Analysis of Anchored Slopes Using the Strength-reduction Method and Limit-equilibrium Method

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ABSTRACT

Ground anchors are used as earth reinforcement to stabilize unstable landslides and slopes. In this paper, the effect of some parameters, such as the anchor inclination angle, position of the anchor and the number of the anchor's rows, was investigated using the finite element strength-reduction method. The results of the finite element strength-reduction method were compared with the results of the limit-equilibrium methods (Bishop's method and Spencer's method). The results of this analysis revealed that the finite element strength-reduction method (FESRM) is very applicable to evaluating the stability of reinforced and unreinforced slopes. The angle ranging from 10° to 15° is the optimum inclination angle to increase the factor of safety and decrease the displacement. The anchor position had a considerable effect on the determination of the critical slip surfaces and the optimum location of the anchor is at (x/L) =0.4, where (x) is the horizontal distance of the anchor head from the slope toe and (L) is the horizontal distance from the toe to the heel of the slope. Also, reinforcing the slope by two rows of anchors increases the factor of safety by about 10%.

KEYWORDS: Anchors, Slope stability, Limit-equilibrium method, Strength-reduction method.

INTRODUCTION

Stability of earth dams' slopes and natural slopes is a very important consideration in design and construction, where catastrophic results can occur due to the failure of slopes (Murthy, 2003). Many factors contribute to the failure of slopes, like geologic, hydrodynamic, geotectonic, erosion and human activities (Massannat, 2011).

Ground anchors are used as earth reinforcement to stabilize unstable landslides and slopes. In the last thirty years, the ground anchors technique has been widely used to sustain the stability of the slopes. The restraint force required to keep the slopes stable can be calculated using limit-state equilibrium (Sabatini et al., 1999).

Stabilizing the slopes and landslides by anchors has many advantages, like reducing cost and reducing construction time (Tan et al., 2019). Abramson et al. (1996) and Bromhead (1992) studied stabilization methods of slopes. Cai and Ugai (2003) studied the anchored slopes using the finite element method, where the factors of safety of anchored slopes were determined and compared with the factors of safety determined by limit-equilibrium methods. Although the finite element method has more advantages than limit-equilibrium methods, most of the engineering codes of slope stability depend on limit-equilibrium methods, because they need fewer geotechnical parameters and give acceptable results in terms of accuracy in comparison with the finite element method (Zheng et al., 2005).

Many researchers have carried out stability analysis of anchored slopes using the finite element method and limit-equilibrium methods (Li et al., 2015; Zhu et al., 2005; Li et al., 2012; Rui et al., 2016). Hryciw (1991) concluded the optimum orientation of anchors in sandy soil using theoretical analysis for infinite slope. Based on the limit-equilibrium method, Yamagami and Yamakawa (1990) developed a method to determine the anchor axial tension required to keep the slopes stable. In the present study, the effect of anchorage parameters on the stability of slopes reinforced by anchors, such as

Received on 12/2/2022.
Accepted for Publication on 9/5/2022.

the anchor inclination angle, position of the anchor and the number of the anchor rows (one row or two rows) was studied. A parametric study using the finite element strength-reduction method (FESRM) was undertaken and the influence of anchors on the factor of safety of slopes, displacement value and critical slip surfaces was observed. The finite element program GEO5 was adopted herein to determine the value of the factor of safety of slopes, displacement values and critical slip surfaces. Also, the results of the finite element strength-reduction method were compared with the results of the limit-equilibrium methods (Bishop's method and Spencer's method).

Limit-Equilibrium Methods (LEM)

Limit-equilibrium methods are widely used to evaluate the factor of safety of anchored slopes. The most common methods of limit-equilibrium are: Sweden arc method, method of slices, Morgenstern-Price method, Bishop simplified method and Spencer method (Liu et al., 2015). The slices method is a suitable method to study the stability of slopes reinforced with anchors. The anchor's force can be simulated as an axial tension force which is assumed to act at one of two places: at the anchor head near the ground surface or at the anchor intersect with the slip surface at the base of the slice.

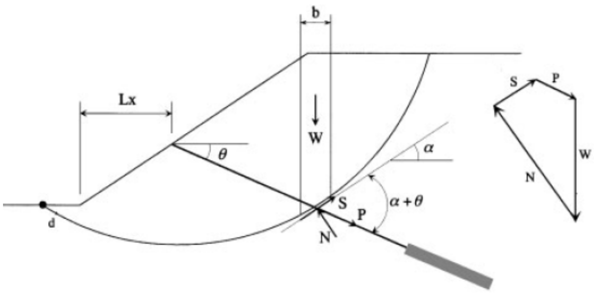


Figure (1): A schematic presentation of the characteristics of anchored slope (Cai et al., 2003)

As shown in Fig. (1), the axial tension force of the anchor at the base of the slide can be decomposed into two components: tangential to the slip surface circle at the base of the slice and vertical on the slip surface circle. Accordingly, the resistance moment can be easily calculated to determine the factor of safety.

In the case of the axial tension force of the anchor assumed to be at the anchor head, the factor safety can be calculated using the same approach (Oasys, Ltd., 2001).

The Finite Element Strength-reduction Method (FESRM)

Study stability of slopes *via* finite element programs can be considered as a mathematical process containing constraint conditions where the riskiest slip surface is determined. This problem can be solved by many intelligent algorithms. Many researchers used the technique of strength-reduction to determine the safety factor of reinforced and unreinforced slopes (Deng et al., 2007; Griffiths et al., 1999; Griffiths et al., 2007).

Cai and Ugai (2003) studied the stability of slopes reinforced by anchors using the strength-reduction method. Also, they applied the same method to analyze and study the parameters affecting the stability of slopes reinforced by piles (Cai, 2000).

When using the strength-reduction method to study the stability of slopes reinforced by anchors, the shear strength parameters which include: cohesion(c) and internal friction angle (ϕ), are reduced gradually to the value of the slope instability (Wei, 2008; Zheng et al., 2008; Shukha et al., 2008; Gurocak et al., 2008).

Therefore, the factor of safety according to this method can be calculated as follows:

$$F.S. = [\tan\phi / \tan\phi^f] = [c / c^f] \quad (1)$$

Where: ϕ= the actual value of soil friction angle.
ϕ^f = value of soil friction angle at failure.
c = the actual value of cohesion.
c^f = value of cohesion of soil at failure.

Rui et al. (2016) analyzed the stability of slopes using (FESRM) with the elastic-plastic stress-strain model and Mohr-Coulomb failure criterion.

Modeling of Anchor and Failure Criterion

The ground anchor is a structural element used to transmit the axial tensile force into the soil. The ground anchor consists of:

- Free length, which is the distance from the head point of the anchor to the point of the intersection between the anchor and the slip surface.
- Root length or bond length, as shown in Fig. (2).

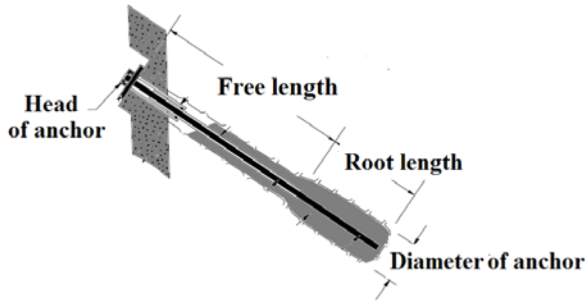


Figure (2): Geometric characteristics of the anchors used

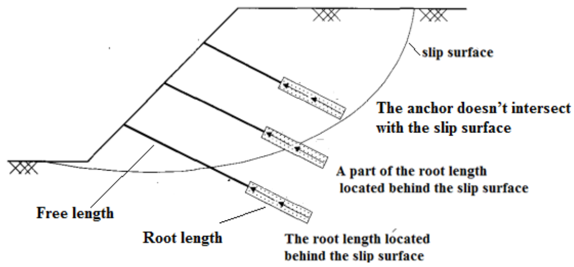


Figure (3): A schematic presentation showing different locations of anchors root length

In finite element modeling, the interaction between the free length without grouting and the soil can be neglected (Rui et al., 2016).

Generally, anchors can be classified into unstressed anchors and prestressed anchors, where the unstressed anchors depend on the downward movement of the soil to resist failure.

To model the anchor *via* the finite element program GEO5, two options are available:

a) Calculation of the length of the anchor. In this case, an infinite length of the anchor is assumed to calculate the free length. After that, the bond length can be placed behind the circle of the slip surface, which leads to increasing the factor of safety of slopes. This method is used when the minimum length of the anchor is required.

b) The length of the anchor is given. In this case, the anchor is taken into account if a part of the root length is located behind the slip surface, where only the anchors whose endpoints of root length are located behind the slip surface are considered. If the slip surface is located completely behind the free length, the anchor force is entirely considered in the calculation. If a part of the root length is located behind the slip surface, the anchor force is reduced linearly during the analysis. This option is used to evaluate the stability of the slopes with previously present anchors. The anchor is being neglected in the calculation of slope stability if the anchor doesn't intersect with the slip surface (The website of GEO5, June 2020), as shown in Fig. (3).

Modeling and Parameters

In this analysis, the model consists of two soil layers, as shown in Fig. (4). Layer (1) is soft clay soil._The model length is 30 m and its height is 7m. To avoid the boundaries effect on the slope behavior, the length of the model was taken far enough. The basic soil parameters considered in this analysis include: the unit weight, saturated unit weight, modulus of elasticity, Poisson's ratio, friction angle, cohesion and dilation angle.

The soil parameters are shown in Table 1.

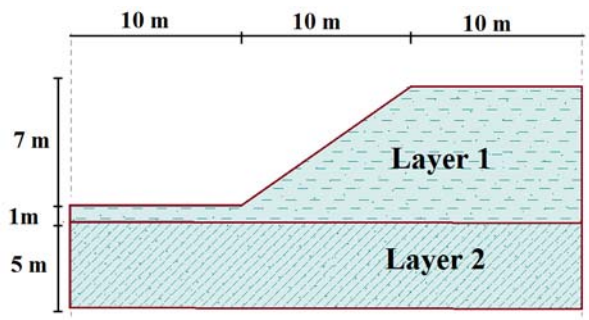


Figure (4): Geometry of the anchored slope

Table 1. Soil properties

Soil parameters	Layer 1	Layer 2
Unit weight, kN/m^3	19	20
Saturated unit weight, kN/m^3	21	22
Modulus of elasticity, MPa	20	250
Poisson´s ratio	0. 25	0. 3
Friction angle, degree	25	36
Cohesion, kPa	10	100

To ensure that there is no volumetric change in the soil when subject to shear stress, the dilatation angle is considered equal to zero for both soil layers.

The anchor properties are shown in Table 2.

Table 2. The anchor parameters

Anchor length, m	15
Inclination of anchor, degree	25°
Anchor spacing, m	1
Anchor diameter, mm	20
Pre-stressing force, kN	50
Modulus of elasticity, Mpa	210000

The head of the anchor in all analyses was taken at the mid of the horizontal dimension of the slope unless otherwise stated during the analysis. The influence of variation in anchorage parameters on the stability of anchored slopes will be investigated in this study.

Finite Element Analysis (FEA)

Analysis of the stability of slopes should be achieved using a non-linear soil model to permit the development of plastic strains. So, Drucker-Prager material model was used in this analysis. The failure surface in the Drucker-Prager model in 3D stress space is a conical failure surface. The mesh generation results according to GEO 5 finite element program are as follows: the number of nodes is 845 and the number of elements is 534 (regions 238, beams 74, interfaces 222), as shown in Fig. (5).

Generally, anchors can be classified into unstressed anchors and pre-stressed anchors, where unstressed anchors depend on the downward movement of the soil to resist failure. Herein, pre-stressed anchors were used to evaluate the stability of anchored slopes.

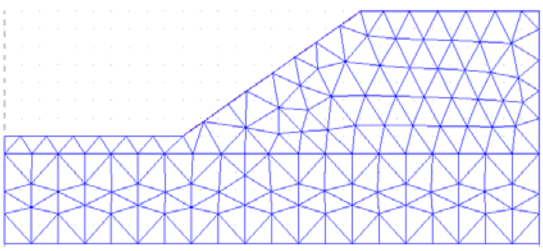


Figure (5): Mesh configuration of the anchored slope

RESULTS AND DISCUSSION

The results of the analysis using the finite element strength-reduction method and limit-equilibrium methods (Bishop and Spencer) can be summarized as follows.

Effect of Reinforcement by Anchor on Stability of Slopes

In this sub-section, the factor of safety of unreinforced slopes and slopes reinforced by anchors was calculated to evaluate the role of reinforcement by one row of anchors on stability. The results given from the finite element strength-reduction method (FESRM) and the limit-equilibrium methods (LEMs) are shown in Table 3.

Fig. (6) and Fig. (7) show the displacement and the critical slip surfaces for the slope with and without anchor reinforcement. The factor of safety given by (FESRMs) for the unreinforced slope was 1. 33 and for the reinforced slope was 1. 5.

The difference between the factor of safety determined by the finite element strength-reduction method (FESRM) and the limit-equilibrium methods (LEMs) for both reinforced and unreinforced slopes is nearly 10%. The factor of safety determined by Bishop method and Spencer method for unreinforced slope is almost equal 1. 5 and 1. 51, respectively. For reinforced slope, Spencer method gives a higher value of factor of safety than Bishop method, where the Spencer method gave up to 5% greater values than the Bishop method. While the limit-equilibrium methods are depending on choosing a number of slip surfaces arbitrarily which gives an approximate value of the factors of safety, the finite element method grants more accurate access to deformations and constraints within the soil.

From Fig. (6) and Fig. (7), it can be seen that the displacement decreased by a remarkable value due to reinforcing the slope by the anchors, where the maximum displacement decreased from 212. 5 mm to 62. 9 mm. In the unreinforced slope, the largest value of displacement was at the top of the slope.

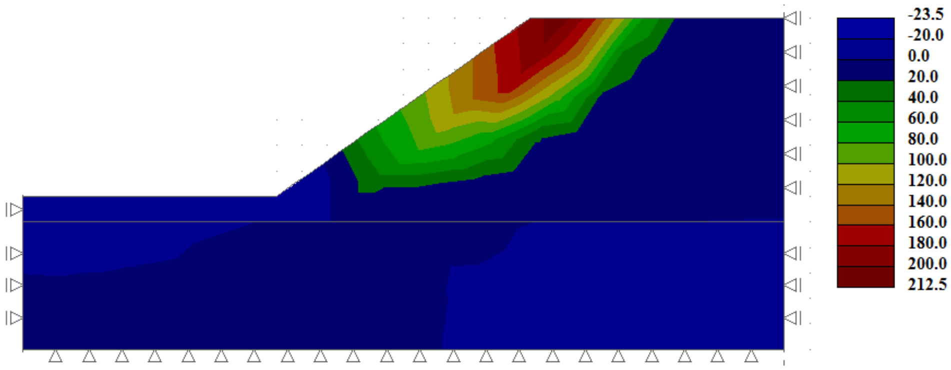


Figure (6): Displacement contour and critical slip surfaces of the slope without anchor reinforcement

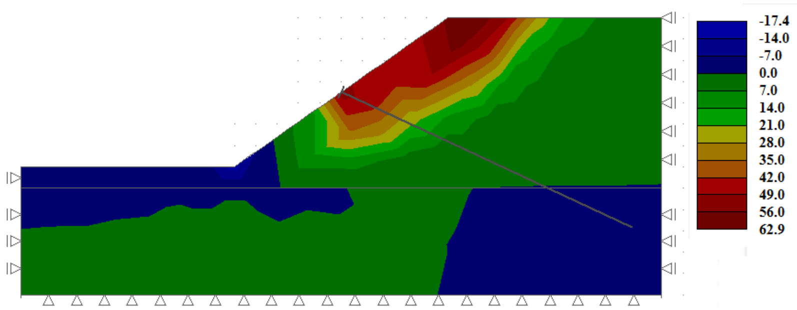


Figure (7): Displacement contour and critical slip surfaces of the slope with anchor reinforcement

Table 3. (FESRM) results and (LEM) results

Slope type	Factor of safety			Max. displacement (mm)
	FESRM	Bishop	Spencer	
Unreinforced	1. 33	1. 5	1. 51	212. 5
Reinforced with anchor	1. 5	1. 67	1. 76	62. 9

Also, in the reinforced slope, the largest value of displacement was at the top of the slope.

In the case of the unreinforced slope, the number of critical surfaces generated is bigger than the number of critical surfaces generated in the case of the reinforced slope.

Also, it was observed that for the reinforced slope, the first critical slip failure extended to the head of the anchor.

Effect of Anchor Inclination

In this sub-section, the effect of the anchor inclination on the factor of safety and the critical slip surfaces was investigated. All anchoring parameters were kept invariant, but the inclination angle varied from $0^{\circ}$to 35°. Fig. (8) shows the results for variation in inclination angle on the stability of the anchored slope according to (FESRM) and (LEM). The analysis using the finite element program gives lower values of the factor of safety than the analysis using Bishop and Spencer methods. Also, the factor of safety decreases as the inclination angle increases according to the analysis *via* (LEM).

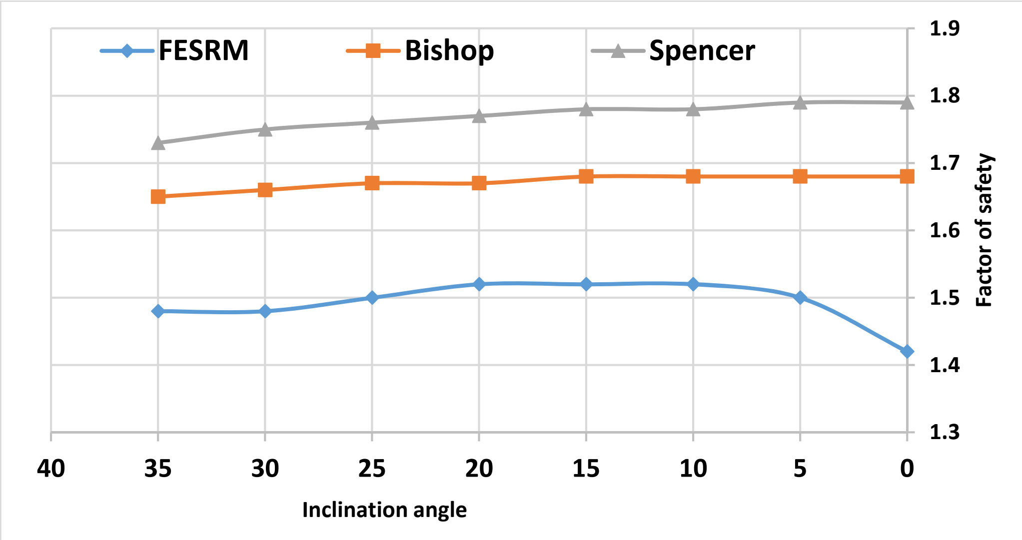


Figure (8): Effect of anchor inclination on stability of anchored slope

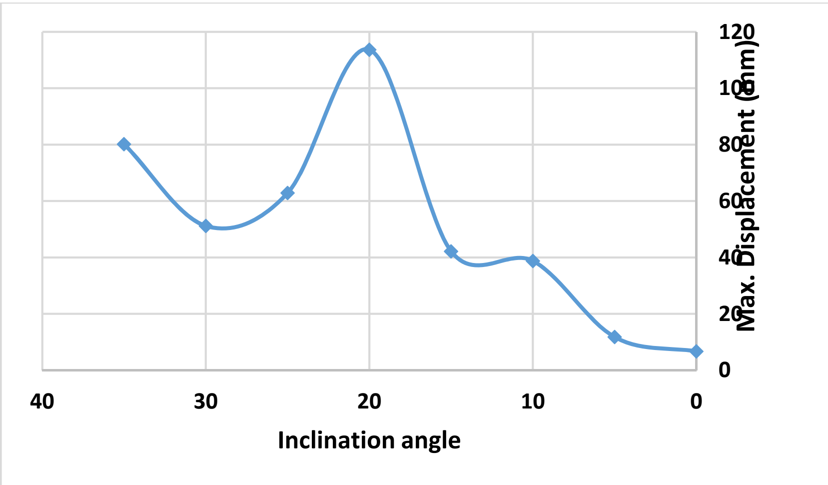


Figure (9): The relation between the inclination angle and maximum displacement

According to FESRM results, it was noticed that when the inclination angle was less than 10°, the factor of safety increases as the inclination angle increases. For inclination angles ranging from 10° to 20°, the factor of safety reaches the highest value (1. 52). For an inclination angle greater than 20°, the factor of safety decreases as the inclination angle increases. According to LEM results, the factor of safety decreases as the inclination angle increases.

From Fig. (9), it can be seen that as the inclination angle increases, the maximum displacement increases. Additionally, the highest value of displacement occurred at an inclination angle of 20°.

For inclination angles ranging from 20° to 35°, the inclination angle increases as the maximum displacement decreases.

According to the previous analysis, the angles ranging from 10° to 15° are the optimum inclination angles to increase the factor of safety and decrease the displacement.

According to the analysis by the FESRM method, the slip surfaces failure is varied with the variation in the anchor inclination and non-circular slip surfaces were generated, as shown in Fig. (10). It can also be seen that the slip surfaces were formed from the slope crest and extended to the toe of the slopes for inclination angles more than zero.

Also, if the inclination angle is equal to zero, the slip surfaces extended to a large area on the crest. When the inclination angle is 30° or more, small circular slip surfaces have formed.

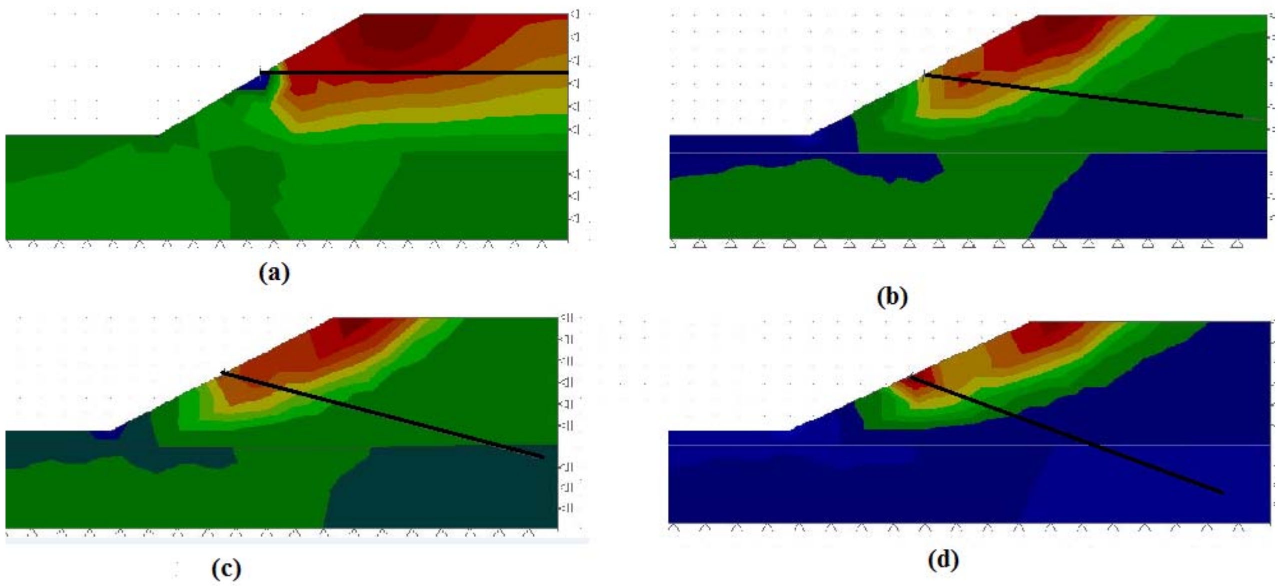


Figure (10): Schematics showing different inclinations of the anchors used a- θ= 0°, b- θ= 10°, c- θ= 20° and d- θ= 30°.

To calculate the safety factor of slopes, the equilibrium of all moments about the center of the slip circle is taken for all slices. The tension force for anchors can be divided into two components: one in the vertical direction (P sin θ) and the other in the horizontal direction (P cos θ); therefore, both of resistance moment value and driving moment value vary with the value of the angle of inclination (θ). Accordingly, the factor of safety increases with increasing the inclination angle from 0° to 20° due to increasing the forces in the vertical direction (P sin θ), which leads to increase the resistance moment. Moreover, increasing the forces in the vertical direction decreases the displacement.

Effect of Anchor Position

In this sub-section, the effect of anchor position on the safety factor was analyzed. The position of an anchor can be expressed by the ratio (x/L), where (x) is the horizontal distance of the anchor head from the slope toe and (L) is the horizontal distance from the toe to the heel of the slope, as shown in Fig (11).

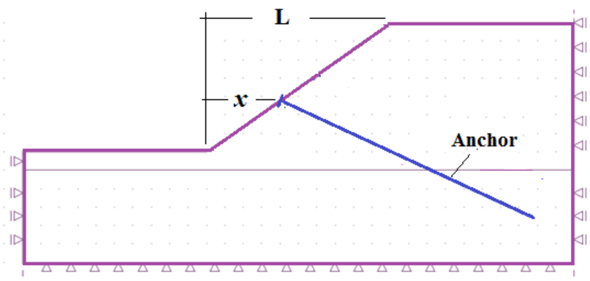


Figure (11): A schematic showing the anchor position relative to slope

The results of (FERSM) and (LEM) are shown in Fig. (12).

As shown in Fig. (12), the analysis using (FERSM) gives lower values of the factors of safety than the analysis using Bishop and Spencer methods. Also, the factor of safety decreases as the ratio (x/l) increases. The maximum factor of safety was attained at (x/L) = 0. 2 according to (FERSM) and at (x/L) = 0. 4 according to (LEM). Generally, the maximum factor of safety was attained at (x/L) < 0.5.

In other words, the factor of safety increases as the anchor position is closer to the slope toe.

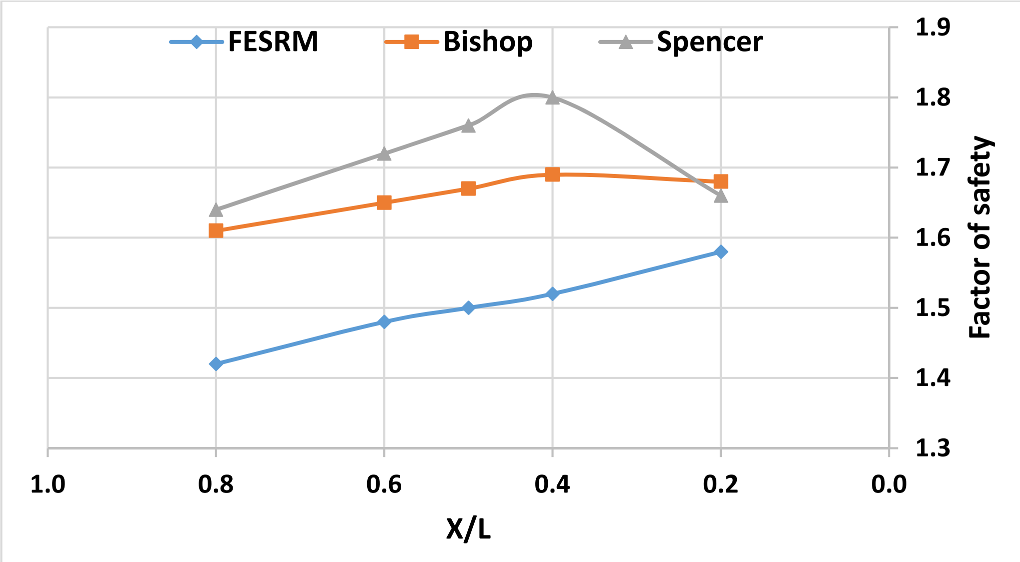


Figure (12): Effect of anchor position on safety factor

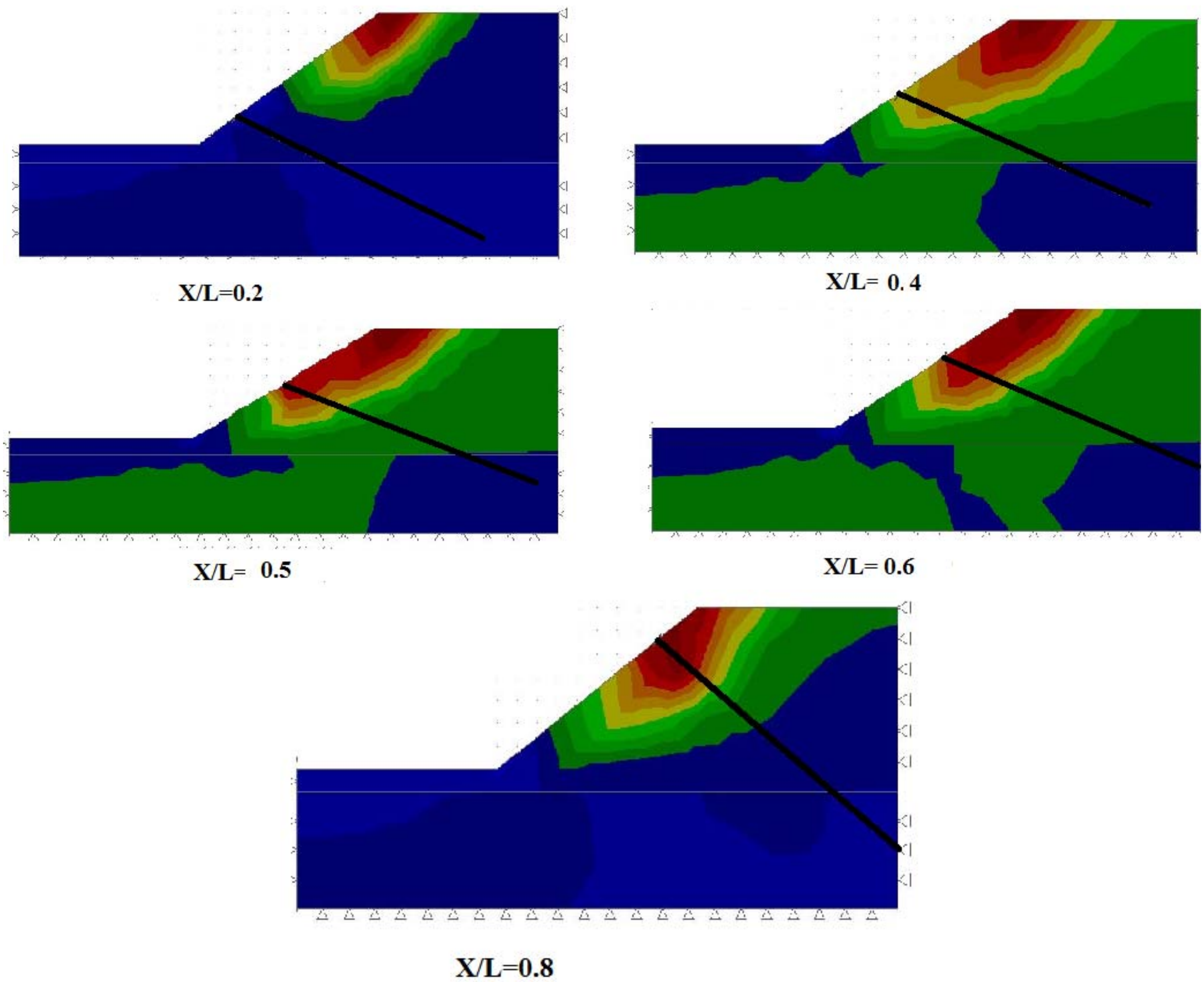


Figure (13): Effect of anchor position on critical slip surfaces

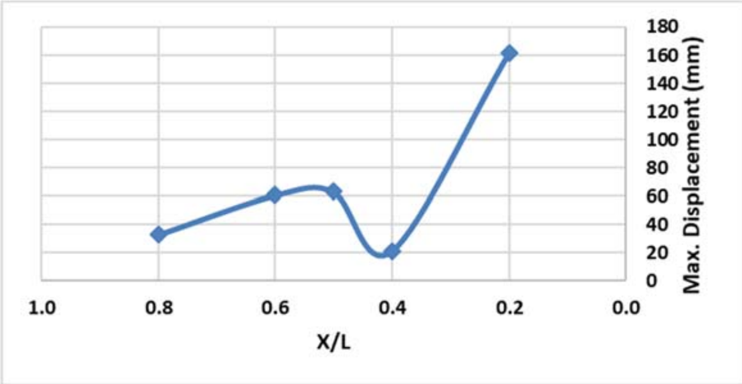


Figure (14): The relation between the ratio (x/L) and the maximum displacement

From FERSM results, it was noticed that at the maximum value of the safety factor, the anchor is located outside the critical slip circle, as shown in Fig. (13), which means that an installation of the anchor in this position leads to move the critical slip surface and increase the factor of safety. Also, when the ratio (x/L)> 0.4, the anchor passed through all the critical slip surfaces and when (x/L) = 0.8, the anchor passed through the center of all critical slip surfaces. Accordingly, the anchor position had a considerable effect on the determination of the critical slip surfaces and factor of safety.

From Fig. (14), it can be seen that the greatest value of displacement was at (x/L) =0.2 and the lowest value was at (x/L) = 0.4. For (x/L) >0.5, the displacement decreases as the ratio (x/L) increases. Accordingly, the optimum location of the anchor is at (x/L) = 0.4.

Effect of Number of the Anchor Rows

In case that one row of anchors was insufficient to reach the required stability, two rows of anchors were used. The influence of using two rows of anchors on the factor of safety was studied.

Two models were used in this analysis. In the first model, the slope was reinforced by two rows of anchors located at x =2m and 4m, respectively.

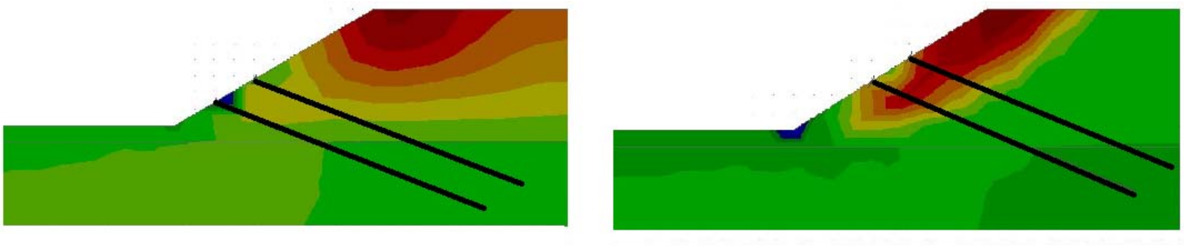
In the second model, the slope was reinforced by two rows of anchors located at x=4m and 6m, respectively. All other anchor parameters remained unchanged.

The results of (FERSM) and (LEM) are listed in Table 4.

The factors of safety of reinforced slope with one anchor at distance (x) =2, 4 and 6 m were 1.58, 1.52 and 1.48, respectively.

Table 4. The safety factors of slope reinforced by two rows of anchors

Distance *X*, (m)	Factor of safety			Displacement (mm) FERSM
	FESRM	Bishop	Spencer	
First anchor= 2m Second anchor= 4m	1. 76	1. 87	2. 15	21. 2
First anchor= 4m Second anchor= 6 m	1. 71	1. 8	2. 0	60. 5



a b

Figure (15): Arrangement of two parallel anchors: (a) x= 2, and 4 mm; (b) x= 4 and 6 mm

From Table 4, it can be seen that the finite element strength-reduction method (FESRM) predicts lower factors of safety as compared to the limit-equilibrium methods ((LEMs). Although the slope was reinforced by two rows of anchors, the factor of safety increased by about 10% only. So, it can be noticed that using two rows of anchors doesn't contribute significantly to increasing the factor of safety.

For the case where the first anchor is at a distance = 2m and the second anchor is at a distance = 4m, the critical slip surfaces moved away from the slope toe and extended to a large distance on the crest, as shown in Fig. (15a). If the first anchor is at a distance =4m and the second anchor is at a distance = 6m, the critical slip surfaces extended to the slope toe and a small settlement occurred at the toe of the slope, as shown in Fig. (15b).

CONCLUSIONS

1. The finite element strength-reduction method (FESRM) is very applicable to evaluate the stability of reinforced and unreinforced slopes. Also, the results of limit-equilibrium methods (LEMs) are fairly in agreement with those of the finite element strength-reduction method (FESRM).
2. The finite element strength-reduction method (FESRM) predicts lower factors of safety as compared to the limit-equilibrium methods (LEMs).
3. An angle ranging from 10° to 15° is the optimum inclination angle to increase the factor of safety and decrease the displacement.
4. The anchor position had a considerable effect on the determination of critical slip surfaces. The optimum location of the anchor is at (x/L) =0. 4.
5. Reinforcing the slope by two rows of anchors increases the factor of safety by about 10%.

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