Simplified Model for Diagonal Cracking Shear Capacity of Slender RC Beams without Web Reinforcement

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ABSTRACT

A simplified shear-moment interaction model is proposed for the shear capacity of reinforced concrete (RC) beams without web reinforcement, considering four basic variables (longitudinal steel ratio, concrete strength, shear span to depth ratio and beam size) and verified by existing experimental results. The proposed model shows that shear strength and mode of failure depend on the shear-moment paths followed to bring the beams to failure. This study shows that the variations in the experimental results of shear capacity of slender RC beams ($a / d \geq 2.5$) defined at the formation of diagonal cracks of beams under the combined action of moment and shear can primarily be explained by the variations of the concrete tensile strength. A design formula based on this shear-moment interaction model is also proposed for shear capacity of slender RC beams.

KEYWORDS: diagonal tension cracking, Ultimate shear strength, Concrete tensile strength, Slender beams, Moment capacity, Interaction diagram.

INTRODUCTION

Although great progress has been achieved in evaluating the shear strength of reinforced concrete (RC) beams through many theoretical and experimental studies over the past several decades, failure under combined action of moment and shear is still not completely resolved, as reflected by the deviation of the results of theoretical analysis from the experimental results and is still open to discussion. One of the major difficulties in developing a theoretical expression for the shear strength of RC beams is due primarily to the indeterminacy of the internal force system of a cracked reinforced concrete member (Krefeld and Thurston, 1966). The beam size is also an important factor affecting the shear strength of RC beams (Leonhardt and Walther, 1962; Kani, 1967; Bhal, 1968; Bazant and Kim, 1984; Bazant and Kazemi 1991; Shioya et al., 1989; Reineck, 1991; Walraven, 1978; Walraven and Lehwalter, 1994; Ramallo et al., 1995; Bazant and Yu, 2005; Collins and Kuchma, 1999). Shear strength of a reinforced concrete beam is also a function of the relative magnitude of the applied moment and shear force, making the problem more complex. Furthermore, the large variation in beam strength even under one particular loading condition on duplicate beams contributes to these difficulties because of the brittle nature of concrete as well as the effect of microcracks (MacGregor, 1997) present in the material that may greatly influence or dominate the effect of other variables. As a result, numerous models, including international codes (such as: ACI318-2011; AASHTO LRFD 2004; CSA A23.3-2004; Bentz and

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Collins, 2004; ACI-ASCE 445-1999) have been suggested for evaluating the shear capacity of RC beams primarily based on rather empirical or semi-empirical considerations, using statistical analyses alone or accompanied by rationalized expressions. Each one of the available models was generally developed to describe, capture or explain an aspect of the failure of RC beams depending on the mode of failure utilized for developing the model. Hence, no single model can explain all modes of failure of RC beams. The experimental work has shown that failure of RC beams under combined action of moment and shear may happen in very different modes (Russo et al., 2005). These modes may include failure due to flexure, diagonal tension cracking, splitting of compression zone, crushing of concrete, anchorage and loss of bond (MacGregor, 1997; Zararis, 2005; Zararis and Papadakis, 2001).

In this study, a simplified model developed based on mechanics analysis is proposed for evaluating the shear capacity of slender RC beams (having shear span to depth ratio $a/d \geq 2.5$) without web reinforcement defined at the formation of diagonal tension cracks. The proposed model considers four basic variables (longitudinal steel ratio ρ, concrete strength $f_c^{'}$, shear span to depth ratio a/d and beam size d) and involves only two "constants". Because slender RC beams fail upon the formation of diagonal tension cracks or shortly afterwards (MacGregor, 1997), the proposed model provides a lower limit for the shear capacity of slender RC beams. Comparison with existing experimental results as well as with existing models (Zararis and Papadakis, 2001; Vecchio and Collins, 1986) supports the validity of the proposed model in predicting and explaining the observed behavior of slender RC beams ($a/d \geq 2.5$) without web reinforcement.

Proposed Shear-Moment Interaction Model

In this section, a simplified model is developed for evaluating the shear capacity of slender RC beams ($a/d \geq 2.5$) without web reinforcement under combined action of moment and shear at the instant at which diagonal tension cracks (*web shear* or *flexural shear* cracks) start to form.

The principal tensile stress f_1 in an element under a state of stresses (Fig. 1) may be expressed in terms of the normal tensile stress f_t (acting parallel to the beam longitudinal axis) and shear stress v (acting on faces parallel and perpendicular to the beam longitudinal axis) using a Mohr circle (Fig. 2) as follows (tension is positive):

$$f_1 = \frac{f_t}{2} + \sqrt{\left(\frac{f_t}{2}\right)^2 + v^2} \qquad (1)$$

The vertical normal stress in the direction of the shear force is assumed to be zero (Fig. 1).

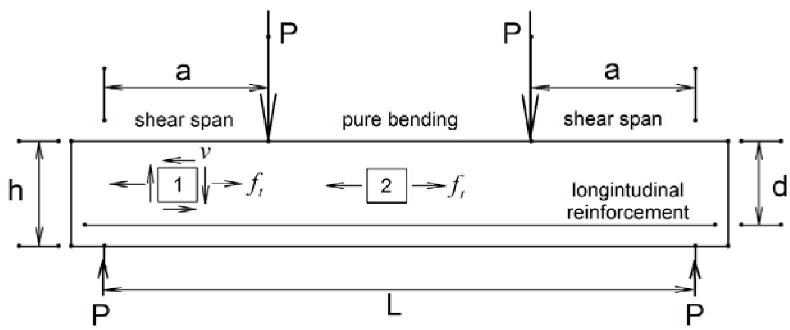

Figure (1): Two elements in the shear span and pure bending span of a beam showing the normal tensile stress f_t and shear stress v acting on these elements

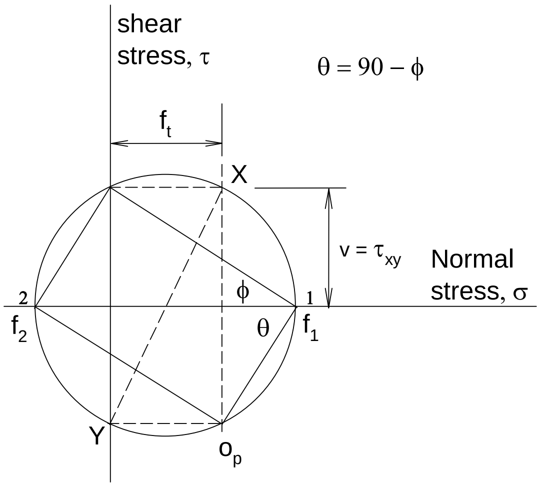

Figure (2): **Mohr circle of stresses for the element in the shear span of a beam showing the orientation of principal tensile stresses (f_1 and f_2) and that of possible inclined cracks**

Diagonal cracks form in RC beams when the principal tensile stress f_1 reaches the concrete strength in tension, f'_{ct}. Diagonal tension cracks may develop before any flexural cracks start to form in which case the cracks are known as *web shear* cracks; these cracks develop at regions in a beam where the shear forces are large relative to moment. These *web shear* cracks form mostly at or near the neutral axis and propagate from that location. Diagonal tension cracks may also form after the development of flexural cracks in which case the diagonal tension cracks are known as *flexural shear* cracks; these cracks develop at regions in a beam where shear and moment are relatively large. These diagonal *flexural shear* cracks, which are the cause of failure of most slender RC beams without web reinforcement, start to develop in the shear span at the tip(s) of the flexural cracks and then extend into the intact (uncracked) concrete between the tip(s) of the flexural cracks and the neutral axis (MacGregor, 1997) as well as between the pre-existing flexural cracks. The intact (uncracked) concrete between the tip(s) of flexural cracks and the neutral axis and between the pre-existing flexural cracks can still carry tensile stresses.

According to Eq. 1, the shear stress v may be expressed in terms of the principal tensile stress f_1 (major principal stress) and tensile stress f_t as follows:

$$v = f_1\sqrt{1-\frac{f_t}{f_1}} \qquad (2)$$

The shear stress v at a point in any cross-section in the beam may be defined as follows:

$$v = k_1\frac{V}{bd} \qquad (3)$$

where k_1 is a dimensionless “constant” introduced to account for stress concentration at a point in any cross-section as contrasted to the average shear stress over the cross-section; V is the shear force at that cross-section.

The tensile stresses f_t at the formation of diagonal tension cracks (i.e., either web shear cracks or flexural shear cracks) may be simplified as follows:

$$f_t = k_2\frac{E_c}{E_s}\frac{M}{\rho bd^2} \qquad (4)$$

where k_2 is a dimensionless “constant” introduced to account for the fact that concrete is generally an inelastic, anisotropic and heterogeneous material and to account for the condition of combined action of moment and shear as contrasted to the elasticity theory for RC beams under pure bending moment. Equation 4 is derived assuming that the uncracked concrete below the neutral axis in both cases of web shear cracks or flexural shear cracks can still carry tensile stresses as stated above.

Diagonal tension cracks can be expected to form when concrete principal tensile stress f_1 reaches the tensile strength of concrete f'_{ct}, which may, according to Nilson and Winter (1993), vary from 3 to $5\sqrt{f'_c}$ in psi units (from 0.25 to $0.42\sqrt{f'_c}$ in MPa units). Hence, the tensile strength of concrete may also be expressed as follows:

$$f_1 = k_3\sqrt{f'_c} \qquad (5)$$

where k_3 is a “constant” that may vary from 2.5 to 5 in psi units (from 0.21 to 0.50 in MPa units). This range is also confirmed below by utilizing extensive experimental data obtained from literature for laboratory test beams.

The modulus of elasticity of concrete may be expressed as follows (Bentz et al., 2006; ACI 318-11):

$$E_c = k_4\sqrt{f'_c} \qquad (6)$$

Based on Eqs. 3 to 6, the shear stress at which diagonal crack start to develop may be expressed as follows:

$$\frac{V_{cr}}{bd} = m_1\sqrt{f'_c}\sqrt{1 - m_2\,\frac{M_{cr}}{E_s\rho\, bd^2}} \qquad (7)$$

where m_1 is a "constant" equal to k_3 / k_1 and m_2 is a dimensionless "constant" equal to $k_2 k_4 / k_3$. V_{cr} and M_{cr} are, respectively, the applied shear force and bending moment at any cross-section of the beam at the formation of the diagonal tension cracks.

Equation 7, which represents a shear-moment interaction model, involves two "constants", m_1 and m_2, to be determined. The value of m_1 may be obtained from Eq. 7 by testing beams with large shear and small or near zero moment (practically pure shear) or considering sections near or close to the supports (Fig. 1). In these cases, experimental results show that the value of m_1 may range from 2.5 to 5 in psi units (from 0.21 to 0.42 in MPa units), as may be deduced from Fig. 3. On the other hand, the value of m_2 may be obtained from Eq. 7 by testing beams using large moment relative to shear (close to the pure bending span of Fig. 1). However, m_2 may be evaluated from the nominal moment capacity of beams in which flexural failure is ensured. Based on Eq. 7, m_2 may be expressed as follows:

$$m_2 = \frac{E_s\rho\, bd^2}{M_{no}} \qquad (8)$$

where M_{no} is the beam nominal moment capacity. For flexural tension failure (i.e., $\rho < \rho_{bal}$; ρ_{bal} is the balance steel ratio), the beam nominal moment capacity may be given by the following expression (ACI 318-11):

$$M_{no} = \rho\, bd^2 f_y\left(1 - 0.59\rho\ f_y / f'_c\right) \qquad (9)$$

Utilizing Eqs. 8 and 9, m_2 may be expressed as follows:

$$m_2 = \frac{E_s}{f_y\left(1 - 0.59\rho\ f_y / f'_c\right)} \qquad (10)$$

Equation 10 can directly be used for determining m_2 for the beam under consideration based on its own values of f_y, f'_c and ρ. Hence, m_1 is the only constant to be determined for evaluating the shear capacity of slender RC beams using the proposed model (Eq. 7).

Based on the possible range of m_1, described earlier, three curves obtained by the proposed model (Eq. 7) using three values of m_1 (i.e., 2.5, 3.5 and 5 in psi units: 0.21, 0.29 and 0.42 in MPa units, representing lowest, average and highest possible values) and an average value of $m_2 = 500$ is compared to experimental results (Bower and Viest, 1960; Cossio and Siess, 1960; ACI-ASCE, 1962) in Fig. 3. Based on this comparison, the following observations may be noted.

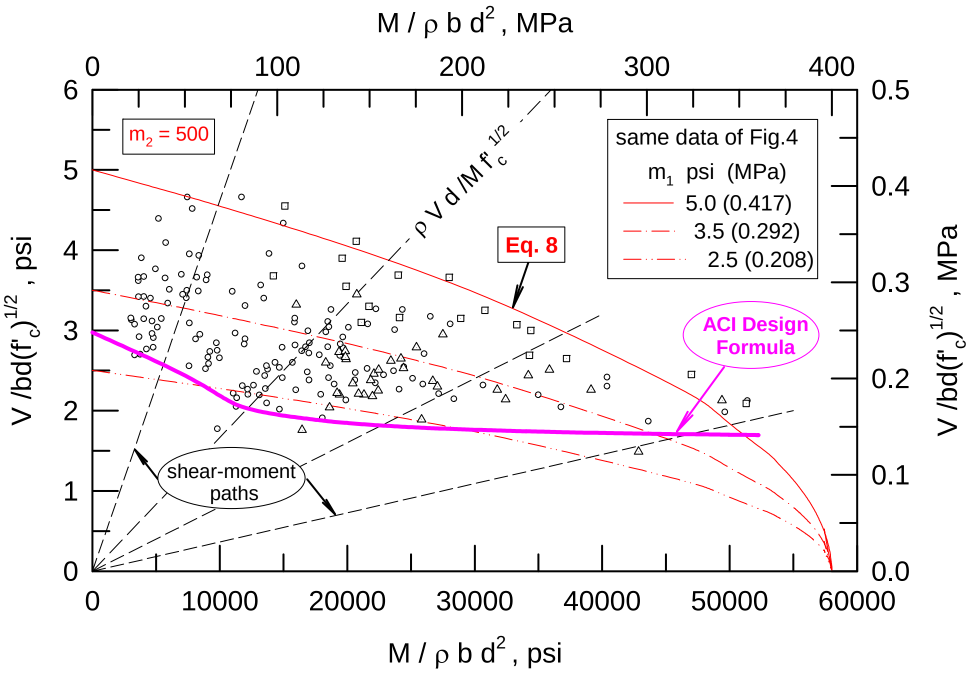

Figure (3): Shear-moment interaction diagram showing variation of $V_u / \sqrt{f_c'}\,bd$ vs. $M_u/\rho\rho b^2$ for different values of m_1 for the proposed model (Eq. 7) and ACI (Eq. 18) with experimental results

Firstly, the majority of the experimental data is within the limits predicted by the proposed model with varying m_1. This implies that the relatively large variations in the shear strength may be attributed to the variation of the concrete tensile strength. ***Secondly***, for a given beam under a specific loading configuration, the value of $\rho Vd / M\sqrt{f_c'}$ becomes constant as depicted by the dashed lines in Fig. 3. These dashed lines represent shear-moment paths followed to bring the beams to failure. A shear-moment path represents the rate of increase in shear relative to the rate of increase in moment during loading a beam to failure. A large value of $\rho Vd / M\sqrt{f_c'}$ for a given beam implies that the applied shear increases more rapidly than the applied moment and thus the shear strength may be reached before the moment capacity is fully mobilized. On the other hand, a small value of $\rho Vd / M\sqrt{f_c'}$ implies that the applied moment increases more rapidly than the applied shear and thus the moment capacity may be reached before the shear strength is fully mobilized. Hence, the rate of increase in shear relative to the rate of increase in moment, reflected in the magnitude of $\rho Vd / M\sqrt{f_c'}$, controls the formation of cracks, shear capacity and failure mode. ***Thirdly***, the proposed model and experimental data show a tendency for the shear strength to decrease below the lower limit set by the ACI code as the steel ratio decreases confirming the observed behavior of slender RC beams.

Diagonal Tension Capacity As a Unique Function of $\rho Vd/M\sqrt{f_c'}$

Equation 7 may be rearranged such that it represents a quadratic equation in terms of $\rho Vd / M\sqrt{f_c'}$ as follows:

$$\left(\frac{V_{cr}}{bd\sqrt{f_c^{'}}}\right)^2 + m_1^2 m_2 \frac{M_{cr}\sqrt{f_c^{'}}}{E_s \rho\, V_{cr} d}\left(\frac{V_{cr}}{bd\sqrt{f_c^{'}}}\right) - m_1^2 = 0 \quad (11)$$

An explicit solution of this quadratic equation may be written in the following form:

$$\frac{V_{cr}}{bd\sqrt{f_c^{'}}} = \sqrt{m_1^2 + \left(\frac{m_1^2 m_2}{2E_s}\left(\frac{\sqrt{f_c^{'}}M_{cr}}{\rho\, V_{cr} d}\right)\right)^2} - \frac{m_1^2 m_2}{2E_s}\left(\frac{\sqrt{f_c^{'}}M_{cr}}{\rho\, V_{cr} d}\right) \quad (12)$$

where m_2 can be calculated by Eq. 10.

Equation 12 shows that, for a given m_1 value, the shear capacity defined as $V/\sqrt{f_c^{'}}bd$ is a unique function of the variable $\rho Vd/M\sqrt{f_c^{'}}$, as demonstrated in Fig. 4. Figure 4 shows that the proposed model can completely predict the trend, variation and range of experimental results utilizing the possible range of m_1 values reported earlier in this study. Figure 4 also shows that, for given values of $\rho Vd/M\sqrt{f_c^{'}}$ and m_2, the variation of $V/\sqrt{f_c^{'}}bd$ is only a function of m_1, which reflects the variation of the concrete tensile strength and stress concentration as described earlier.

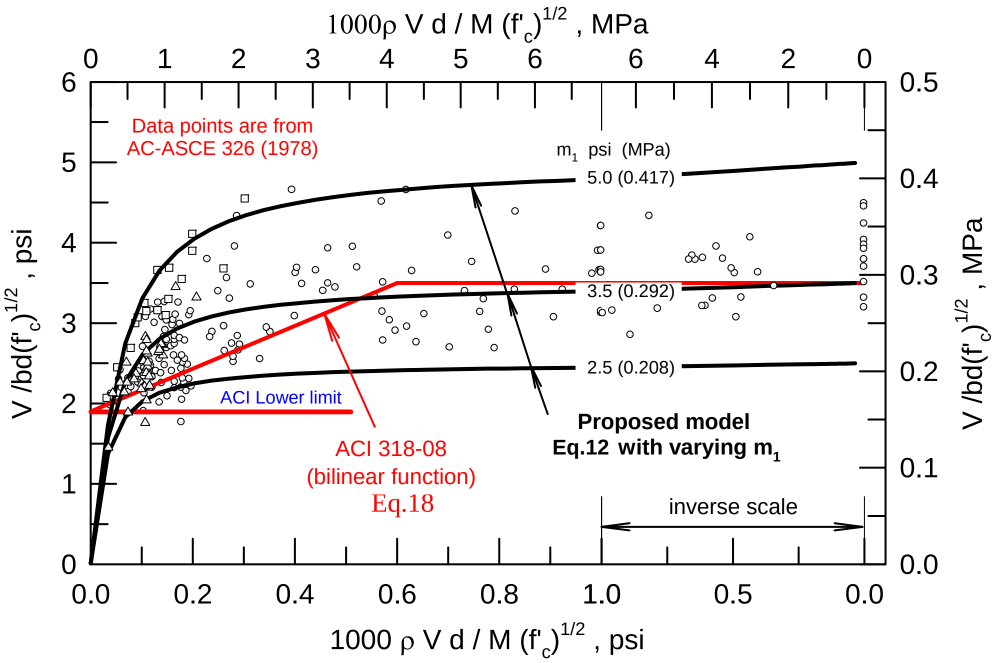

Figure (4): Variation of $V/\sqrt{f_c^{'}}bd$ as a function of $\rho Vd/M\sqrt{f_c^{'}}$ for the proposed model (Eq. 12) and ACI 318 (2011) (Eq. 18) compared to experimental results

Diagonal Tension Capacity As a Function of Shear Span to Depth Ratio *(a/d)*

For RC beams without web reinforcement, loaded with concentrated loads, Eq. 12 may be rearranged in order to express the shear capacity at the formation of diagonal tension cracks in terms of the shear span to depth ratio a/d (where $a = M_{cr}/V_{cr}$), concrete compressive strength f'_c and steel ratio ρ as follows:

$$\frac{V_{cr}}{bd\sqrt{f_c^{'}}} = \sqrt{m_1^2 + \left(\frac{m_1^2 m_2 \sqrt{f_c^{'}}}{2E_s\rho}\left(\frac{a}{d}\right)\right)^2} - \frac{m_1^2 m_2 \sqrt{f_c^{'}}}{2E_s\rho}\left(\frac{a}{d}\right) \quad (13)$$

Diagonal Tension Capacity As a Function of Beam Depth (Size Effect)

Experimental and analytical studies on RC beams of different sizes (Leonhardt and Walther, 1962; Taylor, 1972; Kani, 1967; Bazant and Kim, 1984; Vecchio and Collins, 1986; Shioya et al., 1989; Bazant and Kazemi, 1991; ACI-ASCE 445-1998; Bazant and Yu, 2005) show that there is a very significant size effect on the shear strength of members without web reinforcement. Because of its simplicity, the size effect law (Bazant and Kim, 1984) is utilized herein for modifying the proposed model in order to take the size effect into account. This size effect law can be incorporated in the proposed model such that the basic shear formula (without size effect) of the proposed model (e.g., Eq. 12 or 13) is simply multiplied by the size effect factor ξ as follows:

$$\frac{V_{cr}}{bd\sqrt{f_c^{'}}} = \xi\left(\sqrt{m_{10}^2 + \left(\frac{m_{10}^2 m_2}{2E_s}\left(\frac{\sqrt{f_c^{'}}M_{cr}}{\rho\, V_{cr} d}\right)\right)^2} - \frac{m_{10}^2 m_2}{2E_s}\left(\frac{\sqrt{f_c^{'}}M_{cr}}{\rho\, V_{cr} d}\right)\right) \quad (14)$$

$$\frac{V_{cr}}{bd\sqrt{f_c^{'}}} = \xi\left(\sqrt{m_{10}^2 + \left(\frac{m_{10}^2 m_2 \sqrt{f_c^{'}}}{2E_s\rho}\left(\frac{a}{d}\right)\right)^2} - \frac{m_{10}^2 m_2 \sqrt{f_c^{'}}}{2E_s\rho}\left(\frac{a}{d}\right)\right) \quad (15)$$

$$\xi = \frac{1}{\sqrt{1 + d/d_o}} \quad (16)$$

where d_0 is defined in terms of the maximum aggregate size d_a as $d_0 = 25d_a$ (Bazant and Kim, 1984). Note that m_1 is modified to m_{10} because of introducing the size effect factor ξ . m_{10} corresponds to the value of m_1 for very small beams (i.e., when d approaches zero or ξ approaches unity). Still, the value of m_2 can be directly calculated by Eq. 10.

A comparison of the proposed model (Eq. 14 or 15) with experimental results of geometrically similar beams (e.g., Shioya et al., 1989; Bazant and Kazemi, 1991; Ramallo et al., 1995) generally indicates that m_1 decreases as the beam size increases. This comparison also indicates that m_1 approaches about 5 in psi units (0.42 in MPa units) for the smallest beams. It is thus suggested that m_{10} (to be used in Eq. 14 or 15) is equal to 5 in psi units (0.42 in MPa units) for the slender RC beams considered in this study without web reinforcement and axial forces. This suggestion is supported by the range of m_1 that may be deduced from Fig. 3. It should also be pointed out that the concrete tensile strength in the MCFT (Vecchio and Collins, 1986) is taken as a function of strain such that as the strain approaches zero, the concrete tensile strength approaches $4\sqrt{f_c^{'}}$ in psi units, which is generally attained for small beams.

Hence, the shear capacity of slender RC beams ($a/d \geq 2.5$) without web reinforcement and axial forces that takes the size effect into account may be expressed as follows:

$$\frac{V_{cr}}{bd\sqrt{f_c^{'}}} = \frac{1}{\sqrt{1+d/25d_o}}\left(\sqrt{25+\left(12.5\frac{m_2\sqrt{f_c^{'}}}{E_s\rho}\left(\frac{a}{d}\right)\right)^2} - 12.5\frac{m_2\sqrt{f_c^{'}}}{E_s\rho}\left(\frac{a}{d}\right)\right) \quad psi$$

$$\frac{V_{cr}}{bd\sqrt{f_c^{'}}} = \frac{1}{\sqrt{1+d/25d_o}}\left(\sqrt{0.174+\left(0.088\frac{m_2\sqrt{f_c^{'}}}{E_s\rho}\left(\frac{a}{d}\right)\right)^2} - 0.088\frac{m_2\sqrt{f_c^{'}}}{E_s\rho}\left(\frac{a}{d}\right)\right) \quad MPa \qquad (17)$$

Equation 17 is used in the following section for validating and investigating the effects of the basic variables on the shear capacity of RC beams without web reinforcement.

Validity of the Proposed Model: Comparison with Experimental Results and Existing Models

The diagonal tension cracking capacity and ultimate shear strength of slender RC beams involve four basic variables (longitudinal steel ratio ρ, concrete compressive strength $f_c^{'}$, shear span to depth ratio a/d and beam size d). Several sets of existing experimental results on diagonal tension cracking and ultimate shear strengths are utilized herein in order to show the capability of the proposed model in predicting the observed variations in the experimental results of slender RC beams without web reinforcement. Each set was carefully selected in order to examine the influence of one basic variable at a time. Furthermore, three expressions for computing the shear capacity of slender RC beams without web reinforcement (ACI 318-11; Vecchio and Collins, 1986; Zararis and Padadakis, 2001) have been selected in order to verify and compare the proposed model of this study with these existing models.

ACI Code Formula (ACI 318-11)

The ACI code (ACI 318-11) has accepted the following empirical bilinear expression, depicted in Fig. 3, as the nominal shear strength at which diagonal flexural shear cracking develops in beams without web reinforcement.

$$\frac{V}{bd\sqrt{f_c^{'}}} = \left(1.90 + 2500\frac{\rho V d}{M\sqrt{f_c^{'}}}\right) \le 3.5 \quad [psi]$$

$$\frac{V}{bd\sqrt{f_c^{'}}} = \left(0.16 + 17\frac{\rho V d}{M\sqrt{f_c^{'}}}\right) \le 0.29 \quad [MPa] \qquad (18)$$

This expression sets a lower limit of 1.9 in psi units (0.16 in MPa units) at locations of large moment relative to shear and an upper limit of 3.5 in psi units (0.29 in MPa units) at locations of large shear relative to moment for the value of shear capacity defined as $V/bd\sqrt{f_c^{'}}$. On the other hand, the proposed model reveals that as the value of $\rho Vd/M\sqrt{f_c^{'}}$ decreases, the shear capacity decreases without limits as shown in Fig. 4 and may theoretically approach zero for small values of $\rho Vd/M\sqrt{f_c^{'}}$, which can be attained by large moment compared to shear or by relatively small steel ratios. The ACI code expression may become unsafe at relatively low values of $\rho Vd/M\sqrt{f_c^{'}}$, because the value of $V/bd\sqrt{f_c^{'}}$ may drop below the ACI code lower limit.

Model of Zararis and Papadakis (2001)

Zararis and Papadakis (2001) suggested the following form for the ultimate shear strength of slender RC beams ($a/d \ge 2.5$) without web reinforcement, including size effect.

$$v_u = \frac{V}{bd} = \lambda_d \frac{c}{d} f_{ct} \qquad (19)$$

$$\lambda_d = \left(1.2 - 0.2\frac{a}{d}d\right) \geq 0.65 \quad (20)$$

$$\left(\frac{c}{d}\right)^2 + 600\frac{\rho}{f_c'}\frac{c}{d} - 600\frac{\rho}{f_c'} = 0 \quad (21)$$

$$f_{ct} = 0.30\left(f_c'\right)^{2/3} \quad (22)$$

where d is in meters and f_c' in MPa.

Diagonal Cracking Capacity and Shear Strength Shear Span – Depth Ratio (*a/d*)

Figure 5 compares the variation of $V/\sqrt{f_c'}bd$ as a function of a/d value of the proposed model (Eq. 17) with experimental results of Kani (1967) as well as with the model of Zararis and Papadakis (2001). Other variables (ρ, f_c', and d) are being practically constant as shown in the figures. The shear strength decreases drastically as the a/d value increases from 1 to about 2.5 after which (i.e., $a/d \geq 2.5$) the shear strength decreases more slowly.

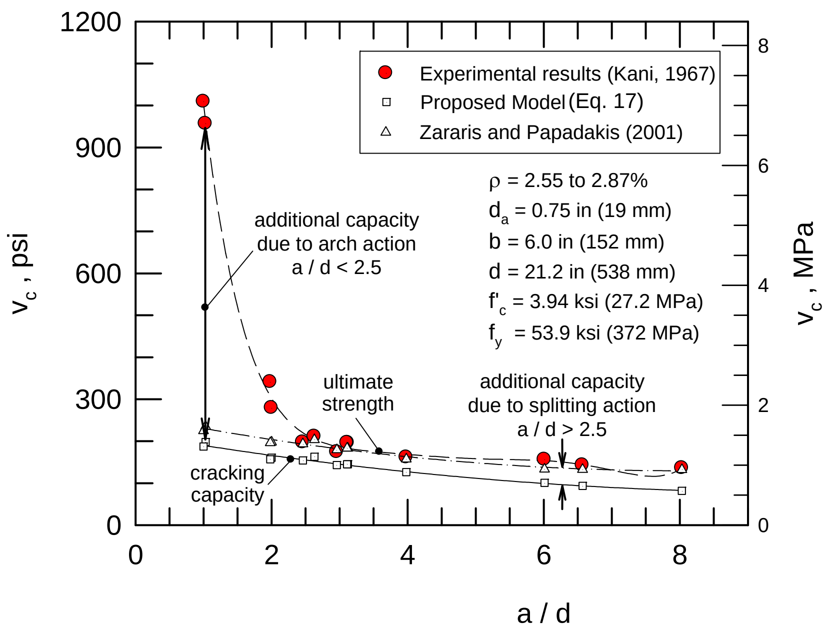

Figure (5): Comparison of the proposed model with experimental results of Kani (1967) for different values of *a/d* from 1 to 8

The proposed model can reliably be used to predict the trend of shear capacity of slender RC beams ($a/d \geq 2.5$) as compared to the experimental results as well as to the model of Zararis and Papadakis (2001). The shear capacity predicted by the proposed model (Eq. 17) at the formation of diagonal tension cracking may generally be considered as a lower limit for the experimental shear strength of slender RC beams ($a/d \geq 2.5$). However, the experimental shear strength is higher than those predicted by the proposed model at diagonal tension cracking by only about 10%-15%. Zsutty (1968) also reported that the shear strength is in the range of zero to 10-15% of cracking capacity and stated that the slender RC beams were not stable following the formation of diagonal tension cracks. For this reason, the shear capacity of slender RC beams ($a/d \geq 2.5$) without web reinforcement is usually taken equal to the inclined cracking shear, because such beams fail when inclined cracking occurs or shortly afterwards (MacGregor, 1997). On the other hand, Fig. 5 shows that deep or short RC beams having $a/d < 2.5$ can carry additional loads in excess to those causing the formation of diagonal tension cracking by arch action. Strut and tie models may generally be used for this purpose (e.g., the strut-and-tie method according to ACI 318-11, Appendix A; ACI-ASCE Committee 445, 1999; Park and Kuckma, 2007).

Influence of Steel Ratio ρ on Shear Capacity (Diagonal Tension Capacity)

Figure 6 compares the variation of $V/\sqrt{f_c^{'}}bd$ as a function of steel ratio ρ obtained by the proposed model (Eq. 17) with experimental results of Krefeld and Thurston (1966) including diagonal cracking capacity and ultimate shear strength, as well as with the model of Zararis and Papadakis (2001). As can be seen, the general trend of the experimental results can be predicted quite reasonably utilizing the two models. However, the proposed model again provides a lower limit for the experimental results as may be expected, since the formation of diagonal cracks generally precedes the beam failure. On the other hand, the Zararis and Papadakis (2001) model, which was developed for ultimate shear strength of slender RC beams, provides an upper limit for the observed shear strength presented in Fig. 6. The experimental results (cracking capacity and ultimate shear strength) lie in the range furnished by the proposed model developed at the formation of diagonal tension cracks and the model of Zararis and Papadakis (2001) developed for ultimate shear strength.

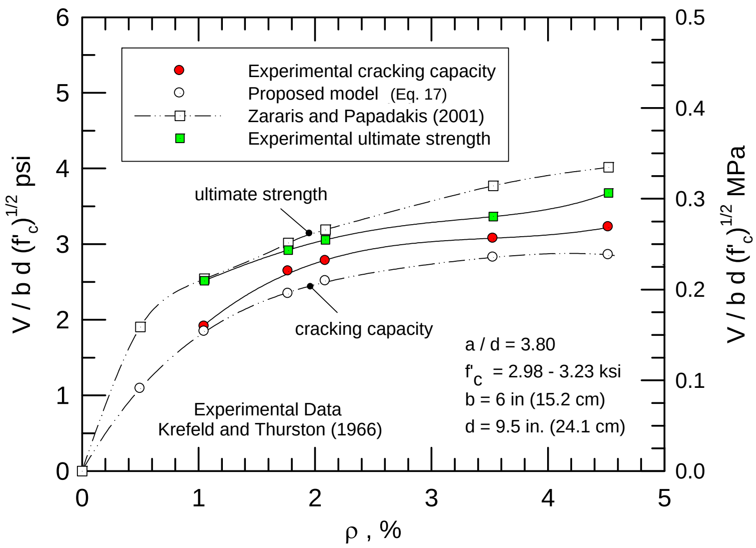

Figure (6): Comparison of the proposed model with experimental results of Krefeld and Thurnston (1966) and the model of Zararis and Papadakis (2001) for varying ρ

Influence of Concrete Compressive Strength $f_c^{'}$ on Shear Strength

Figure 7 compares the variation of $V/\sqrt{f_c^{'}}bd$ as a function of $f_c^{'}$ obtained by the proposed model (Eq. 17) with experimental results of Mphonde and Frantz (1984) as well as with the model of Zararis and Papadakis (2001). Figure 7 includes both low and high concrete compressive strengths in the range $1500 < f_c^{'} < 13500$ psi ($10.5 < f_c^{'} < 95$ MPa). Figure 7 shows that the two models can reasonably predict the general behavior of shear capacity with f_c'. Figure 7 also shows that the shear capacity varies in the range defined by the proposed model developed at the formation of diagonal tension cracking (as a lower limit) and the Zararis and Papadakis (2001) model developed for ultimate shear strength (as an upper limit).

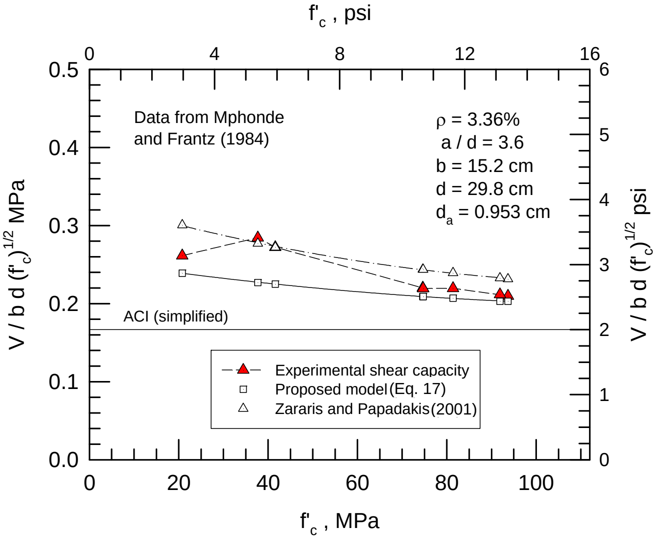

Figure (7): Comparison of the proposed model with experimental results of Mphonde and Frantz (1984) and the model of Zararis and Papadakis (2001) for varying f_c'

Shear Capacity with Beam Size (d)

The variations of $V/\sqrt{f_c'}bd$ with the beam depth d obtained by the proposed model (Eq. 17) are compared with the experimental results of Bhal (1968) with $30 \le d \le 120$ cm in Fig. 8. The proposed model can reasonably predict the general trend of shear capacity with the variation of the depth size d and provides a lower limit for the experimental values of shear strength. The Zararis and Papadakis (2001) model also provides quite comparable results with the experimental values.

Another set of experimental results on a series of geometrically similar beams (Shioya et al., 1989) is utilized in the following section, in order to show the capability of the proposed model in predicting shear capacity of large beams (with d up to 3 m) and in order to compare the proposed model with other existing models (ACI 318-11; Zararis and Papadakis, 2001; MCFT of Vecchio and Collins, 1986).

Proposed Design Formula

In order to be used for design purposes, the proposed model must be modified by assigning appropriate values for m_1, which reflects the variation of the concrete tensile strength, and m_2, which reflects the variation of the nominal flexural strength. By introducing appropriate reduction factors for shear capacity and nominal flexural strength, a design formula for reinforced concrete beams without web reinforcement under combined action of shear and moment may be expressed in a general form that takes all the basic variables (ρ , f_c', M/Vd and ξ) into account as follows:

$$\frac{V_d}{bd\sqrt{f_c'}} = \xi\left(\sqrt{\phi_S^2 m_{10}^2 + \left(\frac{\phi_S^2 m_{10}^2 m_2}{2\phi_M E_s}\left(\frac{\sqrt{f_c'}M}{\rho\, Vd}\right)\right)^2} - \frac{\phi_S^2 m_{10}^2 m_2}{2\phi_M E_s}\left(\frac{\sqrt{f_c'}M}{\rho\, Vd}\right)\right) \qquad (23)$$

where ϕ_S and ϕ_M are reduction factors for shear strength and moment capacity, respectively. If a value of 0.85 for ϕ_S and a value of 0.90 for ϕ_M, similar to those of the ACI code (ACI 318-11), are used in the proposed model, the following formula may be suggested for the design of slender RC beams without web reinforcement ($m_{10} = 5$ in psi units: 0.42 in MPa units):

$$\frac{V_d}{bd\sqrt{f_c^{'}}} = \frac{1}{\sqrt{1+d/25d_o}}\left(\sqrt{18+\left(10\frac{m_2}{E_s}\left(\frac{\sqrt{f_c^{'}}M}{\rho\, Vd}\right)\right)^2} - 10\frac{m_2}{E_s}\left(\frac{\sqrt{f_c^{'}}M}{\rho\, Vd}\right)\right) \quad \text{in psi units}$$

$$\frac{V_d}{bd\sqrt{f_c^{'}}} = \frac{1}{\sqrt{1+d/25d_o}}\left(\sqrt{0.125+\left(0.07\frac{m_2}{E_s}\left(\frac{\sqrt{f_c^{'}}M}{\rho\, Vd}\right)\right)^2} - 0.07\frac{m_2}{E_s}\left(\frac{\sqrt{f_c^{'}}M}{\rho\, Vd}\right)\right) \quad \text{in MPa units} \qquad (24)$$

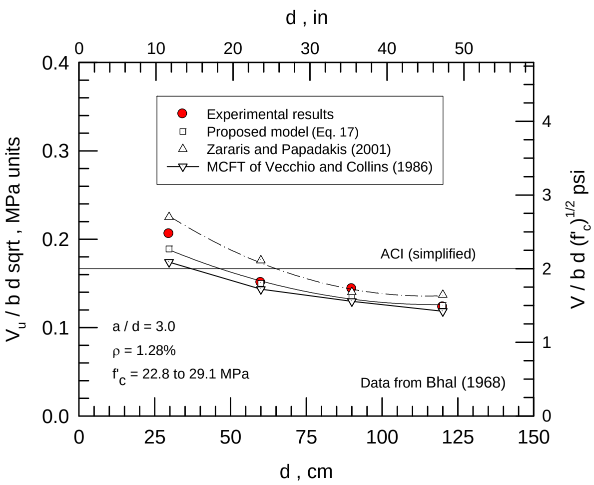

Figure (8) Comparison of the proposed model with experimental results of Bhal (1968), MCFT (1986) and model of Zararis and Papadakis (2001) for different d values

The proposed model (Eq. 24) is compared to experimental results of a series of geometrically similar beams (Shioya et al., 1989), as well as with the MCFT model in Fig. 9, which depicts the variations of the shear capacity $V/bd\sqrt{f_c^{'}}$ as a function of the beam depth d using the properties of each individual beam. As can be seen, the proposed design model gives quite comparable results to those measured by Shioya et al. (1989) and models of Zararis and Papadakis (2001) and MCFT of Vecchio and Collins (1986) and thus can reliably be used for the design of slender RC beams of up to 3 m depth. The MCFT predictions were obtained utilizing a spread sheet using the general methodology described in Bentz and Collins (2004).

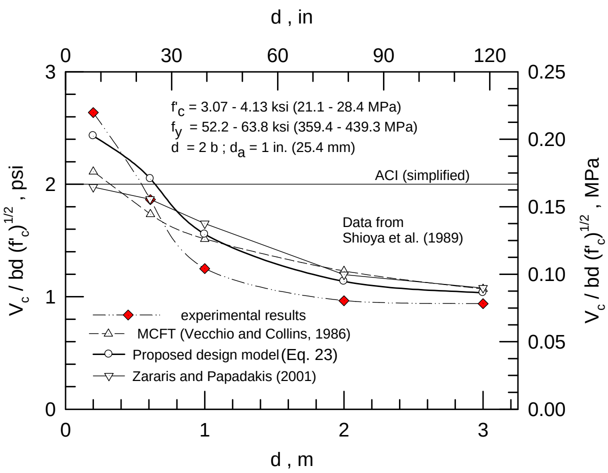

Figure (9): Comparison of the proposed model with experimental results of Shioya et al. (1989) for different values of d

SUMMARY AND CONCLUSIONS

A model derived based on mechanics analysis and verified by existing experimental data is proposed for evaluating the shear capacity of slender RC beams at the formation of diagonal tension cracks and for assessing the effect of four basic variables (steel ratio ρ, concrete strength $f_c^{'}$, shear span to depth ratio a/d and beam size d). This model shows that the shear strength of a beam is controlled by the shear-moment path followed to bring the beam to failure as reflected in the value of $\rho Vd/M\sqrt{f_c^{'}}$, which represents the rate of increase in shear relative to the rate of increase in moment during loading the beam to failure. The proposed model involves only two "constants" that are evaluated using experimental results for two specific well-known cases (pure bending moment and pure shear). Based on the results of the present study, the following conclusions are drawn:

- The proposed model predicts and explains the observed behavior (trend and magnitude) of slender RC beams ($a/d > 2.5$) without web reinforcement at the formation of diagonal cracks for each one of the four different basic variables (ρ, $f_c^{'}$, a/d and d).
- The large variations observed in the experimental results of shear capacity at the formation of diagonal tension cracks may essentially be explained by the variations of the concrete tensile strength.
- The proposed model developed at the formation of diagonal tension cracks and the model of Zararis and Papadakis (2001) developed at ultimate failure as well as the MCFT (1986) predict and explain the observed behavior of slender RC beams ($a/d > 2.5$) without web reinforcement.

The proposed design model (Eq. 23) can be flexible in reflecting the actual properties of beams under consideration by allowing the designer to select appropriate values of m_1 and m_2 in the ranges provided by this study, to ensure the safety of the design.

REFERENCES

AASHTO LRFD. (2004). “Bridge design specification and commentary”. 3rd Edition. American Association of State Highway Transportation Officials, Washington, D.C., , 1264 pp.

ACI 318. (2011). “Building code requirements for structural concrete (ACI318-11) and commentary (318R-11)”. American Concrete Institute, Farmington Hills, Mich., 430 pp.

ACI-ASCE Committee 326. (1962). "Shear and diagonal tension". ACI Journal, Proceedings, 59 (1), 1-30 and 59 (2), 277-344.

ACI-ASCE Committee 445 (1998). "On shear and torsion: recent approaches to shear design of structural concrete". ASCE Journal of Structural Engineering, 124 (12), 1375-1417.

Bazant, Z.P., and Kazemi, M.T. (1991). ‘‘Size effect of diagonal shear failure of beams without stirrups”. ACI Structural Journal, 88 (3), 268-276.

Bazant, Z.P., and Kim, J.K. (1984). "Size effect of longitudinally reinforced beams". ACI Journal, 81 (5), 456-468.

Bazant, Z.P., and Yu, Q. (2005). "Designing against size effect on shear strength of reinforced concrete beams without stirrups: i. formulation and ii. verification and calibration". The Journal of Structural Engineering, 131 (12), 1877-1885 and 1886-1897.

Bentz, E.C., and Collins, M.P. (2006). "Development of the 2004 Canadian Standards Association {CSA} A23.3: Shear provisions for reinforced concrete". Canadian Journal of Civil Engineering, 33, 521-534.

Bhal, N.S. (1968). ‘‘Über den Einfluss der Balkenhöhe auf Schubtragfähigkeit von einfeldrigen Stahlbetonbalken mit und ohne Schubbewehrung’’. PhD Dissertation, Universität Stuttgart, Stuttgart, Germany (in German).

Bower, J.E., and Viest, I.M. (1960). "Shear strength of restrained concrete beams without web reinforcement." ACI Journal, 73-98.

Bukhari, Imran A., and Ahmad, Saeed. (2008). "Evaluation of shear strength of high-strength concrete beams without stirrups". Arabian Journal for Science and Engineering, 33 (2B), 321-336.

Collins, M.P., and Kuchma, D.A. (1999). "How safe are our large, lightly reinforced concrete beams, slabs and footings?" ACI Structural Journal, 96 (4), 282-290.

Cossio, R.D., and Siess, C.P. (1960). "Behavior and strength in shear of beams and frames without web reinforcement". ACI Journal, 695-735.

CSA Committee A23.3. (2004). "Design of concrete structures (CSA A23.3-04)". Canadian Standards Association, Mississauga, 214 pp.

Kani, G.N.J. (1967). "How safe are our large reinforced concrete beams?". ACI Journal, Proceedings, 64 (3), 128-141.

Krefeld, W.J., and Thurston, C.W. (1966). "Studies of shear and diagonal tension strength of simply supported reinforced concrete beams." ACI Journal, 63 (4), 451-476.

Leonhardt, F., and Walther, R. (1962). "Beiträge zur Behandlung der Schubprobleme in Stahlbetonbau". Beton-und Stahlbetonbau (Berlin), March 54-64 and June 141-149.

MacGregor, J.G. (1997). "Reinforced concrete mechanics and design". 3rd Edition. Prentice-Hall, Eaglewood Cliffs, N.J.

Mphonde, A.G., and Frantz, G.C. (1984). ''Shear tests of high- and low-strength concrete beams without stirrups''. ACI Journal, 81 (4), 350-357.

Nelson, A., and Winter, G. (1993). "Design of concrete structures". 11th Ed., McGraw-Hill.

Park, J.W., and Kuckma, D. (2007). "Strut and tie model for strength prediction of deep beams". ACI Structural Journal, 104 (6), 657-666.

Ramallo, J.C., Kotsovas, M., and Danesi, R. (1995). "Unintended out-of-plane actions in size effect tests of structural concrete". Transactions, 13th SMIRT, 13-18 Ago. 351-357, Brasil.

Reineck, K.H. (1991). ''Ultimate shear force of structural concrete members without transverse reinforcement derived from a mechanical model''. ACI Structural Journal, 88 (5), 592-602.

Russo, G., Somma, G., and Mitri, D. (2005). "Shear strength analysis and prediction for reinforced concrete beams without stirrups". ASCE Journal of Structural Engineering, 131 (1), 66-74.

Shioya, T., Iguro, M., Nojiri, Y., Akiayma, H., and Okada, T. (1989). "Shear strength of large reinforced beams". Fracture Mechanics: Application to Concrete, SP-118, V.C.

Taylor, H.P.J. (1972). ''Shear strength of large beams''. Journal of Structural Division, ASCE, 98 (11), 2473-2490.

Vecchio, F.J., and Collins, M.P. (1986). "The modified compression field theory for reinforced concrete elements subjected to shear". ACI Journal, 83 (2), 219-231.

Walraven, J.C. (1978). ''The influence of depth on the shear strength of lightweight concrete beams without shear reinforcement''. Stevin Lab. Rep. No. 5-78-4, Delft University of Technology, Delft, the Netherlands.

Walraven, J., and Lehwalter, N. (1994). "Size effect in short beams loaded in shear". ACI Structural Journal, 91 (5), 585-593.

Zararis, P.D. (2005). "Shear compression failure in reinforced concrete deep beams". ASCE Journal of Structural Engineering, 129 (4), 544-553.

Zararis, P.D., and Papadakis, G. (2001). "Diagonal shear failure and size effect in RC beams without web reinforcement". ASCE Journal of Structural Engineering, 127 (7), 733-742.

Zsutty, T.C. (1968). "Beam shear strength prediction by analysis of existing data". ACI Journal, 65 (11), 943-951.