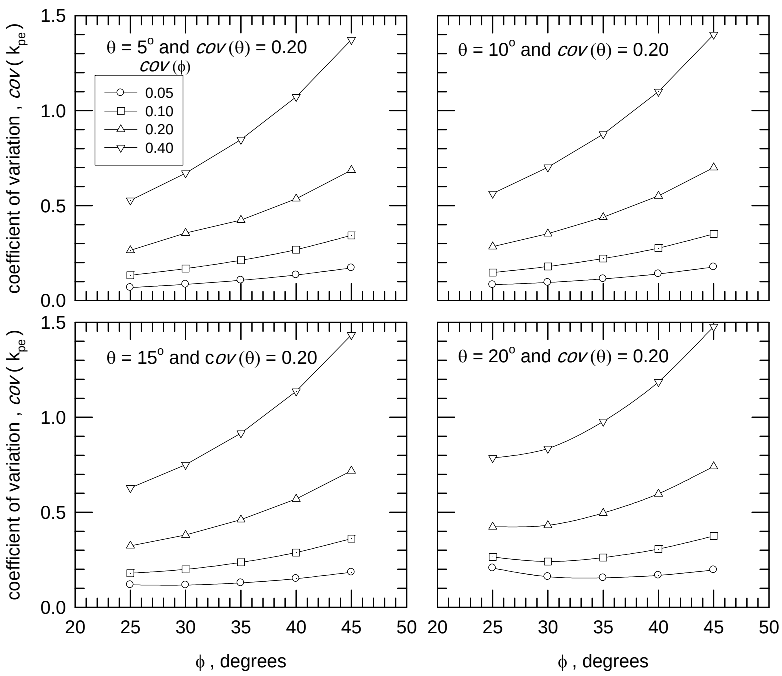

Figure (10): The coefficient of variation of k_{pe} with soil friction angle ϕ for various values of the coefficient of variation of ϕ and seismic angle θ: $\mathrm{cov}(\theta) = 0.20$

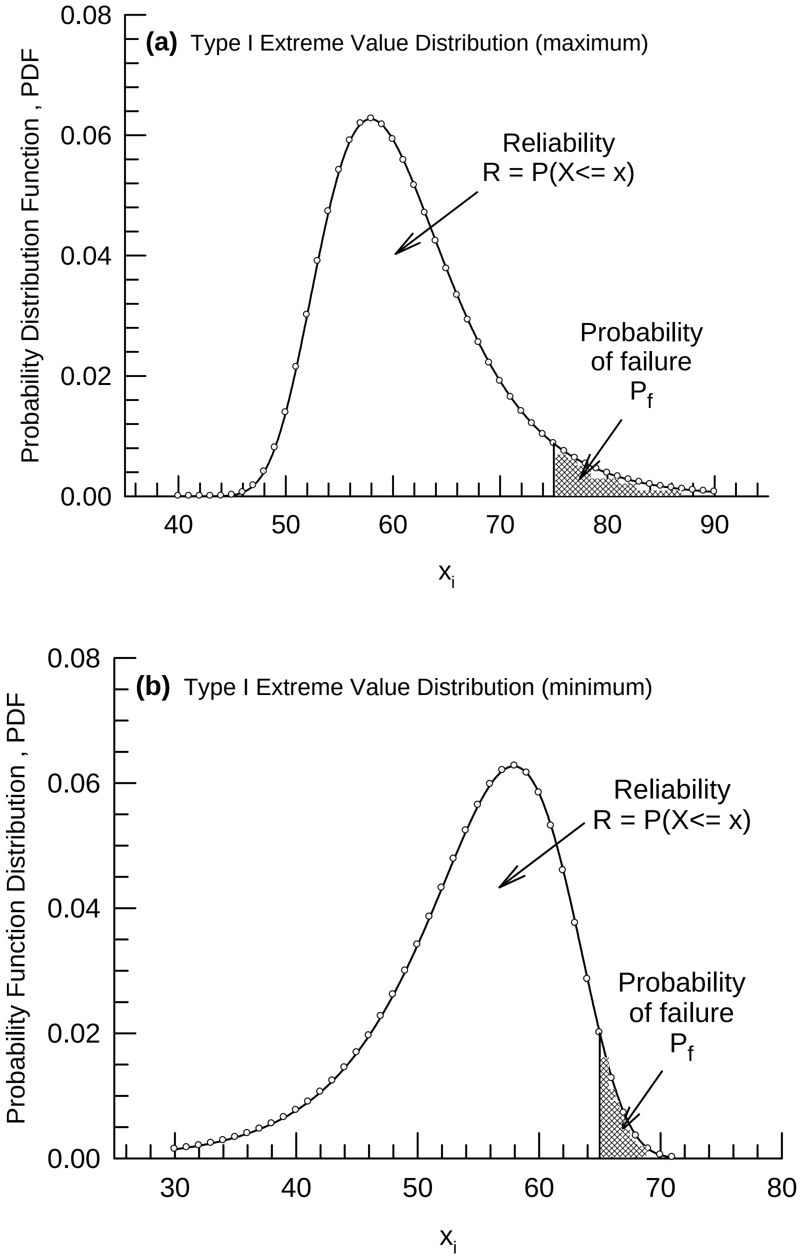

Figure (11): Design reliability with known probability distribution function (PDF).
(a) Type I extreme value distribution (maxima), (b) Type I extreme value distribution (minima)

Probabilistic Analysis and Reliability of Earth Pressure Coefficients

The main goal of any engineering project is to select the most economical and safe design. In the conventional deterministic design approach, a minimum factor of safety is selected arbitrarily based on past experience with structures that failed or those that remained stable under certain loading conditions for adopting design values for the coefficients of lateral earth pressure. If these arbitrarily selected factors of safety are too high, uneconomical designs will be obtained, whereas if these arbitrarily selected factors of safety are too low, unsafe designs will be obtained. Selecting a minimum factor of safety for a structure may depend on the mean values of input parameters (including the properties of the structure and the loading conditions), as well as on their variations that could in many cases be very high. In such cases, probabilistic techniques may provide a valuable design method that takes the mean input values and their variations into considerations in addition to the conventional deterministic approach.

In this study, the following assumptions are made in order to evaluate the design value for the coefficient of lateral earth pressure acting on retaining walls taking into account the seismic effects: (a) the design input values are independent random variables, (b) the active earth pressure coefficient k_{ae} is assumed to have extreme (maxima) type (I) probability distribution function, and (c) the passive earth pressure coefficient k_{pe} is assumed to have extreme (minima) type (I) probability distribution function. The extreme type (I) distribution function has been widely used in many structural designs (Halder and Mahadevan, 2000).

The reliability R of a design can be defined as follows:

$$R = P(X \le x)\,. \tag{11}$$

The reliability R is the probability that the random variable X is less than or equal to a given selected value x. Alternatively, the probability of failure can be defined as follows:

$$P_f = 1 - R = 1 - P(X \le x)\,. \tag{12}$$

If the distribution of the value x is known (i.e., $f_x(x)$ is known), then the reliability R can be determined as shown in Figs. 11(a) and (b).

Since the active earth pressure coefficient (k_{ae}) is assumed to have extreme (maxima) type (I) probability distribution function, then the reliability R_{kae} of a selected value of k_{ae} can be given by the following expression:

$$R_{kae} = F(k_{ae}) = Exp\{-Exp(-\alpha_{kae}[k_{ae} - u_{kae}])\}\,; \tag{13}$$

where

$$\alpha_{kae} = \frac{1.2825}{\sigma_{kae}} \tag{14}$$

$$u_{kae} = \bar{k}_{ae} - \frac{0.5772}{\alpha_{kae}} \tag{15}$$

Similarly, since the passive earth pressure coefficient (k_{pe}) is assumed to have extreme (minima) type (I) probability distribution function, the reliability R_{kpe} of a selected value of k_{pe} may be given as follows:

$$R_{kpe} = F(k_{pe}) = 1 - Exp\{-Exp(\alpha_{kpe}[k_{pe} - u_{kpe}])\}\,; \tag{16}$$

where

$$\alpha_{kpe} = \frac{1.2825}{\sigma_{kpe}} \tag{17}$$

$$u_{kpe} = \bar{k}_{pe} + \frac{0.5772}{\alpha_{kpe}}\,. \tag{18}$$

Normalizing k_{ae} and k_{pe} with respect to the mean and standard deviation and substituting in Eqs. 15 and 16 yield the following:

$$R_{kae} = Exp\{-Exp(-1.282[Z_{kae} + 0.450])\} \tag{19}$$

$$Z_{kae} = \frac{k_{ae} - \bar{k}_{ae}}{\sigma_{kae}} \tag{20}$$

$$R_{kpe} = 1 - Exp\{-Exp(1.282[Z_{kpe} - 0.450])\} \tag{21}$$

$$Z_{kpe} = \frac{k_{pe} - \bar{k}_{pe}}{\sigma_{kpe}}\,. \tag{22}$$

The variation of the reliability R_{kae} is graphically shown in terms of Z_{kae} in Fig. 12(a) and the variation of the reliability R_{kpe} is graphically shown in terms of Z_{kpe} in Fig. 12(b).

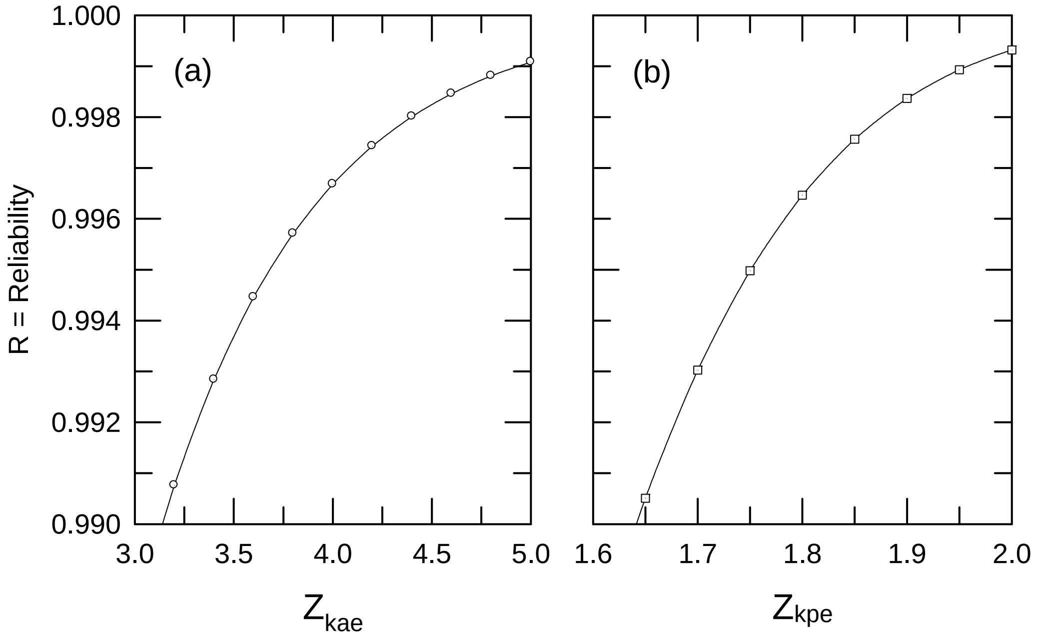

Figure (12): Reliability for active and passive lateral earth pressure coefficients

Table 3. Load factors that can be used in the classical deterministic approach for the active and passive earth pressure coefficients at different levels of reliability

Reliability	Load Factor (LF)	
	active case	passive case
0.99	1.334	1.276
0.995	1.391	1.365
0.999	1.524	1.409
0.9999	1.715	1.455
0.99999	1.906	1.492

Illustrative Examples

The following two examples are presented herein to illustrate the use of the methodology presented in this study for the following conditions: $\alpha = 90^o$, $\beta = 0^o$ and $\delta = 16^o$.

Example (1)

Consider the following properties for the soil and seismic loading: $\phi = 30^o$, $\theta = 5^o$, $\text{cov}(\theta) = \text{cov}(\phi) = 0.10$. The design values for k_{ae} and k_{pe} are 0.60 and 6.0, respectively. What are the reliabilities of these design values? What is the probability of failure for each case?

From Eq. 4 (Table 1), $\overline{k_{ae}} = 0.358$ and Eq. 5 (Table 1), $\overline{k_{pe}} = 4.791$.

From Fig. 6(a), $\text{cov}(k_{ae}) = 0.1157$ and from Fig. 9(a), $\text{cov}(k_{pe}) = 0.1678$ (or Table 2).

Therefore, $\sigma(k_{ae}) = 0.1157 * 0.358 = 0.041$
$\sigma(k_{pe}) = 0.1678 * 4.791 = 0.804$.

If the design values of k_{ae} is 0.60 and k_{pe} is 6.0, then the reliability can be calculated as follows:
$R_{kae} = F(k_{ae})$.

From Eq. 20, $Z_{kae} = \dfrac{0.60 - 0.358}{0.041} = 5.902$, then $R_{kae} = 0.9997$ from Eq. 19 or Fig. 12(a), therefore, $P_f = 0.0003$.

Similarly, for k_{pe} ; $R_{kpe} = F(k_{pe})$

From Eq. 22, $Z_{kpe} = \dfrac{6.0 - 4.791}{0.804} = 1.5037$, then $R_{kpe} = 0.9790$ from Eq. 21 or Fig. 12(b), therefore, $P_f = 0.0210$.

It should be pointed out that the active and passive lateral earth pressure coefficients are considered in this study to be load-related random variables; in this case, design values are assumed larger than their respective mean values. However, the passive lateral earth pressure can also be considered as a resistance-related random variable; in this case the passive coefficient is supposed to be less than its mean value and therefore, Z_{kpe} value is negative.

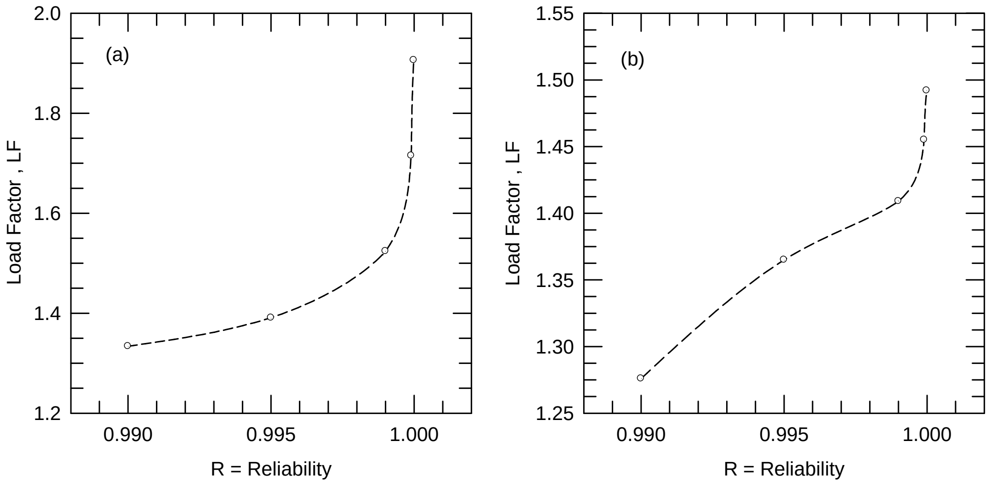

Figure (13): Relationship between the load factor (LF) and reliability (R) for example 2 used in this study (a) for the active case and (b) for the passive case (see Table 3)

Example (2)

Consider the same values for $\phi = 30^o$, $\theta = 5^o$, $\text{cov}(\theta) = \text{cov}(\phi) = 0.10$ as in Example (1). What are the design values of k_{ae} and k_{pe} for a reliability of 0.99?

For R of 0.99, $Z_{kae} = 3.14$ from Eq. 20 (Fig. 12(a)) and $Z_{kpe} = 1.64$ from Eq. 21 (Fig. 12(b)).

From Example (1),
$\sigma(k_{ae}) = 0.041$ and $\sigma(k_{pe}) = 0.804$.

From Eq. 21:

$k_{ae} = \overline{k_{ae}} + Z_{kae}\sigma_{kae}$
$k_{ae} = 0.358 + 3.14 * 0.041 = 0.487$

From Eq. 23:

$k_{pe} = \overline{k_{pe}} + Z_{kpe}\sigma_{kpe}$
$k_{pe} = 4.791 + 1.64 * 0.804 = 6.111$.

Based on these calculations of the lateral earth pressure coefficient, the load factor that can be used in the deterministic approach may be obtained by dividing the design value of the lateral earth pressure coefficient by the mean value as follows (for the active case):

$$LF = \frac{k_{ae}}{\overline{k_{ae}}} = \frac{0.487}{0.358} = 1.360 .$$

Additional values for the load factor LF obtained at different levels of reliability are listed in Table 3. As can be seen from Fig. 13, there is a strong (non-linear) relationship between the load factor and the level of reliability selected for the design values. In other words, the load factor varies depending on the importance of the structure and the variations of the design input values as contrasted to the constant value of load factor used in the conventional deterministic approach.

SUMMARY AND CONCLUSIONS

This study investigates the influence of the mean values of the internal friction angle ϕ and the seismic angle θ and their variations (uncertainties) on the coefficients of the active and passive lateral earth pressure utilizing a first order second moment Taylor's series expansion. The concept of extreme value distributions (maxima and minima) is utilized for evaluating the reliability of these earth pressure coefficients. Based on the results of this study, a non-linear relationship between the load factor utilized in the conventional deterministic approach and the level of reliability is established. The reliability-based design approach for selecting the design values for the active and passive earth pressure coefficients is simplified by using graphical forms for relatively wide ranges of soil friction angle ϕ and seismic angle θ.

This study shows that the mean value of the seismic angle θ and its variation have significant effect on both the active and passive earth pressure coefficients, and hence, on the design values. Therefore, it is recommended to accurately estimate the value of θ based on actual representative data for seismic conditions for the region under consideration.

This study confirms that reliability theory can be applied to geotechnical engineering through simple procedures and need not require more data than is required for conventional deterministic analysis. However, the value of the analysis can be increased considerably by using a relatively small additional effort to perform reliability analysis. Reliability analyses can provide a logical framework for selecting a factor of safety that is appropriate for the degree of uncertainty of the problem being considered. However, it is not advocated in this study that the deterministic approach of using the factor of safety analysis be abandoned in favor of reliability analysis; it is, therefore, suggested that factor of safety and reliability be used together as recommended by Duncan (2000). In other words, the reliability is not a replacement for factor of safety, but is a supplement.

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