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Theoretical End Depth Ratio and End Depth Discharge Relationship for Free Overfall with Different End Lip Shape

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ABSTRACT

A free overfall at the end of the channel is used as a flow measuring device. In this paper, theoretical end depth ratio (EDR) and end depth discharge (EDD) relationships have been studied and obtained for subcritical flow in a rectangular channel with a free overfall and with different end lip shapes; i.e., triangular and skewed lip.

For all models, Froude number at the brink section Fb, EDR and EDD are predicted using the theoretical relationship and compared with the values computed experimentally. The mean absolute error varies from 0.5% to 8.5% and the standard deviation varies from 0.824% to 7.35%.

KEYWORDS: EDR, EDD, Free overfall, Theoretical analysis.

NOTATION

Symbol	Description
A_1	Cross-section area at section 1
A_b	Cross-section area at section 2 (brink end)
θ	Angle of skewed and triangular end lip
F_b	Froud number at section 1
F_n	Froud number at section 2
b	Channel width
C_c	Coefficient of contraction
E, H	Specific energy
g	Acceleration due to gravity
Q	Discharge over free overfall
v_b	Velocity at brink section
v_n	Velocity at section 1
y_b	Water depth at brink section
EDR	The relationship between the brink depth and the normal depth, end-depth-ratio (y_b/y_n).
EDD	End depth, discharge
y_n	Water depth at section 1

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INTRODUCTION

A free overfall occurs when there is a sudden reduction in bed elevation of a channel causing the flow to separate and form a free nappe. When the approaching flow is subcritical, the flow upstream of the free overfall becomes critical. Due to the vertical acceleration that occurs at the brink, the pressure distribution is no longer hydrostatic (Seyed et al., 2011). The relationship between the brink depth (y_b) and the normal depth (y_n) is known as end-depth-ratio (EDR=y_b/y_n).

Several works related to free overfall since 1936 have been studied through laboratory experiments and theoretical studies such as (Rouse, 1936). Some of these studies dealt with fined EDR in free overfall for different channel shapes (circular and semicircular channels) such as (Dey, 1998; Dey, 2000; Dey, 2001; Dy, 2002a; Dey, 2003; Dey et al., 2003; Ahmed, 2005; Mahesh and Arun, 2006; Mohammed, 2008). (Rajaratnam and Muralidhar, 1970; Neogy, 1972; Keller and Fong, 1989; Terzidis and Anastasiadou-Partheniou, 1990; Gupta et al., 1993; Pagliara and Viti, 1995; Ramamurthy et al., 2004) presented studies of free overfall in trapezoidal channels. Ali and Sykes (1972) assumed that the value of EDR varied from 0.673 to 0.798. Dey (2002b) presented a lot of literature review studies for free overfall in different channel shapes. A number of studies investigated roughness effect and slope such as (Raju and Garde, 1970; Bauer and Graf, 1971; Rajaratnam et al., 1976; Rajaratnam et al., 1977; Kraijenhoff and Dommerholt, 1977; Knight and MacDonald, 1979; Ferro, 1992; Davis et al., 1998; Dey, 1998; Dey, 2000; Guo, 2005; Guo et al., 2008; Tigrek et al., 2008; Mohammad et al., 2011; Mohammad et al., 2012). A large number of theoretical and numerical studies have been carried out on free overfall such as (Hager, 1983; George, 1985; Litsa and Evangelos, 1995; Abdullah and Peter, 1996; Davis et al., 1999; Ferro, 1999; Dey, 2000; Ahmad, 2003; Guo, 2005; Ramamurthy et al., 2006; Guo et al., 2008).

Mohammed et al. (2007) studied two models of free overfall: straight vertical and skewed end lip, and found the relationship between brink and critical depth, the discharge equations for the two models and showed that the discharge for the skewed lip model was greater by 13% than for the straight vertical model. Mohammed (2008) presented an experimental study and analysis for the effect of channel slope on straight vertical and skew free overfall for a rectangular channel with different slopes, and found that the discharge over the skewed model was greater by 21% than for the straight vertical model. Mohammed (2009) studied the behaviour of free surface flow on a rectangular free overfall which has a triangular shape. The results revealed that the ratio of brink depth to critical depth at the center line for falls inclined with flow direction was greater by (3%) than for falls in the opposite direction. This value increased to (27%) when Froud number increased from (0.04-0.18). Tigrek et al. (2008) presented an experimental study and found the effects of the bed roughness and slope of the channel on the brink depth.

In this paper, the flow over a free overfall in a rectangular channel is similar to that over a sharp crested weir; so a theoretical analysis based on (Litsa and Evangelos, 1995; Ferro, 1999) yielded a general end depth discharge (EDD) relationship for subcritical flow in a rectangular channel with different end lip shapes.

Discharges predicted using the theoretical relationship are compared with the available experimental data carried out by (Ahmed et al., 2007; Mohammed, 2009).

THEORETICAL ANALYSIS

According to the previous works adopted by Litsa and Evangelos (1995) and Ferro (1999) for rectangular free overfall, the flow over an overfall in a rectangular channel can be assumed to be similar to the flow over a sharp crested weir of the same section with P = 0 and h = y_n, Fig.1.

At brink end section, the streamlines are considerably inclined and curved and the pressure distribution is not hydrostatic.

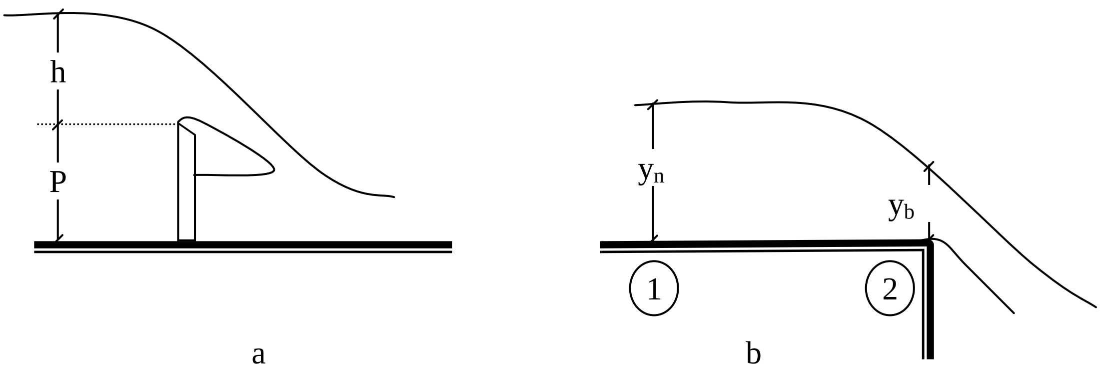


Figure 1: a) sharp crested weir, b) free overfall

The flow velocity is calculated by applying the energy equation:

$$E = y_b + \frac{V_b^{\,2}}{2g} \qquad \text{...........(1)}$$

where

E: specific energy;

y_b: water depth at brink section;

v_b: velocity at brink section;

g: acceleration due to gravity.

In accordance with the theoretical procedure applied to compute the discharge over a sharp crested weir, a zero pressure distribution and parallel streamlines at the brink, neglecting the contraction of the nappe, are initially assumed.

The discharge dQ through an elementary cross-section dA at a vertical distance dz, can be computed applying the following continuity equation:

$$dQ = \sqrt{2g(H - z)}dA \qquad \text{...........(2)}$$

where:

H: the energy head at section 1, and can be computed from:

$$H = E = y_n + \frac{V_n^{\,2}}{2g} \qquad \text{...........(3)}$$

where

y_n: water depth at section 1;

v_n: velocity at section 1.

Under these assumptions of a zero pressure distribution and parallel streamlines at the brink, eq. 2 could be integrated from $z = 0$ to $z = y_n$.

Then, the discharge Q is computed as:

$$Q = \int_0^{yn} C_c \sqrt{2g}(H - z)^{1/2} b.dz \qquad \text{.....................(4)}$$

By integrating eq. 4, the following eq. can be written:

$$Q = \frac{2}{3} b\sqrt{2g} C_c \left[H^{3/2} - (H - yn)^{3/2}\right] \qquad \text{...........(5)}$$

where

b = channel width;

C_c: coefficient of contraction, which can be calculated as:

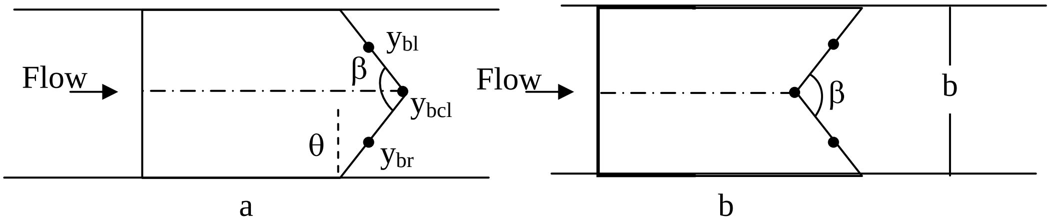
$$C_c = A_b / A_1 \qquad \text{..........................(6)}$$

where

Cross-section area at section 1 is $A_1 = by_n$

Cross-section area at section 2 (brink end), $A_b = \frac{b}{2\cos\theta}.2$, then $A_b = b\sec\theta$ (Figs.2 and 3) where

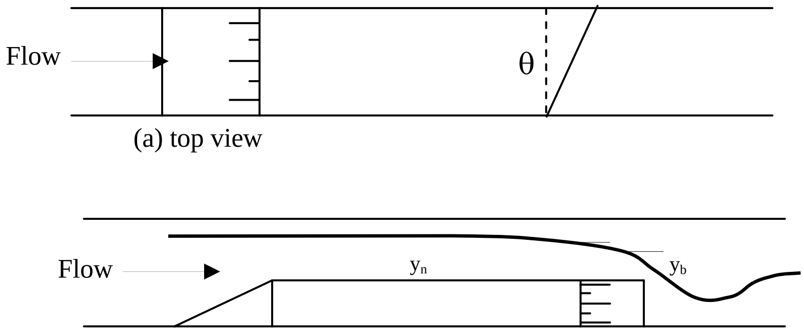
θ : Angle of skewed and triangular end lip.



Triangular end lip with flow direction

Triangular end lip in opposite flow direction

Figure 2: Free overfall sketch with triangular end lip shape (Mohammed, 2009)



(b) side view

Figure 3: Free overfall sketch with skewed end lip shape (Ahmed et al., 2007)

So, eq. 6 can be written as:

$$C_c = \frac{y_b}{y_n} \sec\theta \qquad \text{..............(7)}$$

Substituting eq. 7 into eq. 5 and multiplying by ($1/b\, g^{1/2}\, y_n^{3/2}$)

$$\frac{Q}{bg^{1/2} y_n^{3/2}} = \frac{2}{3} \frac{b\sqrt{2g}\, \sec\theta}{bg^{1/2} y_n^{3/2}} . \frac{y_b}{y_n} \left[H^{3/2} - (H - y_n)^{3/2} \right] \qquad \text{..............(8)}$$

When $\frac{Q}{bg^{1/2} y_n^{3/2}} = F_n$, Froude number, then eq. 8 can be written as:

$$F_n = \frac{2}{3}\sqrt{2}\, \frac{y_b}{y_n} \sec\theta \left[(\frac{H}{y_n})^{3/2} - (\frac{H}{y_n} - 1)^{3/2} \right] \qquad \text{......(9)}$$

Knowing that $E = H = y + \frac{V^2}{2g}$ and $F = \frac{V}{\sqrt{gy}}$,

we can take into account that:

$$\frac{H}{y_n} - 1 = \frac{F_n^{\ 2}}{2} \qquad \text{..................(10)}$$

Then, we can get EDR from eq. 9 as:

$$EDR = \frac{y_b}{y_n} = \frac{3F_n \cos\theta}{(F_n^{\ 2}+2)^{3/2} - F_n^{\ 3}} \quad \text{..............(11)}$$

By applying the continuity equation between sections 1 and 2 for $Q = V_n y_n = V_b y_b$, the following equation can be obtained:

$$\left(\frac{y_n}{y_b}\right)^3 = \left(\frac{F_b}{F_n}\right)^2 sec^2\,\theta \quad \text{..................(12)}$$

From eqs. (11 and 12), the equation below can be obtained:

$$F_b = F_n\left(\frac{(F_n^{\ 2}+2)^{3/2} - F_n^{\ 3}}{3F_n}\right)^{3/2} sec^{1/2}\,\theta \quad \text{......(13)}$$

From the relationship between Froud number and discharge, the theoretical *EDD* can be obtained as:

$$EDD = Q = F_b b g^{1/2} y_b^{\ 3/2} \quad \text{.........(14)}$$

From eqs.(13 and 14), we can get the following equation for calculating *EDD*:

$$Q = F_n\left(\frac{(F_n^{\ 2}+2)^{3/2} - F_n^{\ 3}}{3F_n}\right)^{3/2} sec^{1/2}\,\theta . b . g^{1/2} . y_b^{\ 3/2}$$

.........(15)

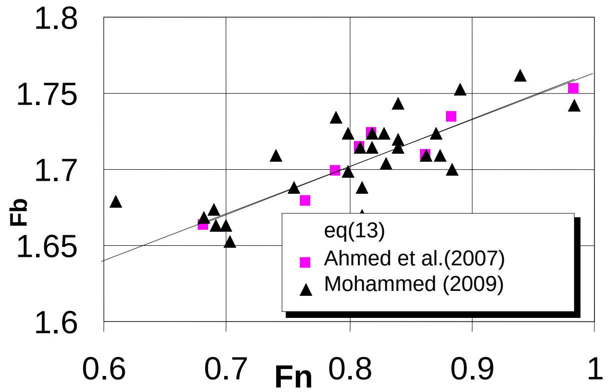


Figure 4: Comparison between experimental Froude number (F_n, F_b) and eq.13

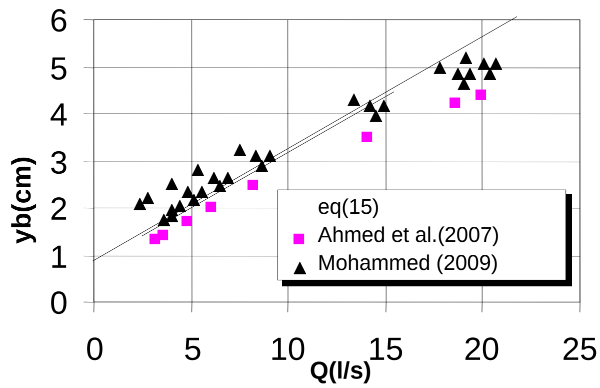


Figure 5: Comparison between experimental (Q, y_b) and eq.15

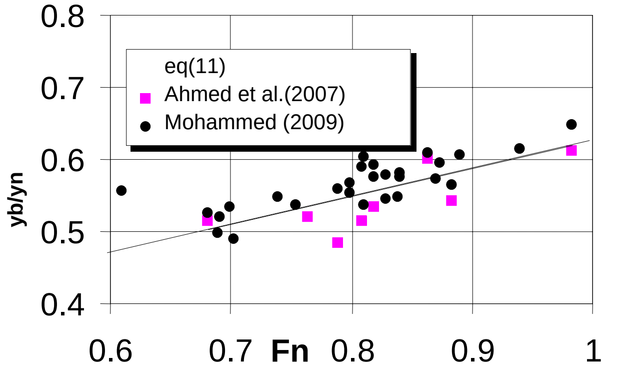


Figure 6: Comparison between experimental (F_n, *EDR*) and eq.11

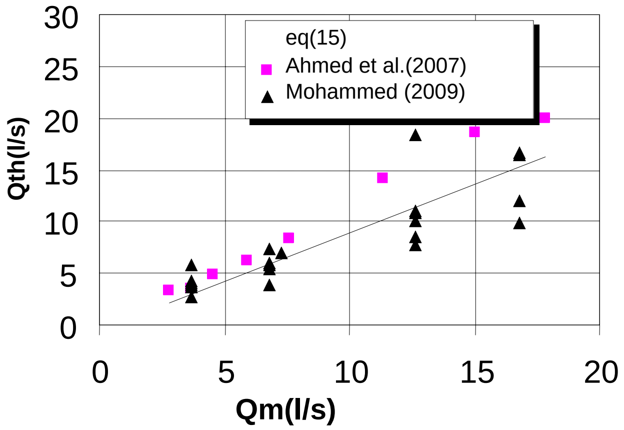


Figure 7: Comparison between experimental discharge data computation and that computed using eq.15

RESULTS

Fig.4 shows the agreement between experimental data of F_n and F_b (Ahmed et al., 2007; Mohammed, 2009) and that computed using eq. 13, the values of F_b computed using this equation have a mean absolute error of 0.5% and a standard deviation of 0.824% .

Fig.5 shows the agreement between experimental data of Q and y_b (Ahmed et al., 2007; Mohammed, 2009) and that computed using eq. 15

Fig.6 shows the agreement between experimental data of F_n and the *EDR* (Ahmed et al., 2007; Mohammed, 2009) and that computed using eq. 11. From statistical programming (SPSS, V.17 and Excel) the values of *EDR* computed using this equation have a mean absolute error of 3.7% and a standard deviation of 4.24%.

The discharge predicted using eq.15 is compared with experimental data (Ahmed et al., 2007; Mohammed, 2009) and is shown in Fig. 7. The percentage error E% and the mean absolute error are estimated. The accuracy of the discharge measurements was within ±1% at low discharge while these values increased at high discharge. The values of discharge predicted using eq.15 have a mean absolute error of 8.5% and a standard deviation of 7.35%.

CONCLUSIONS

In this paper, the free overfall in a rectangular channel with different end lip shapes was investigated as a device for flow measurements. Theoretical equations for calculating end depth ratio, brink Froude number and discharge for rectangular free overfall with triangular and skewed end lip shapes have been predicted (Eqs. 11, 13 and 15). This method provides results that are in agreement with experimental data for the previous works. The mean absolute error varies from 0.5% to 8.5% and the standard deviation varies from 0.824% to 7.35%.

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