Elastic Buckling Solution of SCSC Rectangular Plates under Intermediate and End Uniaxial Loads

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ABSTRACT

This paper is concerned with a new buckling problem of a rectangular plate subjected to combined uniaxial loads. The plate considered has two clamped edges parallel to the load direction, while the other opposite edges are simply supported (SCSC) and subjected to intermediate and end uniaxial loads. The analysis of the plate under study is performed using Rayleigh-Ritz approach and the results are examined using finite element method. The Rayleigh-Ritz approach depends on the assumed deflection function, which satisfies exactly the end conditions and is capable of representing the deflected plate surface. The analysis furnishes an approximate stability solution, presented in curves describing the relation between the ratio of the intermediate load to the end load, the aspect ratio and the location of the intermediate load. These results can be used simply to design plates or walls that have to support intermediate floors/loads. The results of the buckling factor obtained by energy technique and the finite element method are compared also in this paper.

KEYWORDS: Stability, Rectangular plates, Intermediate load, Elastic analysis, Finite element, Rayleigh-Ritz approach.

INTRODUCTION

The stability of thin elastic rectangular plates under uniaxial loads is an important problem in civil engineering applications. Solutions of such plates are documented in all standard texts on plate buckling [for example, Timoshenko and Woinowsky (1987), Szilard (2004) and Timoshenko and Gere (1983)]. Buckling of rectangular plates with different end conditions subjected to various types of loading, such as linearly varying in-plane loads, patch and concentrated loads, has been studied in many papers [for example, Leissa and Kang (2002), Kang and Leissa (2005), Wang et al. (2016) and Miju et al. (2016)].

In some practical applications, plates (or walls) may be subjected to both intermediate load/floor and end loads. Stability of such loaded plates is, however, not available in open literature and has not been studied except for a recent paper by Xiang and Wang (2003) that used the Levy solution approach and the state-space technique to solve such a problem.

Also, Wang et al. (2004) presented some of the important features associated with the buckling of plates under combined uniaxial loads depending on dividing the plate considered into two sub-plates. The exact critical buckling load is the lowest solution of the combinations of each sub-plate solutions. The study by Wang et al. (2004) considered a rectangular plate with two simply supported edges that are parallel to the load direction and different end conditions for the other edges.

Salama (2012) presented an approximate solution for the buckling loads of simply supported plates only in

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case of end and intermediate loads using energy technique.

Buckling solution of rectangular plates under combined uniaxial loads considering the end condition of the edges parallel to the load direction while the other edges are simply supported has not been studied before. So, this paper introduces a buckling solution of SCSC rectangular plate under end and intermediate loads using both Rayleigh-Ritz approach and finite element method.

Model and Assumptions

Consider an isotropic, elastic rectangular thin plate having two simply supported opposite edges perpendicular to the load direction and two clamped edges parallel to the load direction as shown in Figure 1. The plate is subjected to an end uniaxial compressive load N_1 at the left edge (x = 0), beside an intermediate compressive load N_2 at location $x = \beta a$.

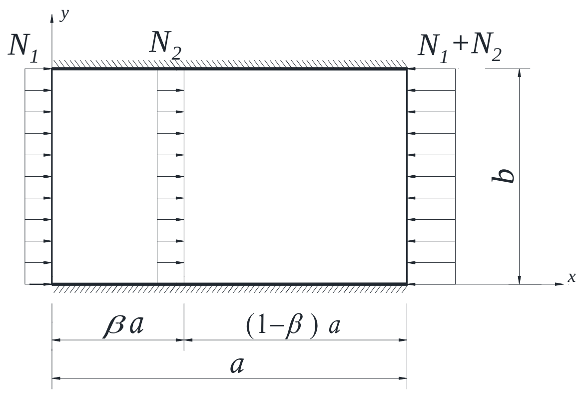


Figure (1): SCSC rectangular plate under intermediate and end uniaxial loads

Method of Analysis and Theoretical Equations

The deflection function of the buckled SCSC rectangular plate can be taken in the form of the following double trigonometric series:

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn} \sin\frac{m\pi x}{a} \sin^2\frac{n\pi y}{b} \qquad (1)$$

where C_{mn} are the unknown constants to be determined.

The general expression in Equation (1) can be simplified by considering the plate buckled into one half-wave in y direction (n=1) and the buckled plate is subdivided along the x-axis into m half-waves. The resulting function can be expressed as:

$$w = \sin^2\frac{\pi y}{b} \sum_{m=1}^{\infty} C_m \sin\frac{m\pi x}{a} \qquad (2)$$

The total strain energy, U, of the plate in bending is:

$$U = \frac{D}{2}\int_0^b\int_0^a \left[\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]\right] dx\,dy \qquad (3)$$

where D is the flexural rigidity of the plate.

Also, the work, T, done by the end uniaxial compressive load N_1 at edge x = 0 and the intermediate compressive load N_2 at location $x = \beta a$ is:

$$T = \frac{(N_1 + N_2)}{2}\int_0^b\int_0^a \left(\frac{\partial w}{\partial x}\right)^2 dx\,dy - \frac{(N_2)}{2}\int_0^b\int_0^{\beta a} \left(\frac{\partial w}{\partial x}\right)^2 dx\,dy \qquad (4)$$

Substituting the deflection expression in Equation (2) into Equations (3 and 4) yields:

$$U = \frac{D\pi^4}{8ab} \sum_{m=1}^{m=\infty} C_m^2 \left(\frac{3m^4 b^2}{4a^2} + \frac{4a^2}{b^2} + 2m^2 \right) \tag{5}$$

and

$$T = \frac{3(N_1 + N_2)\pi^2 b}{32\, a} \sum_{m=1}^{m=\infty} \left[m^2 C_m^2 \right] - \frac{3N_2 \pi^2 b}{32\, a} \left\{ \sum_{m=1}^{m=\infty} \left[m^2 C_m^2 (\beta + S_{2m}) + \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} 2\, i\, j C_i\, C_j\, (S_{j-i} + S_{j+i}) \right] \right\} \tag{6}$$

where $S_k = \dfrac{\sin k\pi\beta}{k\pi}$

Then the total energy (V) of the system is:

$$V = U - T \tag{7}$$

Substituting Equations (5 and 6) into Equation (7) yields:

$$V = \frac{D\pi^4}{8ab} \sum_{m=1}^{m=\infty} C_m^2 \left(\frac{3m^4 b^2}{4a^2} + \frac{4a^2}{b^2} + 2m^2 \right) - \frac{3(N_1 + N_2)\pi^2 b}{32\, a} \sum_{m=1}^{m=\infty} \left[m^2 C_m^2 \right] + \frac{3N_2 \pi^2 b}{32\, a} \left\{ \sum_{m=1}^{m=\infty} \left[m^2 C_m^2 (\beta + S_{2m}) + \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} 2\, i\, j C_i\, C_j\, (S_{j-i} + S_{j+i}) \right] \right\} \tag{8}$$

Using the following expressions to the buckling factors for the end and intermediate loads:

$$A_1 = \frac{N_1\ b^2}{\pi^2 D} \quad \text{and} \quad A_2 = \frac{N_2\ b^2}{\pi^2 D},$$

the total energy (V) can be expressed as:

$$V = \sum_{m=1}^{m=\infty} C_m^2 \left(\frac{3m^4 b^2}{4a^2} + \frac{4a^2}{b^2} + 2m^2 \right) - \frac{3(A_1 + A_2)}{4} \sum_{m=1}^{m=\infty} \left[m^2 C_m^2 \right] + \frac{3A_2}{4} \left\{ \sum_{m=1}^{m=\infty} \left[m^2 C_m^2 (\beta + S_{2m}) + \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} 2\, i\, j C_i\, C_j\, (S_{j-i} + S_{j+i}) \right] \right\} \tag{9}$$

This is a second-degree function with coefficients C_1, C_2, … C_m. These coefficients must be chosen so as to make the total energy (V) a minimum, from which it follows that:

$$\frac{\partial V}{\partial C_1} = 0 \qquad \frac{\partial V}{\partial C_2} = 0 \qquad \cdots \qquad \frac{\partial V}{\partial C_m} = 0 \tag{10}$$

This minimization procedure yields m homogeneous linear equations in C_1, C_2, …, C_m, which can be put in the following form:

$$[K]\ \{C\}\ =\ \{0\} \tag{11}$$

in which $\{C\} = \{C_1, C_2, \ldots, C_m\}^T$ and the (m x m) coefficient matrix [K] will be

$$[K] = [k_{ij}]_{mxm} \tag{12}$$

where

$$K_{ii} = \left(\frac{3m^4 b^2}{4a^2} + \frac{4a^2}{b^2} + 2m^2 \right) - \frac{3}{4} i^2 (A_1 + A_2) + \frac{3}{4} i^2 A_2 (\beta + S_{2i}) \qquad \text{for} \quad i = j,$$

$$K_{ij} = i\, A_2 (S_{j-i} + S_{j+i}) \qquad \text{for} \quad i \neq j, \quad i \prec j$$

$$K_{ij} = j\, A_2 (S_{i-j} + S_{j+i}) \qquad \text{for} \quad i \neq j, \quad j \prec i$$

These equations will be satisfied by putting C_1, C_2,…, C_m equal to zero, which corresponds to the flat form of equilibrium of the plate. For a non-trivial solution, the buckling load of the plate can be obtained by equating to zero the determinant of [*K*]. By selecting the number of trigonometric series (*m*) and assuming the intermediate force N_2, the end compressive force N_1 will be gradually increasing until reaching a value for which one of the coefficients C_1, C_2, …, C_m becomes infinity. The smallest of these values of N_1 is called the end critical value corresponding to the assumed intermediate force N_2. This method of calculating the critical buckling load brings us to a closer and closer approximation as the number (*m*) of the terms of the deflection function series given in Equation (3) increases. By taking (*m*) infinitely large, we obtain an exact solution, where Equation (3) represents the deformed shape exactly.

Also, finite element method is applied to the plate under study. Mode shape for some cases of SCSC rectangular plate under intermediate load N_2 is shown in Figure 3 that describes the buckling behaviour of the plate under study.

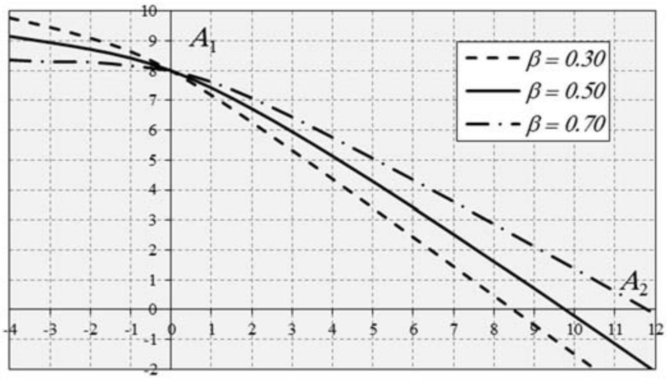
RESULTS AND DISCUSSION

An approximate solution for the elastic stability of SCSC rectangular plate under both intermediate and end loads is presented in Figure (2) (a-c). These results cover various aspect ratios (*a/b* =1, 1.5 and 2) and different intermediate load locations (β = 0.3, 0.5 and 0.7).

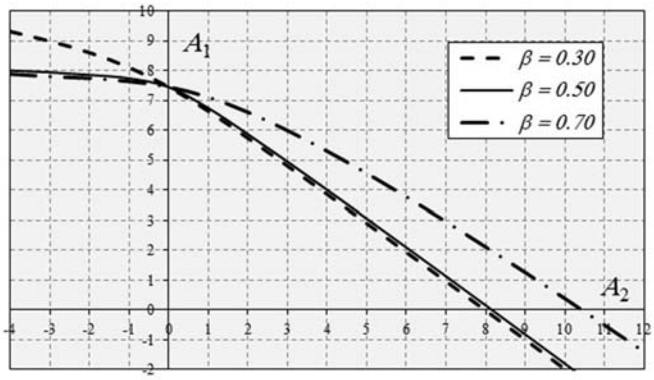
From this figure, it is obvious that when the intermediate in-plane load is positive (N_2>0), the buckling factor A_1 decreases almost linearly as the buckling factor A_2 increases. If the intermediate load is negative (N_2<0), the buckling factor A_1 increases almost linearly as the value of the buckling factor A_2 increases. The increase of A_1 is more pronounced when the intermediate load location factor β is small. It is evident that the stability curves shown in Figure (2) have a highly non-linear portion when the buckling factor A_2 is close to zero.

Also, it can be noticed that when the intermediate in-plane load is absent (N_2=0), the buckling factor A_1 is very close to the exact buckling factor for different aspect ratios.

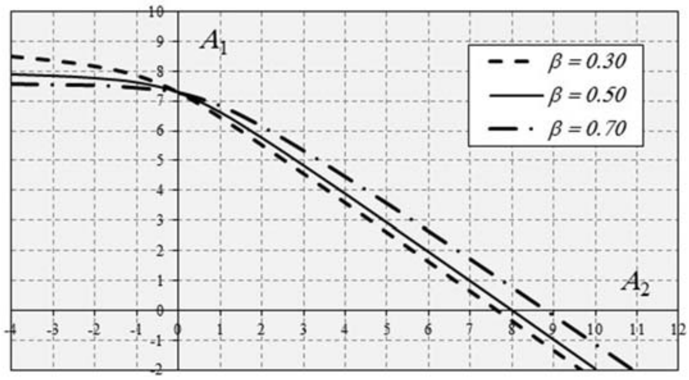
Table 1 shows the percentage of difference between the Rayleigh-Ritz approach and finite element method for the obtained buckling factor A_2 of SCSC rectangular plate subjected to N_2 only. In applying the results obtained for the buckling factor using finite element method, it can be considered very close to the exact solution, especially when using a very large number of elements, but the Rayleigh-Ritz approach gave buckling factors more than the exact solution and the difference reduced with the increase in the number (*m*) of the terms of the deflection function series given in Equation (3). The results of Rayleigh-Ritz approach given in Table 1 are corresponding to (*m* =12).



(a) Rectangular plate [*a/b*=2.0]



(b) Rectangular plate [*a/b*=1.5]



(c) Square plate [*a/b*=1.0]

Figure (2): Stability of SCSC rectangular plate under intermediate load N_2 and end load N_1 with aspect ratio (a) a/b=1.0, (b) a/b=1.5 and (c) a/b=2.0

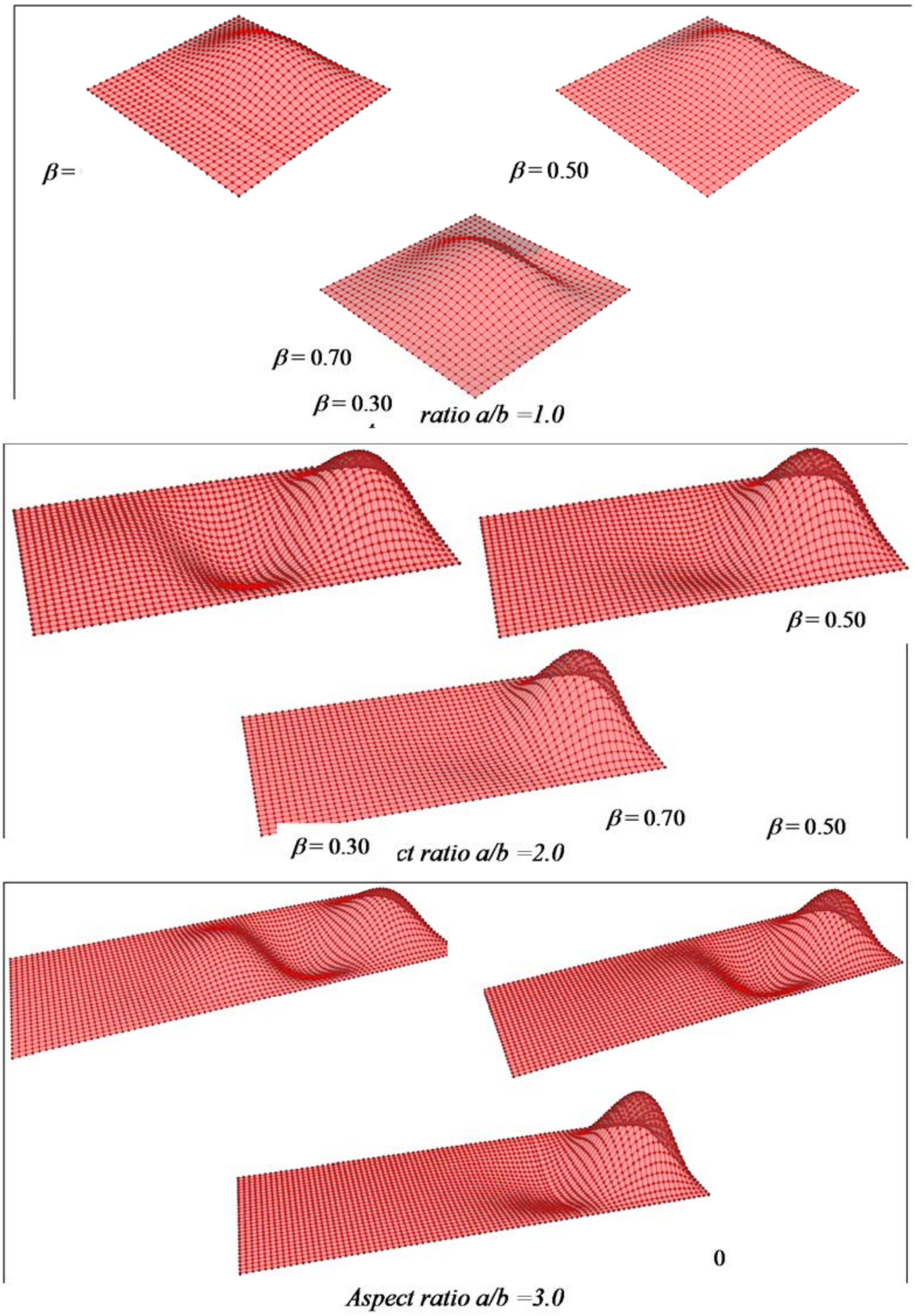


Figure (3): Mode shape for SCSC rectangular plate under intermediate load N_2

CONCLUSIONS

A new buckling problem of SCSC rectangular plates subjected to both end and intermediate in-plane loads is presented in this paper using both of the Rayleigh-Ritz approach and the finite element method.

The great advantage of the present method is the determination of the critical buckling loads of the problem in hand from one solution.

This present method is simple and can be easily used by engineers designing walls or plates that have to support intermediate floor/loads with satisfactory accuracy.

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