

## Dynamic Characteristics of Transmission Tower-Line Coupled System Considering Torsional Effect

Ying Zhou <sup>1)</sup>, Shuguo Liang <sup>1)</sup> and Lianghao Zou <sup>1),2)\*</sup>

<sup>1)</sup> School of Civil Engineering and Structure, Wuhan University, China. E-Mail: whu\_zhy@163.com

<sup>2)</sup> School of Urban Construction, Wuhan University of Science and Technology, China.

E-Mail: 14698870@qq.com \* Correspondence Author: lh Zhou@whu.edu.cn

### ABSTRACT

The multi-particle model is a common method for calculating the dynamic characteristics of typical transmission tower-line coupled systems. The existing multi-particle model often ignores the torsional effect of the transmission tower when performing tower-line coupling. In order to increase the accuracy of the calculation results of dynamic property, for the first time, the torsional effect of the transmission tower is incorporated into the traditional multi-particle model by increasing the degree of freedom of the transmission tower-line system in theoretical formulae. The developed formulae were then used to analyze the inherent frequencies of transmission towers and tower-line coupled systems of a typical tower-line structure. The theoretical calculation results are rather similar to the three-dimensional finite element analysis results and to the experimental results regarding translational motion frequency and torsional frequency. The research results could have civil engineering application value for multi-particle model theoretical research and seismic analysis of vibration characteristics of transmission tower-line systems.

**KEYWORDS:** Transmission tower-line system, Multi-particle model, Dynamic characteristics, Seismic resistance, Torsion.

### INTRODUCTION

High-voltage transmission mode and superhigh-voltage transmission mode are the main modes of electric power supply that are most commonly used all over the world today. According to the statistics of the American Energy Information Administration (EIA), there were 307,000 kilometers of high-voltage transmission lines of 200 kV or more in the United States of America by the end of 2012. In comparison, China had a total length of 20,518 kilometers of ultra-high voltage transmission lines by the end of 2017 with an annual growth rate of more than 5% (Kunjie, 2028). The high-voltage transmission tower-line system is a carrier for high-load power transmission, consisting of a steel lattice tower body used to support large-span transmission lines. This coupled system is not only sensitive to environmental loads, such as earthquakes

and strong winds, but also prone to dynamic fatigue and instability (Gobarah et al., 1996). In 1995, 38 high-voltage lines and 446 distribution lines were damaged in the Hyogo Prefecture Hanshin earthquake (Chen and Zheng, 2010; Qifeng, 1997). Similarly, in 2006, a 5.1 magnitude earthquake in Yanjin County, Zhaotong City, Yunnan Province, China, resulted in damaged wires and collapse of various transmission towers within the 119.2 km transmission network and therefore caused severe economic losses (Meigen Cao et al., 2007).

The evaluation of dynamic characteristics of the transmission tower-line system by determining the inherent frequency and vibration mode of the system is a fundamental part of the structural wind-resistant and seismic design (Irvine, 1981; Danesh, 2019; Shatnawi and Al-Qaryouti, 2018). The mutual coupling effect in the transmission tower-line system makes it a very complicated coupled system and therefore it is quite difficult to accurately evaluate the system's dynamic characteristics subjected to multi-component seismic excitations. The influence of the transmission line on the

---

Received on 4/2/2020.

Accepted for Publication on 19/5/2020.

dynamic characteristics of the transmission tower (Irvine, 1981; Ozono et al., 1988; Ozono and Maeda, 1992) shows the mass effect which tends to reduce the natural frequency of the transmission tower, the stiffness effect which changes the global stiffness matrix of the system and the energy dissipation effect which causes a considerable increase in the damping of the transmission tower.

Researchers generally investigate the dynamic characteristics of transmission tower-line systems through finite element analysis (FEA) model, mechanical model and dynamic test. The FEA model can couple the transmission tower with the transmission line with considerations of six degrees of freedom, but the complicated FEA process and analysis could build a barrier for its engineering application for practical purposes. Also, it is still necessary to comprehensively analyze the measured data from the field (e.g., shaking table test) and the wind tunnel tests for broad understanding (Chunxiang et al., 2009). The limited size of the existing seismic simulation shaking table and the large span of transmission tower-line structure make it impractical to carry out the test on a shaking table. Wind tunnel tests are even more complicated, especially in model design and flow field simulation (Huijun Yin and Weilian Zhai, 2002).

In terms of the mechanical model for the calculation of the dynamic characteristics of the transmission tower-line system, Irvine (1981) proposed a continuum model for calculating static and dynamic characteristics of cable structures. However, the coupling effect in the tower-line system was ignored for the sake of simplicity. Li and his co-authors (Xianxin Wang and Hongnan Li, 1989; Hongnan Li and Xianxin Wang, 1990 and Li, 1999) believed that the long-span transmission system should consider the interaction between the wire and the transmission tower for analyzing the dynamic characteristics of the transmission tower system by using the energy principle theory. A simplified multi-particle model was proposed, the results of which showed that the vibration frequency of the transmission tower system is reduced due to the influence of the wire. Afterwards, Liang and Ma, as well as Liang et al. (2003) proposed a multi-degree-of-freedom calculation method based on the simplified multi-particle model. The influence of the vibration coupling of the tower-line system on the natural frequency and vibration mode of the tower line was analyzed. The results showed that the

mass ratio between the wire and the tower has a minor influence on the natural frequency of the transmission tower system.

The above mentioned multi-particle model is a widely used mechanical analysis model that can reflect the tower-line coupled vibration effect of transmission lines (Liang et al., 2015). The velocity of each mass point of the coupled system can be obtained directly by the speed superposition treatment. The increase of the degree of freedom of the coupled system, however, is ignored in this coupling method. The result leads to the decrease of accuracy in the velocity calculation of each mass point of the coupled system. It is, therefore, reasonable to couple the transmission tower-line system for increasing the degree of freedom. Also, the traditional coupling method simplifies the transmission tower into a cantilever bar, whereby the torsional effect of the large-scale cross-arm in the upper part of the transmission tower is ignored. The translational effect of the transmission tower is considered only through the speed superposition. Currently, there are only few research studies that have been carried out on the torsional response of transmission towers. The main reason is that the cross-arm cantilever length of the tower type studied in the past is small and the torsional effect is insignificant or even negligible (Xia, 2013). However, with the improvement of wire transmission capacity, application of long-cantilever transmission towers has become extensive. The torsion effect on the dynamic response of transmission tower-line system may be more significant, as it could generate a larger error when calculating the dynamic characteristics of the tower-line system.

Based on the multi-particle model, this paper proposed an improved coupling method of the transmission tower-line system by considering the torsion effect on the dynamic characteristics of a representative tower-line structure. The multi-particle model results were then compared to the FEA results and the experimental results regarding the vibration characteristics of the transmission tower line system. As the developed multi-particle model was verified by the experimental results, our research results could provide insight into the accurate theoretical understanding of the multi-particle model of transmission tower-line coupled system. As a result, the seismic design of long cantilever transmission towers could also be improved.

### Improvement of Coupling between Transmission Tower and Wire

Firstly, by combining the traditional multi-particle model theory and the Lagrangian equation (Ghobarah et al., 1996), the basic differential equation of the longitudinal vibration of the transmission line is derived. The calculation methods of the mass matrix and the stiffness matrix of the wire's longitudinal vibration are given. The differences of vibration characteristics of the tower with and without wire are obtained. Then, by adding the degree of freedom, the tower-line coupled method is derived considering the torsional effect of the upper part of the transmission tower. Subsequently, the fundamental equation of the longitudinal vibration of the coupled system is obtained. Finally, the method for solving the vibration characteristics of the coupled system is formulated. The following literature only describes the deduction of the coupling method of the

transmission tower-line system in the plane vibration, because the coupled mode of the plane vibration is similar to that.

### Multi-particle Model

Firstly, the longitudinal vibration characteristics of the transmission line are studied in this section. According to the multi-particle model, the transmission line is simplified to a fixed suspension structure at both ends. Within this structure, multiple degrees of freedom are composed of the deformable connecting rods and concentrated particles. As shown in Figure 1,  $l_i$  indicates the length of  $i$  connecting rod when the suspension cables are vibrating and  $m$  indicates the lumped mass.  $\theta_i$  indicates the horizontal angle of the connecting rod during the vibration of the suspension cables.  $S$  is the horizontal length of the connecting rod when the cable is stationary.

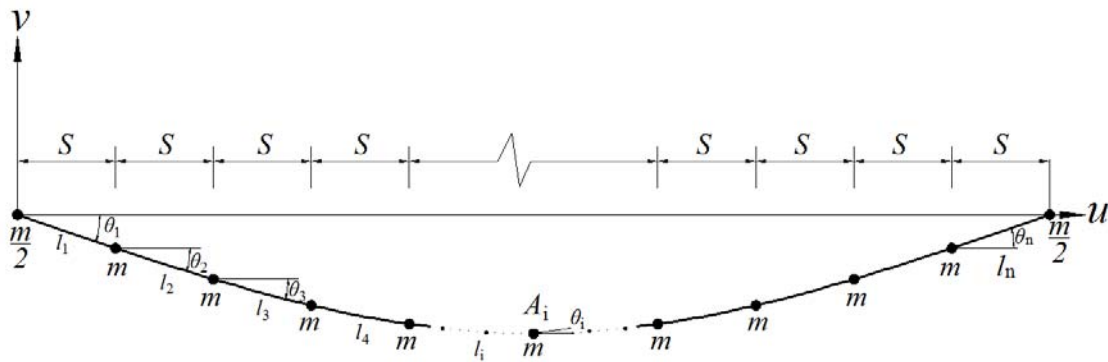


Figure (1): Logitudinal plane vibration of the transmission line with multiple degree of freedom system

### Constraints

By considering the tensile stiffness of the cable, the connecting rods should satisfy the geometric relationship when the cable is stationary.

$$\begin{cases} S = (l_{i0} + l_{is})\cos\theta_{i0} \\ H_i = (l_{i0} + l_{is})\sin\theta_{i0} \end{cases} \quad (1)$$

where  $l_{i0}$  is the original length of the connecting rod;  $l_{is}$  is the static elongation of the connecting rod caused by the lumped mass when the suspension cable is stationary;  $\theta_{i0}$  is the horizontal angle of the connecting rod when the suspension cable is stationary;  $H_i$  is the vertical length of the connecting rod when the cable is stationary.  $S$  is the horizontal length of the connecting

rod when the cable is stationary.

We reasonably assume that the angle  $\theta$  is positive in the clockwise direction, the elongation of the connecting rod is positive and the cable during vibration would satisfy the following equation.

$$\begin{cases} \sum_{i=1}^n l_i \cos\theta_i = L \\ \sum_{i=1}^n l_i \sin\theta_i = 0 \end{cases} \quad (2)$$

where  $L$  is the span of the transmission line;  $n$  is the number of links.

### Vibration Differential Equation

For the transmission line with  $n$  connecting rods and  $(n + 1)$  particle hinges, the system freedom is  $(2n - 2)$ .

After the system is disturbed, it vibrates near the static equilibrium position. We would combine  $\xi_i = \theta_i - \theta_{i0}$ ,  $\delta_i = l_i - l_{i0} - l_{is}$ ,  $\xi_i$  and  $\delta_i$  for generalized coordinates of the system's vibrational differential equations. Therefore,  $\xi_1$  and  $\xi_n$  can be expressed as:

$$\begin{cases} \xi_1 = \sum_{i=2}^{n-1} \frac{\partial \theta_1}{\partial \theta_i} \xi_i + \sum_{i=1}^n \frac{\partial \theta_1}{\partial l_i} \delta_i \\ \xi_n = \sum_{i=2}^{n-1} \frac{\partial \theta_n}{\partial \theta_i} \xi_i + \sum_{i=1}^n \frac{\partial \theta_n}{\partial l_i} \delta_i \end{cases} \quad (3)$$

where  $\frac{\partial \theta_1}{\partial \theta_i}$ ,  $\frac{\partial \theta_n}{\partial \theta_i}$ ,  $\frac{\partial \theta_1}{\partial l_i}$  and  $\frac{\partial \theta_n}{\partial l_i}$  satisfy:

$$\begin{cases} \frac{\partial \theta_1}{\partial \theta_i} = -\frac{l_n \cos \theta_n l_i \sin \theta_i - l_n \sin \theta_n l_i \cos \theta_i}{l_n \cos \theta_n l_1 \sin \theta_1 - l_n \sin \theta_n l_1 \cos \theta_1} \\ \frac{\partial \theta_n}{\partial \theta_i} = -\frac{l_1 \sin \theta_1 l_i \cos \theta_i - l_1 \cos \theta_1 l_i \sin \theta_i}{l_1 \sin \theta_1 l_n \cos \theta_n - l_1 \cos \theta_1 l_n \sin \theta_n} \\ \frac{\partial \theta_1}{\partial l_i} = \frac{\cos \theta_i \cos \theta_n + \sin \theta_i \sin \theta_n}{l_1 (\sin \theta_1 \cos \theta_n - \cos \theta_1 \sin \theta_n)} \\ \frac{\partial \theta_n}{\partial l_i} = -\frac{\cos \theta_i \cos \theta_1 + \sin \theta_i \sin \theta_1}{l_n (\sin \theta_1 \cos \theta_n - \cos \theta_1 \sin \theta_n)} \end{cases} \quad (4)$$

The system vibration differential equation is established by the Lagrange equation (Ghobarah et al., 1996), excluding the higher-order infinitesimal and therefore the velocity of each particle in the horizontal direction is:

$$\begin{cases} \dot{u}_{A_{j+1}} = \sum_{i=1}^j (-H_i \xi_i + \cos \theta_i \delta l_i) \\ \dot{u}_{A_{n+1-j}} = \sum_{i=1}^j (H_{n+1-i} \xi_{n+1-i} - \cos \theta_{n+1-i} \delta l_{n+1-i}) \end{cases} \quad j = 1, 2, \dots, \text{int}(n/2) \quad (5)$$

The vertical velocity of each particle is:

$$\begin{cases} \dot{v}_{A_{j+1}} = \sum_{i=1}^j (-S \xi_i + \sin \theta_i \delta l_i) \\ \dot{v}_{A_{n+1-j}} = \sum_{i=1}^j (S \xi_{n+1-i} + \sin \theta_{n+1-i} \delta l_{n+1-i}) \end{cases} \quad j = 1, 2, \dots, \text{int}(n/2) \quad (6)$$

where  $\text{int} (*)$  in Equations (5) and (6) represents a step function.

**Solution of Plane Vibration Characteristics of the Transmission Line**

System kinetic energy  $T$  of the transmission line is:

$$T = \sum_{i=2}^n \frac{1}{2} m_i (\dot{u}_i^2 + \dot{v}_i^2) = T \left( \sum_{i=2}^{n-1} \xi_i, \sum_{i=1}^n \delta l_i \right) \quad (7)$$

Excluding the infinitesimal that is not less than second-order, the potential energy of the transmission line  $U$  is:

$$U = - \sum_{i=2}^n \{ m_i g [ \sum_{j=i-1}^i (l_{j0} + l_{js} + \delta l_j) \sin \theta_j ] \} + \sum_{i=1}^n \frac{EA}{2l_{i0}} (l_{is} + \delta l_i)^2 \quad (8)$$

The system kinetic energy of the transmission line constitutes the mass matrix of the first-order partial derivatives of the degrees, whereas the system potential energy constitutes the stiffness matrix of the first-order partial derivatives of the degrees. The eigenvalues and eigenvectors of the system mass matrix and the stiffness matrix were then solved to obtain the frequency of

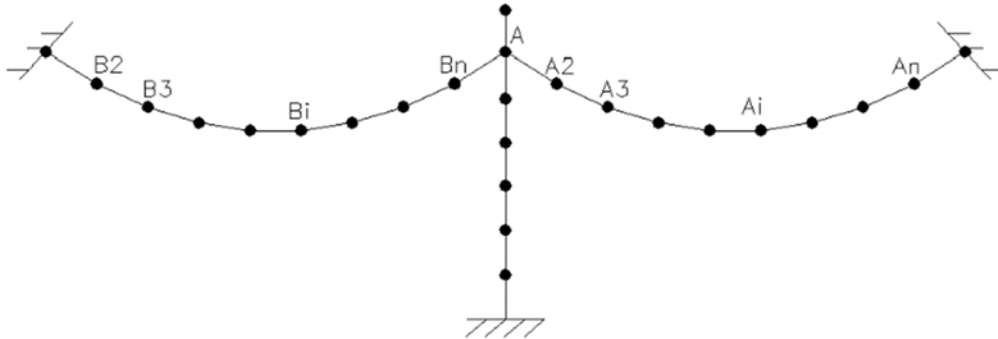
vibration mode and vibration mode of the transmission line at different orders.

If the influence of the tensile stiffness of the wire on the dynamic characteristics of the wire is not considered, the simplified multi-particle model only needs to take  $\xi_2, \xi_3, \dots, \xi_{n-1}$  as the generalized coordinates of the system. The degree of freedom of the simplified model

is reduced. Equation (3) is changed accordingly and the

rest of the calculation process is unchanged.

**Transmission Tower-line System Coupling**



**Figure (2): A multi-particle model of a tower and a two-line transmission tower-line system**

**Traditional Coupling Method for Multi-particle Model**

When the tower-line coupled system is calculated in the traditional multi-particle model, the stiffness matrix is uncoupled, whereas the mass matrix is coupled. As shown in Figure 2, for a typical one-tower and two-line multi-particle model, the particles connected to the

tower and the wire move together with the tower. According to the symmetry, the speed of corresponding nodes is transmitted by each mass point of the wires near the tower part. The vertical speed of the wire is constant and the horizontal speed can be expressed as:

$$\begin{cases} \dot{u}_{A_{j+1}} = u_A + \sum_{i=1}^j (-H_i \xi_i + \cos \theta_i \delta l_i) \\ u_{B_{n+1-j}} = u_A + \sum_{i=1}^j (H_{n+1-i} \xi_{n+1-i} - \cos \theta_{n+1-i} \delta l_{n+1-i}) \end{cases} \quad j = 1, 2, \dots, \text{int}(n/2) \quad (9)$$

Comparing Equation (9) to Equation (5), the coupling method of the traditional multi-particle model is to superimpose the speed of the corresponding points of the transmission tower directly on the speed of the original transmission line. The change in vibration mode

or speed caused by the increase in the degree of freedom of the system after coupling is not considered.

Further, the kinetic energy of the wire in the transmission tower-line coupled system is:

$$T = \sum_{i=1}^j \left[ \frac{1}{2} m_{A_{i+1}} (\dot{u}_{A_{i+1}}^2 + \dot{v}_{A_{i+1}}^2) + \frac{1}{2} m_{B_{n+1-i}} (\dot{u}_{B_{n+1-i}}^2 + \dot{v}_{B_{n+1-i}}^2) \right] + \frac{1}{2} m_A \dot{u}_A^2 \quad j = 1, 2, \dots, \text{int}(n/2) \quad (10)$$

After obtaining the kinetic energy of the conductors in the coupled system, partial derivatives of the wire free-degrees are taken to obtain the coupling term of the mass matrix. Then, the transmission tower, the transmission conductor mass matrix and the stiffness matrix are combined into a global mass matrix and a global stiffness matrix. Finally, vibration mode and frequency of the overall system are obtained after the

eigenvalues and eigenvectors are calculated.

**Improvement of Multi-particle Model Coupling Method**

The above analysis shows that the movement of the transmission tower would cause an overall translation of the transmission line, which in turn would increase the translational freedom of the system. In addition, the torsional effect of the large-scale cross-arm in the upper

part of the transmission tower also causes a change in the velocity of each point of the wire. Therefore, when performing the wire stiffness matrix and the mass matrix calculation, two degrees of freedom can be added. The vibration differential equations for the mass points of the

$$\begin{cases} \xi_1 = \sum_{i=2}^{n-1} \frac{\partial \theta_1}{\partial \theta_i} \xi_i + \sum_{i=1}^n \frac{\partial \theta_1}{\partial l_i} \delta_i + \frac{\partial \theta_1}{\partial \theta_A} \theta_A + \frac{\partial \theta_1}{\partial u_A} u_A \\ \xi_n = \sum_{i=2}^{n-1} \frac{\partial \theta_n}{\partial \theta_i} \xi_i + \sum_{i=1}^n \frac{\partial \theta_n}{\partial l_i} \delta_i + \frac{\partial \theta_n}{\partial \theta_A} \theta_A + \frac{\partial \theta_n}{\partial u_A} u_A \end{cases} \quad (11)$$

where  $u_A$  is the horizontal displacement of point A on the transmission tower,  $\theta_A$  is the upper twist angle of the transmission tower and it satisfies:

$$\begin{cases} \frac{\partial \theta_1}{\partial \theta_A} = \frac{l_d \cos \theta_n \sin(\theta_1 - \theta_n)}{l_1} \\ \frac{\partial \theta_n}{\partial \theta_A} = \frac{l_d \cos \theta_1 \sin(\theta_n - \theta_1)}{l_n} \\ \frac{\partial \theta_1}{\partial u_A} = \frac{\cos \theta_n \sin(\theta_n - \theta_1)}{l_1} \\ \frac{\partial \theta_n}{\partial u_A} = \frac{\cos \theta_1 \sin(\theta_1 - \theta_n)}{l_n} \end{cases} \quad (12)$$

$\frac{\partial \theta_1}{\partial \theta_i}$ ,  $\frac{\partial \theta_n}{\partial \theta_i}$ ,  $\frac{\partial \theta_1}{\partial l_i}$  and  $\frac{\partial \theta_n}{\partial l_i}$  are calculated according to Equation (4).

The system vibration differential equation is established by the Lagrange equation, whereas the horizontal velocity and vertical velocity of each mass point of the system are calculated according to Equations (5) and (6), respectively. The kinetic energy and potential energy of coupled system wires are respectively obtained according to Equations (7) and (8). The coupling terms of the mass matrix and the stiffness matrix of the system can be obtained by deriving the respective degrees. Then, the mass matrix and the stiffness matrix for the transmission tower and the transmission line are combined into an overall mass matrix and an overall stiffness matrix. The vibration mode and frequency of the overall system are obtained by calculating the eigenvalues and eigenvectors.

coupled system transmission line can be established based on the one-column and two-line multi-particle model in Figure 2 by adding the two degrees of freedom derived in Equation (3) as:

The coupling of the traditional multi-particle model keeps the system degrees of freedom constant and only changes the kinetic energy of the wire by superimposing the translational velocity of the corresponding particle on the speed of the transmission tower. Our new coupling method increases the degrees of freedom of the coupled system. Starting from the vibration differential equation of the system, the influence of the transmission tower translational motion frequency and the upper torsion can be simultaneously considered. Thus, the coupled dynamic characteristics of the system can be better reflected compared to the traditional multi-particle model.

## RESULTS

### Engineering Background

In order to demonstrate the accuracy of the improved multi-particle model coupling method, we selected a typical power transmission project as a prototype for analysis. The transmission tower model is ZBV63-75 with a tower height of 80.5m and a span of 750m. The structural members of the transmission tower are made of angle steel, whereas the connection forms of the components are bolted and welded. The main profiles are shown in Table 1. The transmission line uses four split wires and the relevant parameters are shown in Table 2. In order to simplify the calculations the actual four-split wire is assumed into a single wire whereas the single wire, density and cross-sectional area are considered equivalent to those of the four-split wire.

**Table 1. Main component profiles of transmission towers**

Numbering	Lever specification	Profile use position	Numbering	Lever specification	Profile use position
1	L200×16	Tower main column within 28m	6	L140×10	Cross-arm main material
2	L180×16	Tower main column in the range of 28~34.6m	7	L125×10	Cross-arm main material, tower main column, tower body bracing
3	L180×14	Tower main column in the range of 34.6~45.9m	8	L125×8	Tower body bracing
4	L160×14	Tower main column in the range of 45.9~60.8m	9	L110×10	Tower head material
5	L140×12	Main column of tower head above 60.8m	10	L110×8	Tower body bracing, tower head auxiliary column

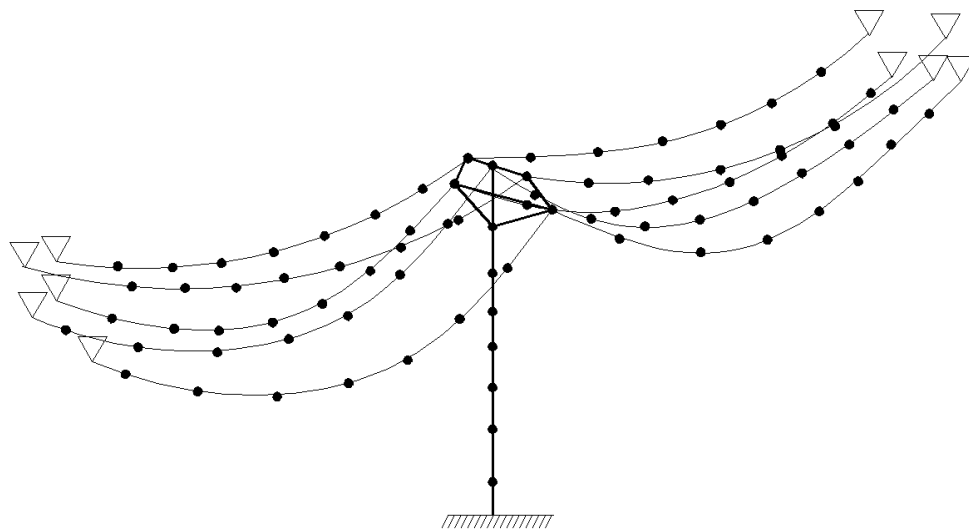
**Table 2. Parameters of transmission line material**

Span / m	750	Average wire running tension / N	36869
Ground line density / (kg / km)	570.3	Wire cross-sectional area / mm <sup>2</sup>	666.55
Wire density / (kg / m)	2060	Ground cross section / mm <sup>2</sup>	121.21
Ground sag / m	10.22	Wire elastic modulus / GPa	63
Wire sag / m	38.5	Ground linear modulus / GPa	103
Ground line average running tension / N		38431	

**Multi-particle Model Analysis**

Based on the multi-particle model, the dynamic coupling characteristics of the transmission tower-line

coupled system are calculated by the improved coupling method. The multi-particle model of the coupled system is shown in Figure 3.



**Figure (3): Transmission tower-line system multi-particle model**

For the transmission line, a multi-particle model simplifies the wire into 10 mass points and nine link structures similarly to the model established in Figure 1. A tandem multi-particle model is also established for the transmission tower as shown in Figure 4. The transmission tower is divided into nine layers, each of

which considers the translation in two directions and the rotation around the vertical reference axis. The mass center height, lumped mass and moment of inertia of the mass points of the transmission tower are shown in Table 3.

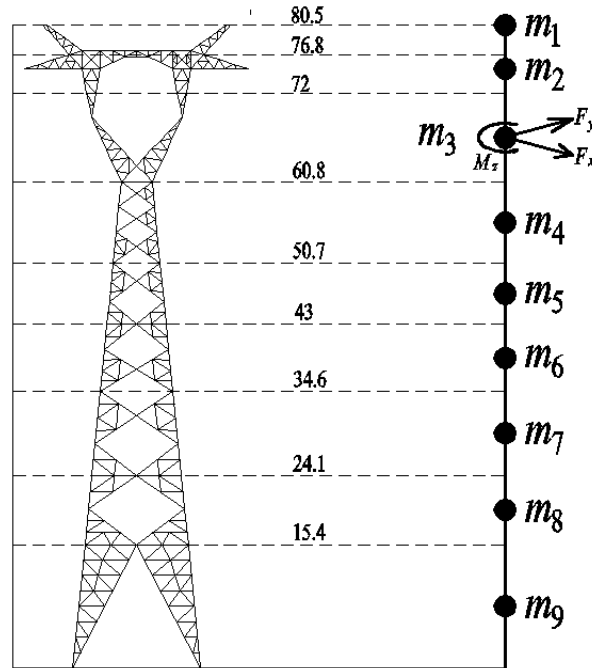


Figure (4): Transmission tower multi-particle model

Table 3. The parameters of the calculation model for the transmission tower

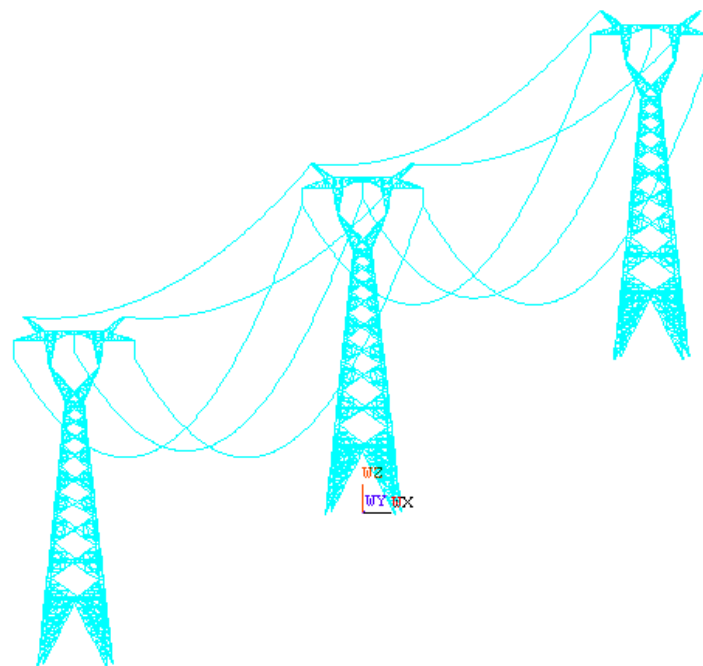
Number of layers	Center of mass height / m	Mass / kg	Moment of inertia/ (kg/ m <sup>2</sup> )
1	80.5	1.35E+03	1.40E+05
2	75.0	4.24E+03	6.25E+05
3	66.4	3.77E+03	9.19E+04
4	55.8	3.53E+03	1.54E+04
5	46.9	2.65E+03	2.35E+04
6	38.8	4.16E+03	5.11E+04
7	29.4	5.07E+03	1.04E+05
8	19.8	6.02E+03	1.65E+05
9	7.7	1.14E+04	4.19E+05

**FEA and Experimental Testing**

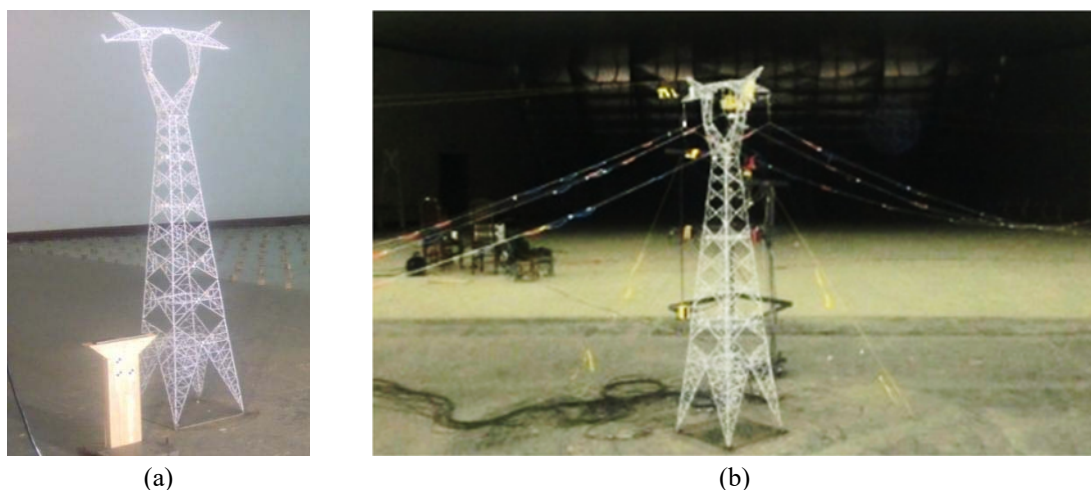
In order to verify the accuracy of the simplified model developed for calculating the dynamic characteristics of the transmission tower-line system, the results are compared with the analysis results of the FEA

model (Figure 5) and the experimental model (Figure 6). It is worth noting that the experimental model is an aeroelastic model that was tested in wind tunnel in this study.





**Figure (5): FEA model of transmission tower line system**



**Figure (6): The experimental model of (a) transmission tower and (b) tower-line system (photograph)**

The finite element model of transmission tower-line system with six degrees-of-freedom is established by ANSYS. The tower adopts BEAM188 rod elements, whereas the transmission line adopts link10 unit. The insulator is replaced by a rigid rod. The most end line node applies UX, UY, UZ constraint. Through the sensitivity analysis of the unit size to the local stress, the model uses a small unit. The material parameters are shown in Table 2. Based on the FEA model of the transmission tower-line system, the dynamic characteristics of the coupled system after the transmission line is connected are calculated. In order to

show the effect of the tower line coupling on the results of dynamic characteristics, we also analyzed and compared the transmission characteristics of the transmission tower without hanging the line.

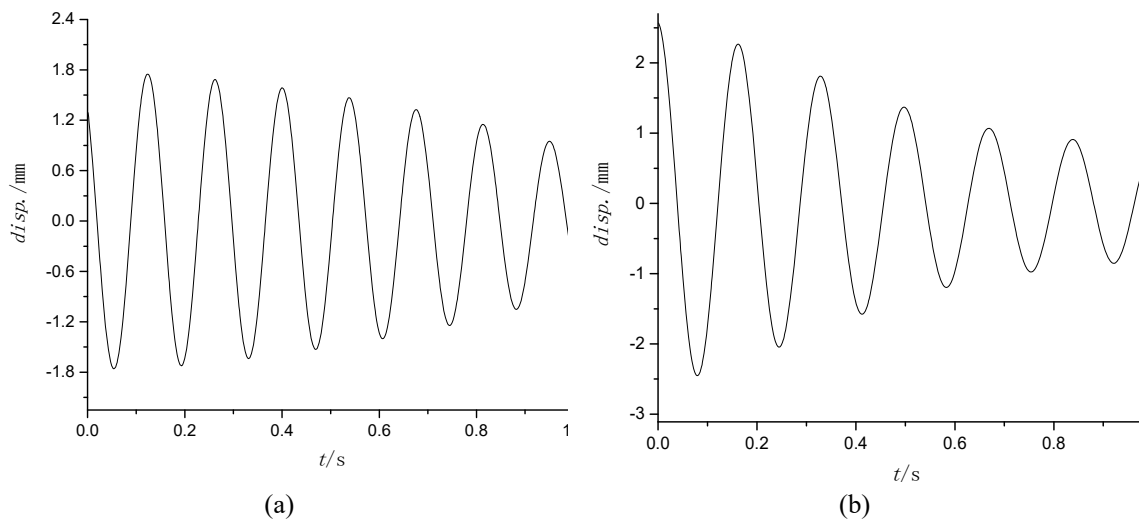
To ensure that the experimental model has a good real structure representation and that it can be properly tested in wind tunnel, a similarity theory was used to scale the structure and wind flow. The similarity scale ratios for the length, wind speed, frequency, Young's modulus and mass are presented in Table 4. It is worth noting that the experimental work of this project is rather expensive and only the first vibration model results

could be measured in this study. Figure 7 shows the free vibration deformations of the tower-line system in X and

Y directions.

**Table 4. The scale ratios for different parameters**

Parameter	Length $\lambda_L$	Wind speed $\lambda_U$	Frequency $\lambda_n$	Young's modulus $\lambda_{EA}$	Mass $\lambda_m$
Scale equation	$n$	$n^{0.5}$	$n^{-0.5}$	$n^3$	$n^3$
Scale ratio	1:40	1:6.32	6.32:1	1:64000	1:64000



**Figure (7): The free vibration deformations off he tower-line system in (a) X direction and (b) Y direction**

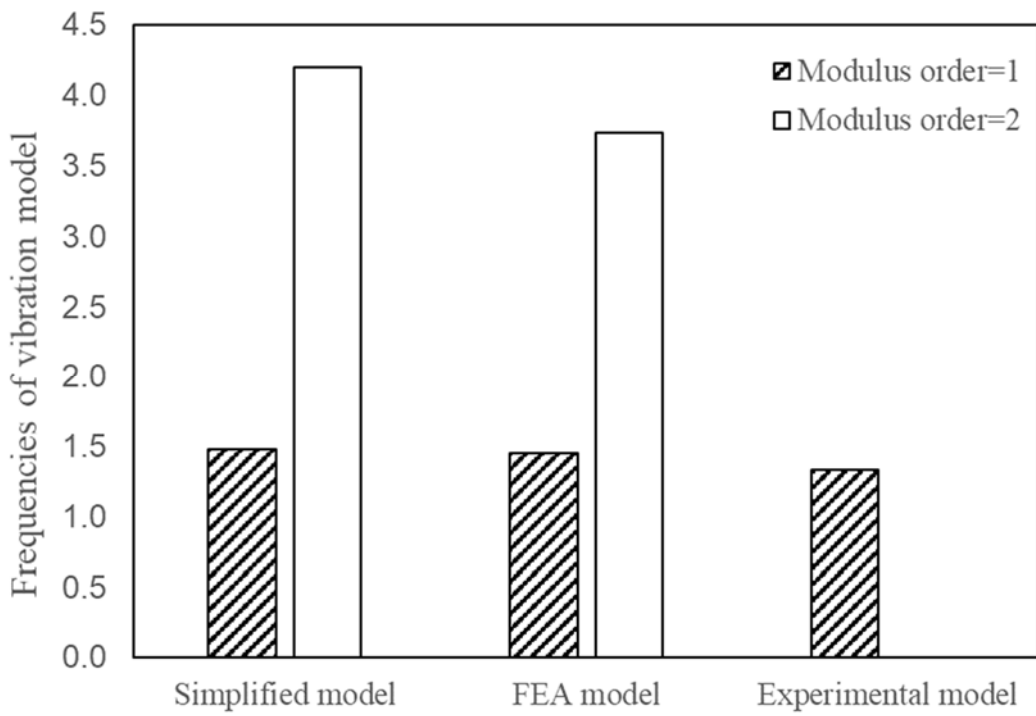
**Calculation Results and Discussion**

Based on the simplified multi-particle model and the improved algorithm of the derived dynamic characteristics, the first- and second-order mode frequencies of the transmission tower and tower-line system were calculated. The theoretical results were then compared to the FEA and experimental results.

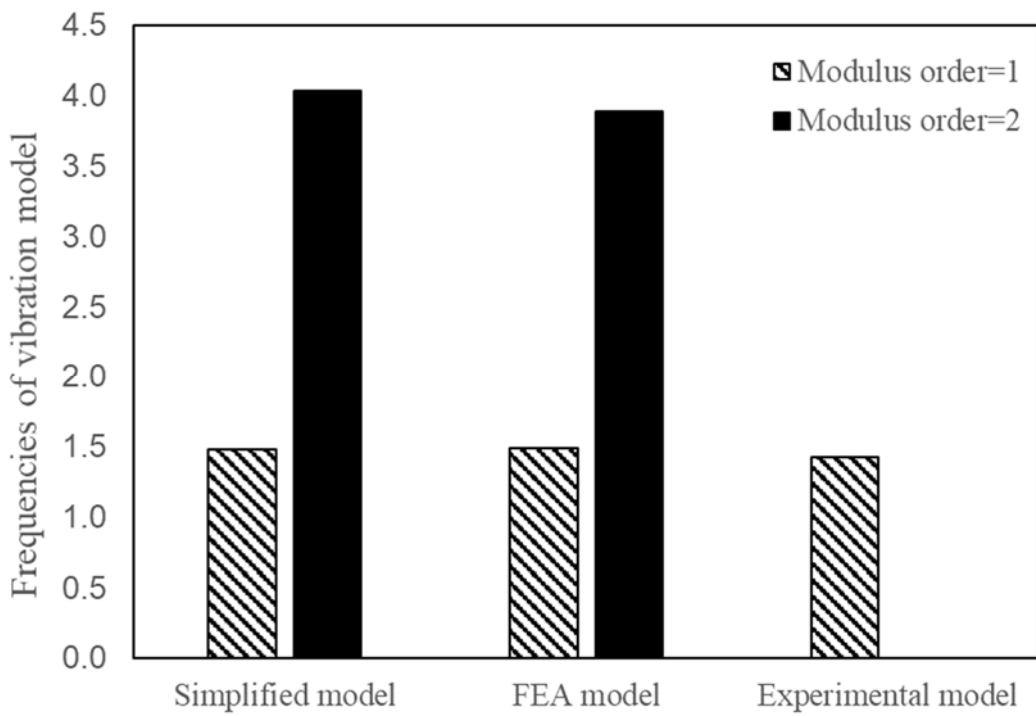
**Power Transmission Tower Dynamic Characteristics**

Figure 8 shows the calculation results of the first- and second-order modes of the simplified model, FEA model and the experimental model of the transmission tower, where  $f_x$  and  $f_y$  are the translational motion frequency of vibration mode in the X and Y directions

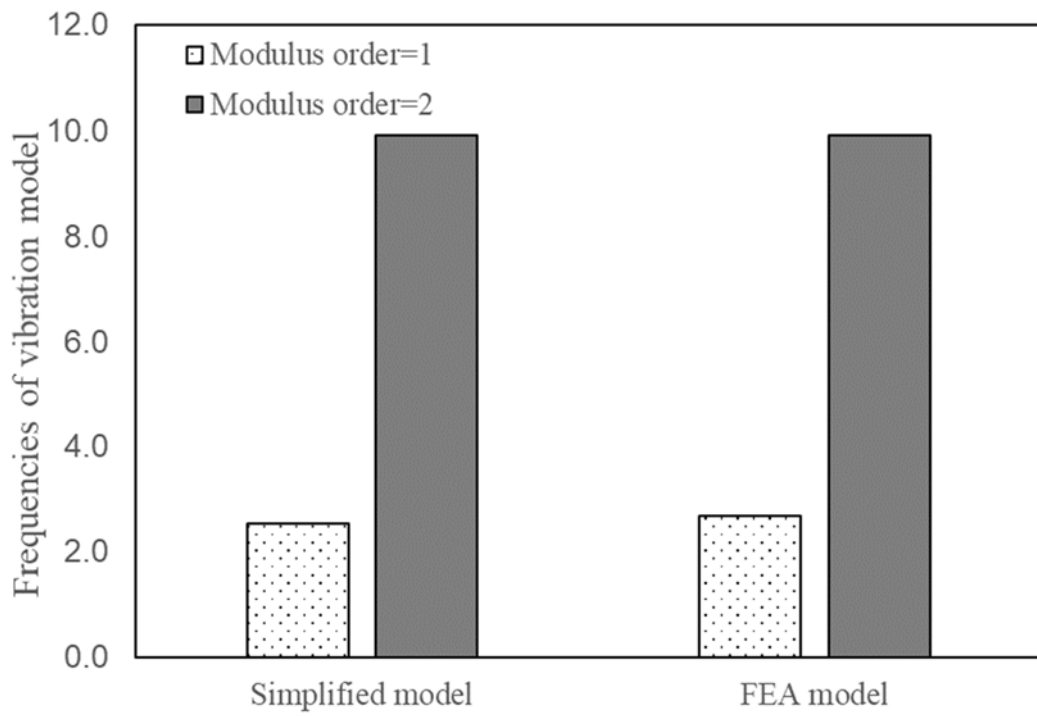
and  $f_z$  is the torsional frequency. The results show that the inherent frequency calculated by the simplified model is rather close to that of the FEA model and the experimental model. Compared to the FEA results, the maximum errors between the first-order and the second-order frequency of vibration mode are only 6.1% and 12.7%, respectively. Compared to the experimental results, the errors for the first order in the X and Y directions are 11% ad 4%, respectively. The coupling causes the torsional freedom and the corresponding vibration frequency to appear within the model. Figure 9 shows the first-order mode of the transmission tower-line system.



(a)  $f_x$



(b)  $f_y$



(c)  $f_z$

Figure (8): Frequencies of vibration mode of the transmission tower

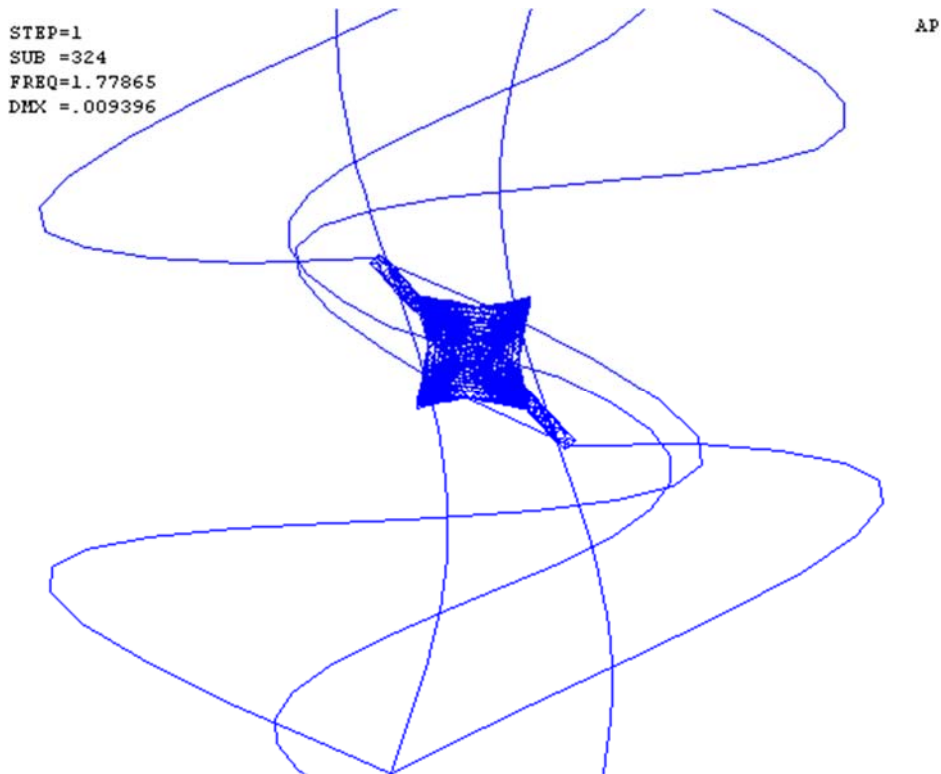
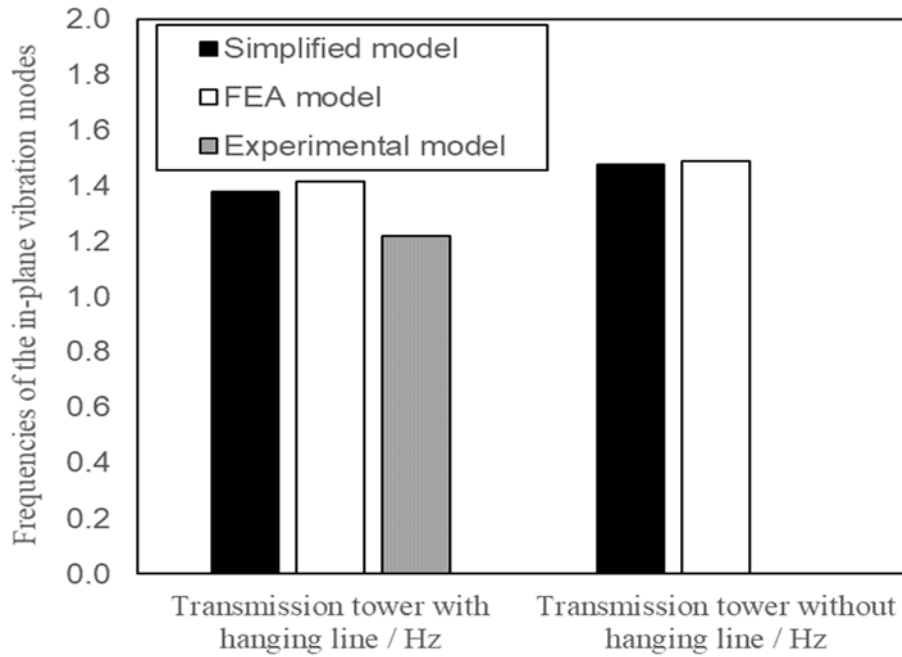


Figure (9): First-order vibration mode of the tower-line system

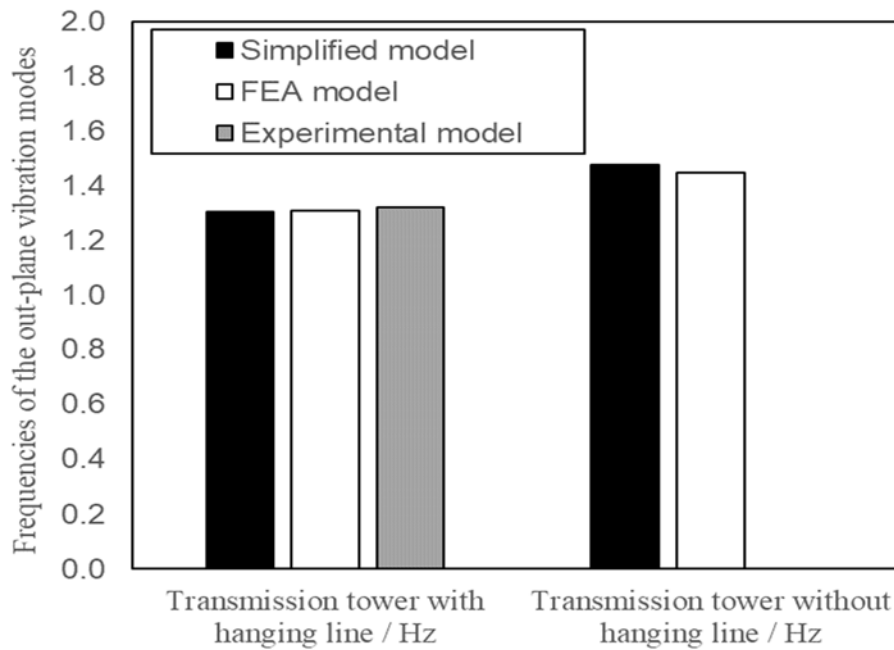
**Dynamic Characteristics of the Coupled System**

Figures 10-12 show the results of the first-order natural frequency of the multi-particle model developed for the transmission tower-line coupled system at in-plane (X direction), out-plane (Y direction) and torsion direction. The difference between the simplified model

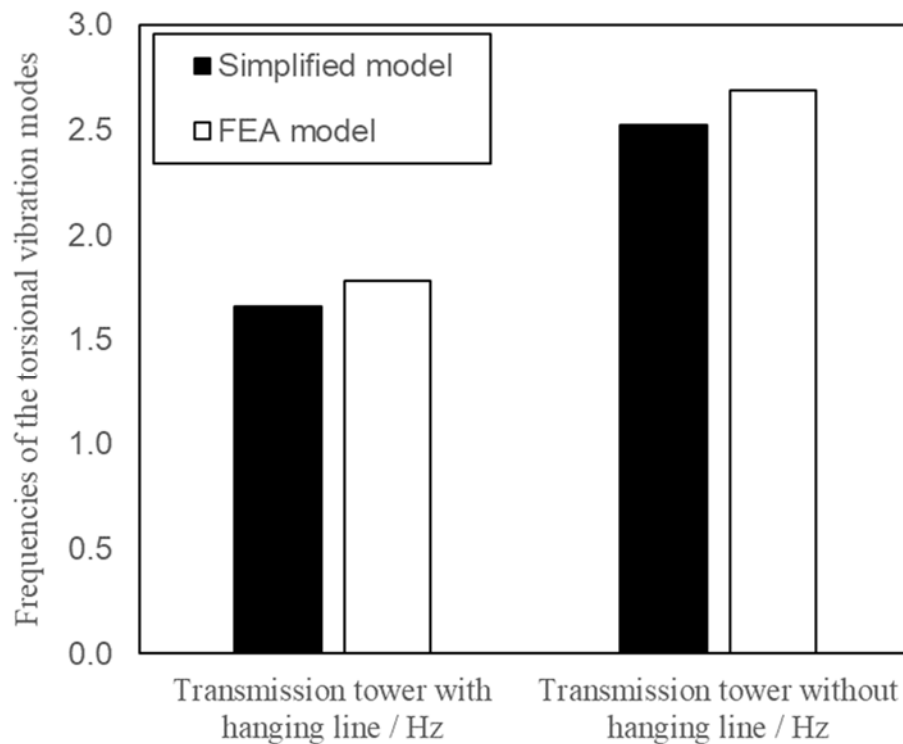
results and the other two models (FEA and experimental) can also be studied in the figures. Also, we compared the different calculation results between the single tower and the coupling effect of the tower-line.



**Figure (10): Frequencies of the in-plane vibration modes of the transmission tower system (Y direction)**



**Figure (11): Frequencies of the out-plane vibration modes of the transmission tower system (X direction)**



**Figure (12): Frequencies of the torsional vibration modes of the transmission tower system**

For the transmission tower-line system, by comparison of the self-vibration frequency of the wire under the condition of hanging and non-winding wire, it can be concluded that the influence of the transmission line on the frequency of vibration mode of the transmission tower is insignificant. However, the self-vibration frequency of the wire has a significant impact on the torsional direction. After twisting the line, the torsion frequency of the transmission tower is reduced by about 34%. Therefore, the transmission tower-line coupled system considering the torsion effect could more accurately reflect the dynamic characteristics of the transmission tower system. If the torsional effect is not considered, the frequency of the transmission tower would be overestimated, making the calculations inaccurate. Since this torsional effect is mainly caused by power transmission lines, the change in dynamic characteristics due to torsion may be rather great for long cantilever transmission tower projects. Therefore, it is necessary to consider the torsional effects while designing the span for long cantilever transmission towers.

Figures 10-12 show that the in-plane, out-plane and torsional natural frequencies of the transmission tower calculated by the simplified model are generally close to

FEA and experimental results. The experimental values are slightly lower than the simplified model values by 13% (Y direction) and 1% (X direction), whereas the simplified multi-particle model and the FEA model have a maximum error of less than 7%. These results indicate that the improved calculation method for the single-tower multi-particle model is reliable and can be used to analyze the dynamic characteristics of the tower-line coupling situation and therefore providing a theoretical reference and calculation method for the precise engineering design of the transmission tower. In seismic design, seismic waves with unilateral degrees of freedom are often required. Since two-way coupled load is likely to cause structural torsional effects (Calvi et al., 2005), the multi-particle model calculation method considering the torsional effect can accurately reflect the seismic performance of the transmission tower-line system (e.g., failure and ductility capacity). Thus, it is beneficial to civil engineering design such as the design of damper that performs the vibration control of a power transmission tower (Tian et al., 2017). It is worth noting that our simplified model achieves a better simplicity than the traditional FEA method and can be potentially used for engineering estimation.

## CONCLUSIONS

Based on the exploration of the multi-particle model of the transmission tower-line system, a simplified model of the transmission tower-line coupled system is established in this paper. Considering the cross-arm's torsional effect on the upper part of the transmission tower, the coupling method has been improved by increasing the degree of freedom. The dynamic characteristics of the transmission tower and transmission tower-line coupled system have been calculated and compared to FEA results and experimental results. The conclusions are as follows:

- The frequency of vibration mode of the tower-line coupled system calculated by the simplified multi-particle model established in this paper has a higher accuracy as compared to the FEA and experimental results.
- By increasing the degree of freedom, we can consider the coupling effect of transmission tower

by twisting the transmission line. Thus, the dynamic characteristics of the transmission tower-line system could be accurately reflected.

- Considering the tower-line coupled vibration, the in-plane, out-plane and torsional frequencies of the transmission tower under vibration mode are lower than those of the single tower (without a hanging line). It is also concluded that the hanging line has a significant influence on the torsional frequency under vibration mode.
- During the design process of the transmission tower-line architecture, if the tower-line coupling effect is not considered, the frequency of the transmission tower would be overestimated, making the structural design inaccurate.

## Acknowledgements

**Funding:** This research was funded by the National Natural Science Foundation of China, grant number 51078296.

## REFERENCES

- Calvi, G.M., A. Pavese, A. Rasulo, and D. Bolognini. (2005). "Experimental and numerical studies on the seismic response of RC hollow bridge piers". *Bulletin of Earthquake Engineering*, 3, 267-297; doi:10.1007\_s10518-005-2240-0.
- Chen, Z., and X. Zhang. (2010). "Seismic response analysis of multispan bridge using FEM". In: *ICCTP 2010: Integrated Transportation Systems: Green, Intelligent, Reliable*, 3102-3109.
- Chunxiang, L., J. Li, and Z. Yu. (2009). "A review of wind-resistant design theories of transmission tower-line systems". *Journal of Vibration and Shock*, 28, 15-25, 222-223, (In Chinese).
- Danesh, M. (2019). "Evaluation of seismic performance of PBD optimized steel moment frames by means of neural network". *Jordan Journal of Civil Engineering*, 13 (3).
- Ghobarah, A., T. S. Aziz, and M. El-Attar. (1996). "Response of transmission lines to multiple support excitation". *Engineering Structures*, 18, 936-946; doi: 10.1016/s0141-0296(96)00020-x.
- Hongnan, Li, and M.L., Qianxin Wang. (1990). "Simplified aseismic calculation of high voltage system consisting of long span transmission lines and their supporting towers". *Earthquake engineering and engineering vibration*, 10, 73-87, (In Chinese).
- Huijun, Yin, B.C., and Weilian Zhai. (2002). "Research advances of transmission tower-line system's vibration responses". *Journal of Huazhong University of Science and Technology (Urban Science Edition)*, 3, 79-82, (In Chinese).
- Irvine, H. M. (1981). "Cable structure".
- Kunjie, R. (2018). "Vibration control of power transmission tower-line system under strong seismic conditions". Master thesis, Shandong University.
- Li, H. (1999). "Response spectral method of aseismic calculation for coupled system of long-span transmission lines and their supporting towers". *Special Structures 1996*, 13 (2), 18-22 (In Chinese).
- Liang, S., and Z. Ma. "An analysis of wind-induced response for Dashengguan electrical transmission tower-line system across the Yangtze river." *Proceedings of the 10<sup>th</sup> International Conference on Wind Engineering*.

- Liang, S., J. Zhu, and L. Wang. (2003). "Analysis of dynamic characters of electrical transmission tower-line system with a big span". *Earthquake Engineering and Engineering Vibration*, 23, 63-69, (In Chinese).
- Liang, S., L. Zou, D. Wang, and H. Cao. (2015). "Investigation on wind tunnel tests of a full Aeroelastic model of electrical transmission tower-line system". *Engineering Structures*, 85, 63-72; doi:10.1016/j.engstruct.2014.11.042.
- Meigen, Cao, Q. Z., Zenglu and Mo et al. (2007). "Current research status of HV transmission line earthquake prevention and disaster relief and earthquake countermeasures". *Electric Power Construction*, 28, 23-27, (In Chinese).
- Ozono, S., and J. Maeda. (1992). "In-plane dynamic interaction between a tower and conductors at lower frequencies". *Engineering Structures*, 14, 210-216; doi: 10.1016/0141-0296(92)90009-F.
- Ozono, S., J. Maeda, and M. Makino. (1988). "Characteristics of in-plane free vibration of transmission line systems". *Engineering Structures*, 10, 272-280; doi: 10.1016/0141-0296(88)90049-1.
- Qianxin Wang, M.L., and Hongnan Li. (1989). "Reasonable computation schema for earthquake-resistant analysis of the system consisting of transmission line and its supporting towers". *Earthquake Engineering and Engineering Vibration*, 9, 81-90, (In Chinese).
- Qifeng, L. (1997). "Damages to life-line systems caused earthquake in southern Hyogoken, Japan and their recovery". *Journal of Catastrophology*, 12, 43-48, (In Chinese).
- Shatnawi, A.S., and Al-Qaryouti, Y.H. (2018). "Evaluating seismic design factors for reinforced concrete frames braced with viscoelastic damper systems". *Jordan Journal of Civil Engineering*, 12 (2).
- Tian, L., K. Rong, P. Zhang, and Y. Liu. (2017). "Vibration control of a power transmission tower with pounding tuned mass damper under multi-component seismic excitations". *Applied Sciences*, 7, 477, (In Chinese).
- Xia, L. (2013). "The research on wind effects of power transmission tower with long cross-arms". Master Thesis, Zhejiang University, Hangzhou.