Effect of Water Table Fluctuation on Barometric Pumping in Soil Unsaturated Zone

Talib R. Abbas

Ministry of Science and Technology, Environment and Water Research and Technology Directorate, Baghdad - Iraq

ABSTRACT

An analytical one-dimensional model for vertical transmission of barometric pressure in soil unsaturated zone has been developed to examine the effect of free boundary caused by water table. The model is applicable to a special case in which the water table moves in according to the pressure with high barometric efficiency. Ambient sinusoidal pressure fluctuation is assumed at the soil surface.

The model with the numerical simulations results for an unsaturated zone with typical soil settings show a significant difference between the simulated pressure amplitudes for the case in which the water table is considered as a free boundary and the case in which the water table is considered as a fixed boundary. Maximum difference occurs at the water table. This result suggests the consideration of water table as a free boundary for cases in which the barometric efficiency of water table is relatively high.

KEYWORDS: Barometric pumping, Unsaturated zone, Mathematical modeling, Free boundary.

INTRODUCTION

Natural atmospheric pressure fluctuations associated with diurnal temperature variations and of longer time periods, associated with the passage of weather fronts, are transmitted through the unsaturated subsurface. These pressure waves are damped and delayed in phase to a degree dependent on the effective vertical permeability of the formation. As a result, at a given time, the atmospheric pressure at the surface and soil gas pressure in the subsurface are different (SRNL, 2006). The result is the barometric pumping in which the permeable subsurface "breathes" in response to pressure change at the surface (Auer et al., 1996; DOE, 2000).

Although records of atmospheric pressure variation show inherently random meteorological conditions, a time series analysis may be used to capture the main features of the variation. Daily variation component and weather front passage component can be captured. A one-dimensional model is useful to capture the main features of gas pressure transmission or gas flow phenomena induced in the upper region of the unsaturated zone.

Many previous studies have dealt with gas flow in the unsaturated zone due to barometric pumping (Baehr and Hult, 1991; Shan at al., 1992; Shan, 1995; Chen and Gosselin, 1998; Hunt and Massmann, 2000; Yeung et al., 2002) or with the effect of barometric pumping on contaminant transport in the unsaturated zone (Massmann and Farrier,1992; Auer et al., 1996), considering the lower boundary, water table, as a fixed and impermeable boundary. However, other studies have shown that the lower boundary, water table, may

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act as a free boundary moving up and down in accordance to the pressure variations at this boundary.

Many studies show that a well penetrating unconfined aquifer can behave very much like an ideal barometer. The change in water level from barometric fluctuations is caused by changes in air pressure transmitted down the open well. The water level fluctuates in response to barometric pressure changes and the observed barometric efficiency was about 94% (Hare and Morse, 1997; Spane, 1999; Hubbell et al., 2003; USGS, WRIR 00-4014). Furthermore, Turk (1975) showed that the shallow water table at the Bonneville Salt Flats, Utah, experienced daily fluctuations. The fluctuations are attributed to temperature-related atmospheric pressure changes acting on the capillary zone.

The purpose of this study is to develop an analytical solution for vertical transmission of barometric pressure in the soil unsaturated zone. The solution is applicable to a case in which the lower boundary is a water table moving upward and downward in proportion to the gas pressure variations at this boundary.

MATHEMATICAL DEVELOPMENT

To study the gas pressure in the unsaturated zone in response to barometric pressure changes, some of the complexity of real atmospheric pressure fluctuations can be ignored assuming that these pressure patterns can be represented by a few simple Fourier components (Auer, 1996). Sinusoidal atmospheric pressure changes were also used by Massmann and Farrier (1992) and Choi et al. (2005) to investigate the effects of atmospheric pressures on gas transport in the vadose zone. It is important to note that the pressure fluctuations are always very much smaller than the average pressure, so this phenomenon can be analyzed using linear theory (Fukuda, 1955; Auer et al., 1996; Neeper, 2002; Massman, 2006). The one-dimensional propagation of small variations of gas pressure in a porous medium is described by a diffusion equation in pressure:

$$\frac{\partial P'}{\partial t} = \alpha \frac{\partial^2 P'}{\partial x^2} \tag{1}$$

where

$$\alpha = \frac{k \cdot P_a}{n \cdot \mu} \quad (\mathrm{m^2/sec}) \tag{2}$$

P_a is the mean pressure (Pa)
n is the soil porosity, k is the soil permeability (m^2), μ is the gas viscosity (Pa.sec).

Then, the total pressure P can be defined as:

$$P = P_a + P' \tag{3}$$

Two analytical solutions are derived for Eq. (1) in two different cases.

Case 1: Fixed Reflective Boundary

The system considered in this case is conceptualized as shown in Fig.(1). Uniform sinusoidal pressure fluctuation is assumed at the upper boundary. No flow is assumed at the lower boundary:

$$P'(l,t) = A \cdot Sin(w \cdot t) \tag{4}$$

A is the amplitude of barometric pressure variations (Pa).

$$w = \frac{2\pi}{T_o} \tag{5}$$

T_o = time for complete pressure variation period (sec).

$$\left(\frac{\partial P'}{\partial x}\right)_{x=0} = 0 \tag{6}$$

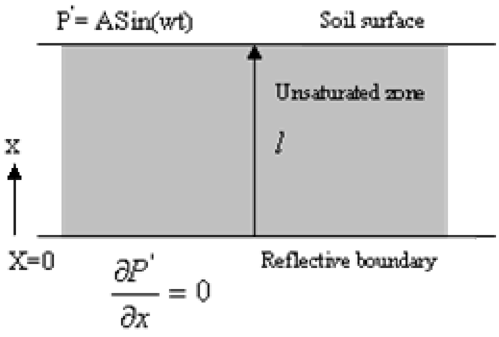

Figure 1: Flow System Considered in Case 1

Eq.'s (1, 4 and 6) can be solved by assuming a solution of the form:

$$P'(x,t) = Im\left[P'(x)\cdot Exp(i\cdot w\cdot t)\right] \quad (7)$$

where

$$i = \sqrt{-1}$$

Then, the steady periodic solution of the pressure for the system considered in this case is:

$$P'(x,t) = A'\cdot Sin(w\cdot t+\phi) \quad (8)$$

$$A' = A\cdot\left|\frac{Cosh[k'\cdot x\cdot(i+1)]}{Cosh[k'\cdot l\cdot(i+1)]}\right| \quad (9)$$

$$\phi = arg\left\{\frac{Cosh[k'\cdot x\cdot(i+1)]}{Cosh[k'\cdot l\cdot(i+1)]}\right\} \quad (10)$$

$$k' = \left(\frac{w}{2\alpha}\right)^{\frac{1}{2}} \quad (11)$$

The quantities A' and ϕ which are the amplitude and phase of the steady pressure oscillation at the point x are functions of the two dimensionless quantities x/l and $k'l$.

Case 2: Free Reflective Boundary

The system considered in this case is conceptualized as shown in Fig.(2). Uniform sinusoidal pressure fluctuation is assumed at the upper boundary, Eq.(4). Free reflective boundary with barometric efficiency of 100% at the bottom of the unsaturated zone is considered, in which the lower boundary moves $\Delta x(t)$ upward and downward around the mean water table in proportion to the gas pressure at this boundary; the capillary effects are neglected (Li and Jiao, 2005):

$$\Delta x(t) = \frac{P'(z,t)}{\rho_w\cdot g}\Big|_{z=WT} \quad (12)$$

ρ_w is the water density (kg/m^3).
g is the gravitational acceleration (m/sec^2).

In order to derive the boundary condition at the bottom free boundary, consider mass balance over control volume dV swept by the free boundary during time increment dt as follows:

$$a\cdot\rho_a\cdot q(z,t)\Big|_{z=WT} = \frac{\partial(\rho_a\cdot n\cdot V(t))}{\partial t}\Big|_{z=WT} \quad (13)$$

a is the unit area (m^2).

where $q(0,t)$ is the isothermal vertical flux at the free boundary.

According to Darcy's law (Rathfilder, 1991):

$$q(z,t)\Big|_{z=WT} = -\frac{k}{\mu}\cdot\frac{\partial P'}{\partial z}\Big|_{z=WT} \quad (14)$$

ρ_a is the gas density (kg/m^3), calculated from the ideal gas law:

$$\rho_a = \frac{(P_a+P')\cdot M}{R\cdot T} \quad (15)$$

M is the gas molecular mass (kg/kg.mole).
R is the universal gas constant (kg.m^2/ºK.sec^2.mole).
T is the absolute temperature (ºK).
V is the volume of soil swept by the free boundary

$$V(t) = a\cdot\Delta x(t) \quad (16)$$

$$V(t) = a\cdot\frac{P'(z,t)}{\rho_w\cdot g}\Big|_{z=WT} \quad (17)$$

Combining Eq.'s (13, 14, 15 and 17) gives:

$$\left(P_a + P'\right)\cdot\left(-\frac{k}{\mu}\cdot\frac{\partial P'}{\partial z}\right)\Big|_{z=WT}$$

$$= \frac{n}{\rho_w \cdot g}\cdot\frac{\partial\left(\left(P_a + P'\right)\cdot P'\right)}{\partial t}\Big|_{z=WT} \qquad (18)$$

Since $P' << P_a$, Eq. (18) can be reduced to:

$$-\frac{k\cdot\rho_w\cdot g}{n\cdot\mu}\cdot\frac{\partial P'}{\partial z}\Big|_{z=WT} = \frac{\partial P'}{\partial t}\Big|_{z=WT} \qquad (19)$$

or

$$k''\cdot\frac{\partial P'}{\partial z}\Big|_{z=WT} = \frac{\partial P'}{\partial t}\Big|_{z=WT} \qquad (20)$$

where

$$k'' = -\frac{k\cdot\rho_w\cdot g}{n\cdot\mu}. \qquad (21)$$

Compared to the unsaturated zone thickness *l*, the fluctuations in water table may be small and the actual unsaturated zone thickness can be therefore approximated by the initial aquifer thickness *l*. The pressure at the free boundary is hence replaced by the pressure at the mean water table. Then Eq.(20) becomes:

$$k''\cdot\frac{\partial P'}{\partial z}\Big|_{z=0} = \frac{\partial P'}{\partial t}\Big|_{z=0} \qquad (22)$$

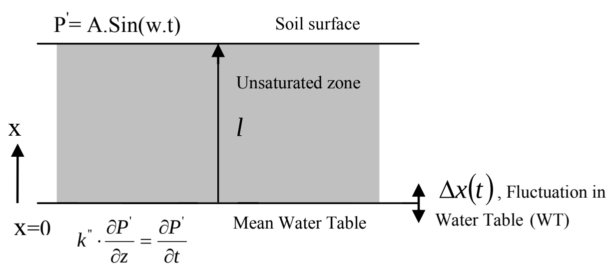

Figure 2: Flow System Considered in Case 2

Similar to case1, Eq.'s (1, 4 and 22) can be solved by assuming a solution of the form:

$$P'(x,t) = Im\left[P'(x)\cdot Exp(i\cdot w\cdot t)\right]. \qquad (23)$$

Then, the steady periodic solution of the pressure for the system considered in this case is:

$$P(x,t) = A'\cdot Sin(w\cdot t+\phi) \qquad (24)$$

where

$$A' = A\cdot\left|B\cdot Exp\left[k'\cdot(i+1)\cdot x\right] + C\cdot Exp\left[-k'\cdot(i+1)\cdot x\right]\right| \qquad (25)$$

$$\phi = arg\left|B\cdot Exp\left[k'\cdot(i+1)\cdot x\right] + C\cdot Exp\left[-k'\cdot(i+1)\cdot x\right]\right| \qquad (26)$$

and

$$B = \frac{1}{\left(\dfrac{k'\cdot(i+1) - i\dfrac{w}{k''}}{k'\cdot(i+1) + i\dfrac{w}{k''}}\right)\cdot Exp\left[-k'\cdot(i+1)\cdot L\right] + Exp\left[k'\cdot(i+1)\cdot L\right]} \qquad (27)$$

$$C = \frac{1}{Exp\left[-k'\cdot(i+1)\cdot L\right] + \left(\dfrac{k'\cdot(i+1) + i\dfrac{w}{k''}}{k'\cdot(i+1) - i\dfrac{w}{k''}}\right)\cdot Exp\left[k'\cdot(i+1)\cdot L\right]} \qquad (28)$$

It can be simply proved that Eq.(25) reduces to Eq.(9) as k'' approaches ∞.

For both cases 1 and 2, if the surface pressure can be represented by the Fourior series:

$$P'(l,t) = \sum_{m=1}^{\infty} A_m \cdot Sin(m\cdot w\cdot t), \qquad (29)$$

then the steady periodic solution is:

$$P'(x,t) = \sum_{m=1}^{\infty} A'_m \cdot Sin(m \cdot w \cdot t + \phi_m). \qquad (30)$$

RESULTS AND DISCUSSION

Fig.(3) shows the way in which the amplitude of gas pressure, in an unsaturated zone with typical soil settings, varies in response to a daily hypothetical sinusoidal barometric pressure at the soil surface. It is clear that there is a significant difference between the simulated pressure amplitudes with and without considering lower boundary, water table, as a free boundary. Maximum difference occurs at this boundary.

The soil permeability is a governing factor of the difference between the simulated pressure amplitudes with and without considering lower free boundary. Maximum simulated pressure difference occurs at soil permeability of 10^{-11} m^2. A decrease in soil permeability below 10^{-12} m^2 results in an insignificant pressure amplitude difference between the two cases. The analytical model can also show the associated phase angle for the two cases with and without considering lower free boundary.

CONCLUSIONS

The analytical model presented in this paper provides a way to examine the effect of free lower boundary, water table, on barometric pressure transmission in soil unsaturated zone. The water table considered in this study moves upward and downward in proportion to the gas pressure at gas-water interface. A hypothetical daily sinusoidal barometric pressure is assumed at the soil surface.

The model and the numerical simulation results show that considering the water table as a free boundary rather than a fixed boundary has a significant effect on the pressure amplitude at any depth in the unsaturated zone in response to barometric pressure variations at the soil surface.

The results of this study may have an importance for studies dealing with barometric pressure transmission through soil unsaturated zone. It is suggested that the water table can be considered as a free boundary for cases in which the barometric efficiency of water table is relatively high.

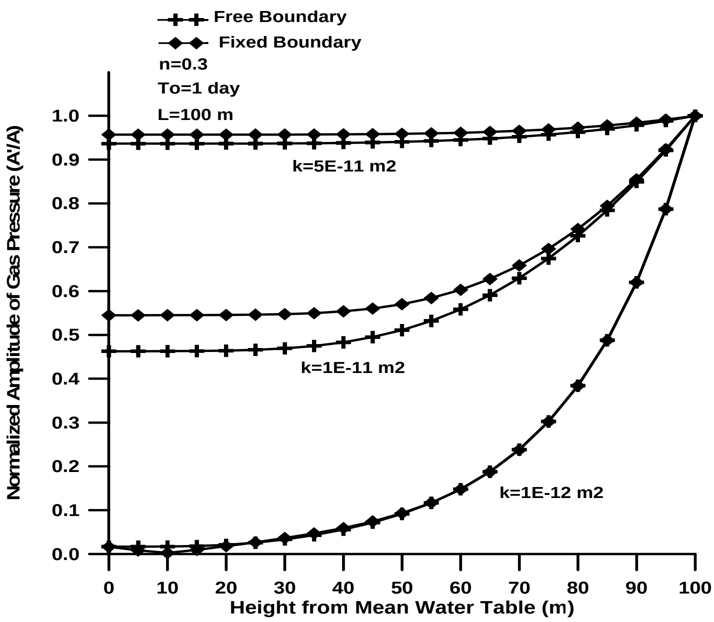

Figure 3: Variation of Amplitude of Gas Pressure with Height from Mean Water Table

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