Comparative Study of SDP and DP Methods for Optimal Water Release from Reservoirs: A Case Study on the Dez Dam, Iran

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ABSTRACT

Due to increase in population and occurrence of extreme droughts in recent years, correct management and planning of water resources are essential and considered vital needs in the Middle East countries. Optimal water release from reservoirs is part of water resource management. For optimization of water release from reservoirs, different methods can be applied. In this research, dynamic programming (DP) method (a discrete method for optimization) and stochastic discrete programming (SDP) method (a stochastic discrete method for optimization) are considered for optimal operation of Dez dam reservoir. The Dez dam locates in the south-west of Iran. Useful storage of the reservoir of the Dez dam is 2993.27 Mcm. This dam was constructed in 1963.

This research shows that reliability and resiliency of SDP method are higher than those of DP method, whereas vulnerability of SDP method is less than that of DP method. Also, SDP method can show months in which deficits occur correctly. The number of deficits in SDP method is less than that in DP method. In addition, in this research, variations of the number of deficits are evaluated in relation to variations of inflow to the reservoir. Sensitivity of DP method to variations of inflow to the reservoir is higher than that of SDP method.

KEYWORDS: Dynamic programming method, Optimization, Stochastic programming method.

INTRODUCTION

Supply water demands, such as drinkable water demand, irrigation water demand, industrial water demand,… etc., form a principle necessary for development of societies. In recent years, by global earth warming, melting of polar ice, rising of sea water level and extreme droughts occurred in Middle East countries. Extreme droughts produced many economic, political and social problems. In these conditions, correct planning and programming for water resource management are very important. Water resource management has different aspects. An aspect of water resource management is the optimization of water resources and supply water demands. For optimization of water resources and supply water demands, two stages must be accomplished (optimization and simulation). By these stages, system reliability, resiliency and vulnerability are determined.

Marino and Mohammadi (1983) used a combination of linear programming (LP) and dynamic programming (DP) for volume optimization of parallel multi-objective reservoirs. Their case study was conducted on Shasta and Folsom dams in California Valley project. Also, Becker et al. (1976) combined LP and DP methods for solving a system with 22 decision variables. They utilized small time (hourly and daily) steps in Central

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Valley project. Bogle and O'sullivan (1979) applied DP method for determination of the value of water demand in the future. Kumar and Baliarsingh (2003) developed the folded dynamic programming (FDP) method. This method is applied for optimization of multi-reservoir systems. This method does not need primary path for finding global optimum. Therefore, it does not converge to local optimums. Also, the number of iterations in this method is less than the number of iterations in dynamic programming for reaching global optimum. Bhaskar and Whitlatch (1987) extracted monthly optimization scenarios for Hoover reservoir in Central Ohio. They used dynamic-regression programming and LP with chance restriction. The results of operation scenarios of the two methods were compared by simulation methods. Mean yearly damage of operative scenarios in LP with chance restriction was less than mean yearly damage of operative scenarios in dynamic regression programming. Also, Karamouz et al. (1992) used DP for optimization of multi-objective reservoirs. Teixeira and Marino (2002) applied a forward dynamic programming (FDP) model for optimization of reservoir operation and irrigation scheduling. They considered two reservoirs and three irrigation districts and forecasted crop transpiration, reservoir evaporation and inflows to reservoirs.

Huang and Loucks (2000) developed an inexact two-stage stochastic programming (ITSP) model for water resource management. This model was a hybrid model. ITSP model reflected not only uncertainties expressed as probability distributions, but also those being available as intervals. Luo et al. (2007) used an SDP model for optimization of water resource systems. Shapiro (2008) evaluated computational complexity and risk averse approaches of two-stage and multi-stage stochastic programming problems. He concluded that the complexity of multi-stage programs grows fast with the increase in the number of stages. Faber and Stedinger (2001) applied a Sampling Stochastic Dynamic Programming (SSDP) model for optimization of reservoir operation. This model can predict stream flow. Shih and ReVelle (1994) stated a method for management of water demands under drought conditions. This method was a combination of optimization and simulation procedures. Tingsanchali and Boonyasirikul (2006) developed an optimization model. This model included dynamic programming, stochastic dynamic programming and trial and error adjustment of risk coefficient. They applied this model for determination of optimal operation policy of the proposed Kok-Ing-Nan transbasin diversion system in Thailand. Galelli and Soncini-Sessa (2010) developed a new approach for the design of release policies in reservoir systems serving irrigation districts. They combined metamodelling and SDP method. In recent years, Dias et al. (2013) modified SDP method by parallel processing techniques. They applied this method for long-term operation planning of electrical power systems. El-Shafie and El-Manadely (2011) presented a model based on Neural Network (NN) theory. This decision-making model can be used with a stochastic dynamic programming approach to produce an NN-SDP model. They applied this model in Aswan High Dam (AHD) for determination of optimal release policy. Pan et al. (2015) introduced a robust optimization (RO) approach. Iterative Linear Decision Rule (ILDR) was utilized to provide a tractable approximation for a multi-period hydropower generation problem. They compared results of this model with results of the sampling stochastic dynamic programming (SSDP) policy derived from historical data. Results showed that the RO method is as good as SSDP when implemented on the original historical inflows and outperforms SSDP policy when tested on generated inflows with the same mean and covariance matrix as those in history.

In this research, water release from the reservoir is optimized using DP and SDP methods. For optimization of water release from the reservoir, water demands, such as irrigation and drinkable water demands, can be considered. Also, the optimal number of classes of discrete volume of the reservoir and discrete inflow to the reservoir are determined. These optimum numbers of classes minimize vulnerability and number of deficits. At the end, the best optimized method is distinguished. This method (SDP method) has the lowest vulnerability and the highest reliability and resiliency.

MATERIALS AND METHODS

The Dez Dam

The Dez dam was constructed on the Dez River in 1963. The area of its watershed is 17430 km^2. Based on data of Khuzestan Water and Power Authority (KWPA) from 1983 to 2013, the annual average inflow to the reservoir is 8804.755 Mcm. This dam is located at 25 km north-east of Dezful city and 23 km north-east of Andimeshk city. This dam supplies an irrigation water demand of 125000 hectares of farms of Andimeshk and Haft Tapeh. Also, its hydropower plant can produce 1783 GWh of electrical energy in a year. This dam is a two-arch concrete dam. Its height from bottom, length of crest, width of crest and width of foundation are: 203m, 212m, 5.4m and 27m, respectively. The total volume of the dam reservoir is 3460 Mcm. The bottom elevation of the reservoir and its normal water surface elevation are 310m and 352m, respectively. The volumes of useful and dead storage of the reservoir are equal to 2993.27 and 466.73 Mcm, respectively. The position of the Dez dam is illustrated in Figure 1.

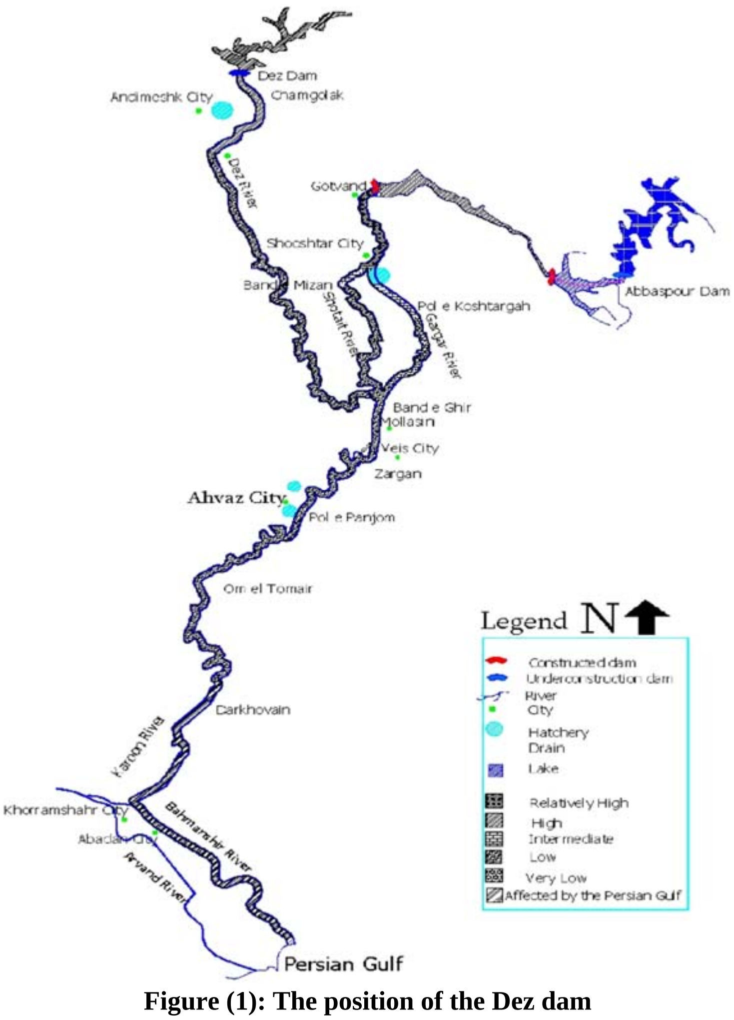

Figure (1): The position of the Dez dam

The volume- area relation of the Dez dam is shown by the formula:

$$A = 8.7054 + 0.0207S \quad (1)$$

where:

A: The area of the reservoir (km^2).

S: The volume of the reservoir (Mcm).

The volumes of drinkable water demand and irrigation water demand are illustrated in Table (1).

Table 1. Volume of drinkable water demand and irrigation water demand

Month	January	February	March	April	May	June
Drinkable water demand (Mcm)	0.496	0.526	0.773	0.4645	0.4	0.3725
Irrigation water demand (Mcm)	59.79	127.96	322.32	432.087	336.65	358.25
Month	**July**	**August**	**September**	**October**	**November**	**December**
Drinkable water demand (Mcm)	0.375	0.255	0.305	2.371	1.78	0.496
Irrigation water demand (Mcm)	345.43	503.16	490.04	421.3	273.36	146.56

The height of monthly evaporation from the Dez dam reservoir is illustrated in Table (2).

Table 2. The height of monthly evaporation from of the Dez dam reservoir

Month	January	February	March	April	May	June
The height of monthly evaporation (m)	0.061	0.087	0.138	0.219	0.365	0.465
Month	**July**	**August**	**September**	**October**	**November**	**December**
The height of monthly evaporation (m)	0.481	0.424	0.314	0.203	0.11	0.063

The mean volume of inflow to the reservoir of the Dez dam is illustrated in Table (3).

Table 3. The mean volume of inflow to the Dez dam reservoir

Month	January	February	March	April	May	June
The mean volume of inflow to the reservoir (Mcm)	637	833	1231	1760	1510	778
Month	**July**	**August**	**September**	**October**	**November**	**December**
The mean volume of inflow to the reservoir (Mcm)	464	311	230	190	300	562

RESEARCH METHODOLOGY

This research has two steps.

1- Optimization of water release from the Dez dam reservoir by DP method.

DP method converts a multi-state, multi-variable problem into several one-state, one-variable problems. Therefore, this method considerably reduces the time of problem solution. In this research, time is the stage variable, while the state variable is the storage of the reservoir at the beginning of month t and the decision variable is the storage of the reservoir at the end of month t. In this method, storage of the reservoir divides into several classes. In other words, storage of the reservoir converts into several discrete variables. The objective function is:

Objective function= min [loss] (2)

$$Loss = ((R_t - D_t)/D_{max})^2 \quad (3)$$

where:

R_t: Water release from the reservoir in month t.

D_t: Water demand in month t.

D_{max}: Maximum monthly water demand.

Restrictions of DP methods are:

$$S_{min} < S < S_{max} \quad (4)$$

where:

S_{min}: Minimum storage (dead storage) of the reservoir.

S_{max}: Maximum storage (dead storage + useful storage) of the reservoir.

Water release from the reservoir is calculated by the formula.

$$R_{klt} = S_{kt} + Q_t - S_{lt+1} - E_{klt} \quad (5)$$

where:

S_{kt}: Storage of the reservoir at the beginning of month t and k is the index of class of this storage.

S_{lt+1}: Storage of the reservoir at the end of month t and l is the index of class of this storage.

Q_t: The volume of inflow to the reservoir in month t.

R_{klt}: Water release from the reservoir in month t (k is the index of class of storage in month t and l is the index of class of storage in month t+1).

E_{klt}: The volume of evaporation from the reservoir in month t.

If $R_t < D_t$, this month is a deficit. This research utilizes the backward propagation method. In stage one and the last period (t=T), damage function is:

$$f_T^1(k) = Min\ [Loss_{klT}] \quad (6)$$

In stage two and t=T-1, damage function is:

$$f_{T-1}^2(k) = Min\ [Loss_{klT-1} + f_T^1(l)] \quad (7)$$

And in stage n and t=h, damage function is:

$$f_h^n(k) = Min[Loss_{klh} + f_{h+1}^{n-1}(l)] \quad (8)$$

If $f_t^n(k) - f_{t+1}^{n-1}(l) = f_{t+1}^{n-1}(k) - f_{t+2}^{n-2}(l)$, optimum storage of the reservoir for different months will be determined. After determination of optimum storage for different months, water release from the reservoir will be calculated by attention to inflows to the reservoir and volume of evaporation from the reservoir.

2- Optimization of water release from the reservoir of the Dez dam by SDP method.

This method used the following damage function:

$$f_t^n(k,i) = \min\left\{NB_{kilt} + \sum_j P_{ij}^t \left| f_{t+1}^{n-1}(l,j)\right.\right\} \quad (9)$$

where: k is the index of the class of reservoir storage in month t, l is the index of class of the reservoir storage in month t+1, i is the index of class of inflow to the reservoir in month t, j is the index of class of inflow to the reservoir in month t+1, n is the number of stages of the solution and P'_{ij} is the transfer probability between inflows to the reservoir in month t and in month t+1. If the indices are k, i and l, the value of damage function in month t will be equal to NB_{kilt}. $f_t^n(k,i)$ is the summation of values of damage functions from stage 1 to stage n. Damage function is shown in the object function:

$$\min \sum_{t=1}^{T} ((R(t) - D(t)) / D_{\max})^2 \qquad (10)$$

If $f_t^n(k,i) - f_{t+1}^{n-1}(k,i) = f_{t+1}^{n-1}(k,i) - f_{t+2}^{n-2}(k,i)$, optimum storage of the reservoir (L=L(k, i, t)) will be determined. In SDP model, constraints of optimization of water release from the reservoir are:

$$R_{kilt} = S_{kt} + Q_{it} - E_{klt} - S_{l,t+1} \qquad (11)$$

$$S_{min} \leq S_t \leq S_{max} \qquad (4)$$

For comparison of DP and SDP methods, three criteria are applied in this research:

a) Reliability is:

Number of successes/ total number of months.

Number of successes = total number of months - number of deficits.

b) Resiliency is:

Number of deficits that convert into success/number of deficits.

c) Vulnerability is given by the damage function (Equation 3).

These criteria state the performance of algorithms. Increasing of reliability and resiliency and reduction of vulnerability prove better performance of algorithms.

RESULTS

Results of DP Model

In this method, the volume of reservoir divides into several discrete values (classes). Optimum number of classes minimizes the objective function (Eq. (2)). First, by increasing the number of classes, the objective function reduces. But, by increasing of number of classes from a distinguished number, the objective function does not reduce. This distinguished number is the optimum number of classes. The variation of objective function in relation to number of classes is illustrated in Figure 2.

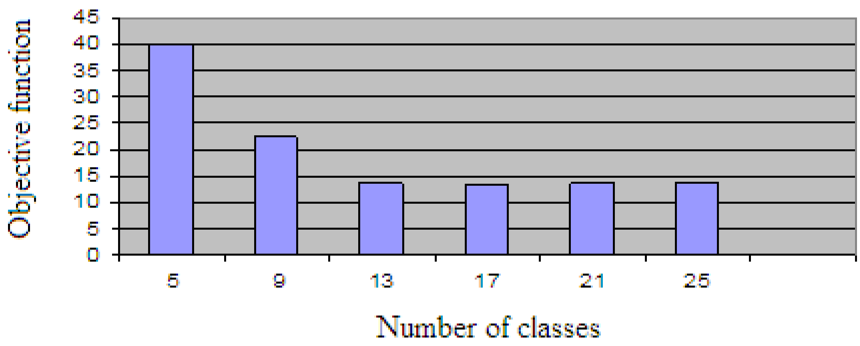

Figure (2): The variation of objective function in relation to number of classes (DP method)

Figure 2 shows that 13 is the optimum number of classes. The volume of the reservoir for 13 classes is illustrated in Table (4).

The variation of number of deficits in relation to variation of inflow to the reservoir (Q= 8804.755 Mcm) is illustrated in Table (5). In this table, Q is the mean year inflow. Also, results of analysis are illustrated in this table for drought years (0.5Q, 0.6Q, 0.7Q, 0.8Q and 0.9Q) and wet years (1.1Q, 1.2Q, 1.3Q, 1.4Q and 1.5Q).

Table 4. Volume of the reservoir for different classes (DP method)

Class	Volume of reservoir (Mcm)
1	0
2	118.181
3	354.545
4	590.909
5	827.272
6	1063.636
7	1300
8	1563.364
9	1772.727
10	2009.091
11	2245.455
12	2481.818
13	2600

Table 5. The variation of number of deficits in relation to variation of the inflow to the reservoir (DP method)

Inflow	0.5Q	0.6Q	0.7Q	0.8Q	0.9Q
Number of deficits	103	86	73	64	56
Inflow	**Q**	**1.1Q**	**1.2Q**	**1.3Q**	**1.5Q**
Number of deficits	44	39	29	23	17

Results of SDP Model

In this method, the volume of the reservoir and inflow to the reservoir divide into several discrete values (classes). The variation of objective function in relation to number of classes of volume of the reservoir is illustrated in Figures 3 to 5 (for discrete inflow to 5, 7 and 9 classes).

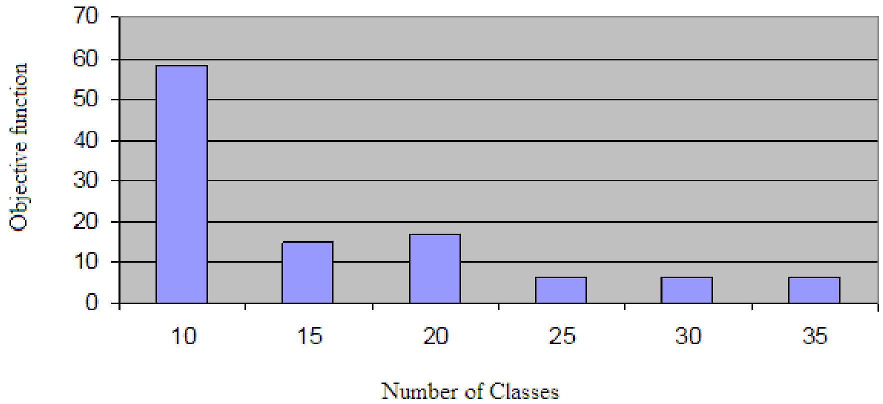

Figure (3): The variation of objective function in relation to number of classes of the reservoir volume (for discrete inflow to 5 classes) (SDP method)

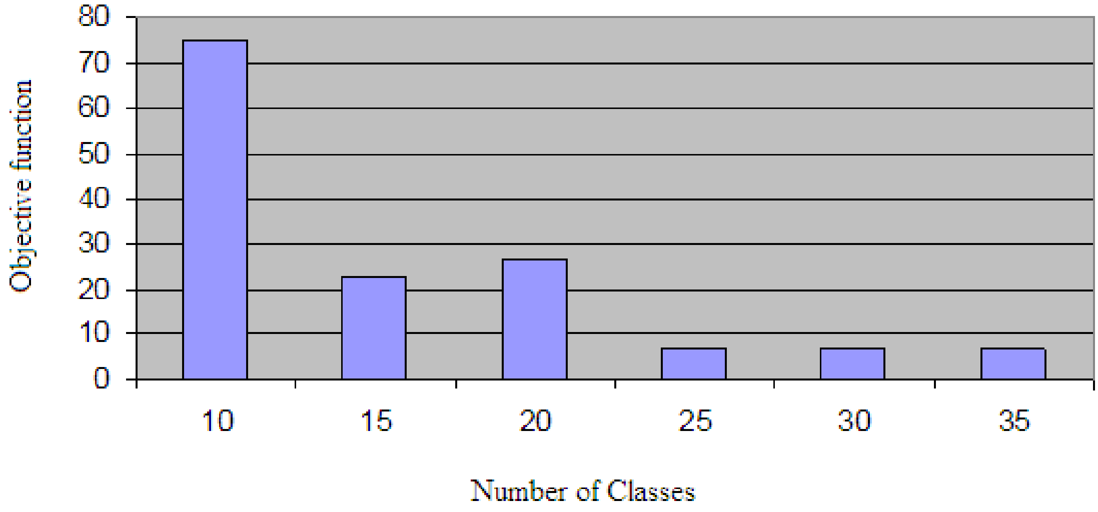

Figure (4): The variation of objective function in relation to number of classes of the reservoir volume (for discrete inflow to 7 classes) (SDP method)

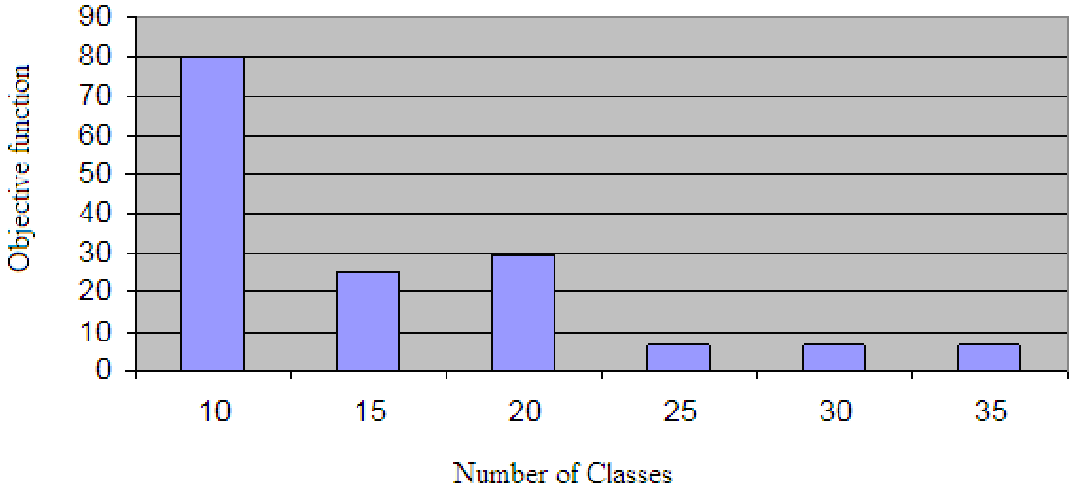

Figure (5): The variation of objective function in relation to number of classes of the reservoir volume (for discrete inflow to 9 classes) (SDP method)

Figures 3 to 5 show that 25 is the optimum number of classes. The reservoir volume for 25 classes is illustrated in Table (6).

Table 6. Volume of the reservoir for different classes (SDP method)

Class	Volume of the reservoir (Mcm)
1	0
2	56.521
3	169.565
4	282.608
5	395.652
6	508.695
7	621.739
8	734.782
9	847.826
10	960.869
11	1073.913
12	1186.957
13	1300
14	1413.043
15	1526.087
16	1639.13
17	1752.174
18	1865.217
19	1978.261
20	2091.304
21	2204.348
22	2317.391
23	2430.345
24	2543.478
25	2600

First, by reduction of number of classes, number of deficits reduces. But, by reduction of number of classes from a distinguished number, number of deficits does not reduce. This distinguished number is the optimum number of classes. The variation of number of deficits in relation to number of classes of inflow to the reservoir is illustrated in Table (7).

Table 7. The variation of number of deficits in relation to number of classes of inflow to the reservoir

Number of classes	3	5	6	7	9	11
Number of deficits	17	17	35	42	49	55

Table (7) shows that 5 is the optimum number of classes of inflow to the reservoir. P'_{ij} transfer probability between inflows to the reservoir in month t and in month t+1 has 25 states (5 classes of inflow in month t *5 classes of inflow in month t+1). These states are shown in Table (8) for different months.

Table 8. P'_{ij} transfer probability between inflows to the reservoir in month t and month t+1

Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
0.8	0.6	0.583	0.727	0.727	0.75	0.857	1	0.429	1	0.75	0.667
0.2	0.2	0.33	0.091	0.273	0.25	0.143	0	0.429	0	0.25	0.19
0	0.067	0	0.182	0	0	0	0	0.143	0	0	0.048
0	0.067	0.083	0	0	0	0	0	0	0	0	0.048
0	0.067	0	0	0	0	0	0	0	0	0	0.048
0.33	0.2	0.33	0.364	0	0	0.056	0	0.167	0.929	0.6	0.125
0.33	0.7	0.33	0.455	1	0.941	0.778	1	0.444	0	0.4	0.375
0.33	0	0.25	0.182	0	0.059	0.167	0	0.278	0	0	0.375
0	0	0	0	0	0	0	0	0.056	0	0	0
0	0.1	0.083	0	0	0	0	0	0.056	0.071	0	0.125
0.2	0.25	0	0	0	0	0	0	0	0.571	0	0
0.6	0.25	0	0.4	0.714	0	0	0.5	0.75	0.429	0	0
0.2	0.25	1	0.2	0.286	1	1	0.5	0	0	1	1
0	0.25	0	0	0	0	0	0	0.25	0	0	0
0	0	0	0.4	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.5	0	0
1	0	1	0	0	0	0	0	0	0.5	0	0
0	0	0	1	0	0	0	1	0	0	0	0
0	1	0	0	1	1	1	0	1	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0.5	0	0.667	1	0	0	0	0	0	1	0	0.667
0.5	0	0.33	0	0	0	0	0	1	0	0	0.33
0	0	0	0	1	0	0	0	0	0	0	0
0	1	0	0	0	1	1	1	0	0	1	0

The variation of number of deficits in relation to variation of inflow to the reservoir is illustrated in Table (9). In this table, Q is the mean year inflow. Also, results of analysis were illustrated in this table for drought years (0.5Q, 0.6Q, 0.7Q, 0.8Q and 0.9Q) and wet years (1.1Q, 1.2Q, 1.3Q, 1.4Q and 1.5Q).

Table 9. The variation of number of deficits in relation to the variation of inflow to the reservoir (SDP method)

Inflow	**0.5Q**	**0.6Q**	**0.7Q**	**0.8Q**	**0.9Q**
Number of deficits	59	56	56	46	22
Inflow	**Q**	**1.1Q**	**1.2Q**	**1.3Q**	**1.5Q**
Number of deficits	17	9	2	2	2

Comparison of Results of DP and SDP Methods

The number of deficits in DP method is 44, while in SDP method it is 17. Reliability, resiliency and vulnerability of DP and SDP methods are illustrated in Table (10).

Table 10. Reliability, resiliency and vulnerability of DP and SDP methods

Method	DP	SDP
Reliability	87.7%	96%
Resiliency	48%	54%
Vulnerability	3632.289	353.893

Months in which deficits occur are illustrated in Table (11) for DP and SDP methods.

Table 11. Number of deficits in different months

Month	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Number of deficits (DP method)	6	0	10	0	13	15
Number of deficits (SDP method)	0	6	6	5	0	0

By attention to Tables (1) and (3), it is observed that inflow to the reservoir is lower than water demands in August, September and October, while it is higher than water demands in July, November and December.

DISCUSSION

Because SDP method considers wet and dry periods by transfer probability, this method can link inflow and storage of the reservoir better than DP method. In SDP method, reservoir saves water in wet periods and releases it in dry periods, while DP method considers mean inflow and can not specify wet and dry periods. Therefore, the reservoir does not save water in wet periods and deficits increase in dry periods.

Also, by reduction of number of classes of volume of the reservoir in DP method, difference between volumes of different classes increases. On the other hand, water release is function of volume of classes. Therefore, water release increases in wet periods and number of deficits in dry periods and value of objective function grow.

By increasing number of classes of inflow in SDP method, the range of inflows of dry periods decreases. Therefore, water release increases in periods that are not dry and number of deficits in dry periods and value of objective function grow. On the other hand, deficits transfer to periods with inflows near to inflows of dry periods (increase of deficits). This subject is very important for arid countries, as Middle East countries, with severe droughts occurring in these regions.

CONCLUSIONS

Ability of SDP method is better than ability of DP method for increasing of reliability (number of successes to number of data) and resiliency (number of deficits that convert into successes to number of deficits) and reduction of vulnerability (damage function). Also, calculated water release by SDP method can supply water demands better than calculated water release by DP method. Nature of DP method is alternative, because this method converts a multi-stage, multi-variable problem into several one-stage, one-variable problems. Therefore, DP method can not supply water demands suitably. On the other hand, DP can not predict months in which deficits occur correctly. DP method that has an alternative nature showed that deficits occur at alternate months of the year (July, September, November and December), while SDP method showed that deficits occur from August to October. In these months, volume of inflow to the reservoir is low, while water demands are high.

Because SDP method follows a stochastic approach and considers transfer probability between consecutive months, this method can supply water demands better than DP method (DP method follows a deterministic approach and can not consider variations of inflow to the reservoir).

SDP method increased reliability to 9.5% and resiliency to 12.5% and reduced vulnerability to 90% in comparison to DP method. Also, SDP method reduced the number of deficits to 61%.

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