Consolidation Analysis by the Extended Taylor Method (ETM)

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ABSTRACT

The Taylor $\sqrt{t}$ method is extended for improving the estimation of the end-of-primary (EOP) settlement δ_p and coefficient of vertical consolidation c_v. The extended Taylor method (ETM) utilizes the same graphical procedure of the conventional Taylor method, but this procedure is repeated twice at any two U values greater than 52.6% instead of the one used in the Taylor method at 90%; U is the average degree of consolidation. The ETM explicitly relates c_v to the slope of the initial linear portion of the δ_t - $\sqrt{t}$ curve as well as the EOP settlement δ_p; δ_t is the settlement at time t during consolidation. The ETM can be applied with a minimum of four consolidation data points for estimating δ_p and c_v. Two types of EOP settlements are distinguished; firstly, local EOP settlement δ_{pi} obtained by matching experimental results to theory at a particular U value; secondly, global EOP settlement δ_p obtained by extrapolation using two or more local EOP settlements. The ETM explains the observed differences in c_v values estimated by the existing methods in terms of the differences in δ_p values obtained by these existing methods. Experimental results are used to validate and compare the extended Taylor method with existing methods.

KEYWORDS: Taylor method, Casagrande method, Coefficient of vertical consolidation, End of primary settlement, Initial compression, Secondary compression.

INTRODUCTION

Consolidation analysis utilizing the Terzaghi one-dimensional consolidation theory along with the results of oedometer tests is still widely used in settlement analysis of soils. The Terzaghi theory was developed only for the primary consolidation assuming constant coefficient of vertical consolidation and ignoring time compressibility during primary consolidation; whereas the observed compression-time curve exhibits initial compression, primary consolidation and secondary compression (Terzaghi et al., 1996). Hence, to properly apply the Terzaghi theory in settlement analysis, the primary consolidation must be recognized by identifying the initial and secondary compressions and then matched with the Terzaghi theory at a particular U value or over a range of U, where U is the average degree of vertical consolidation.

The application of the Terzaghi theory to settlement analysis requires reliable values for the coefficient of vertical consolidation c_v and end-of-primary (EOP) settlement δ_p. Numerous methods were developed for estimating c_v and δ_p values (Casagrande and Fadum, 1940; Taylor, 1948; Scott, 1961; Cour, 1971; Sivaram and Swamee, 1977; Asaoka, 1978; Parkin, 1978; Sridharan and Rao, 1981; Sridharan et al., 1987; Robinson and Allam, 1996; Robinson, 1997, 1999; Mesri et al., 1999a; Feng and Lee, 2001; Al-Zoubi, 2008a, 2008b, 2010, 2014). The most widely used methods for estimating c_v and δ_p are the standard $\log t$ method (Casagrande and Fadum, 1940) and the standard $\sqrt{t}$ method (Taylor, 1948). The $\log t$ method

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computes c_v at 50% consolidation and requires the determination of the initial and final compressions that correspond to 0% and 100% consolidation, respectively. The $\sqrt{t}$ method computes c_v at 90% consolidation and requires the determination of the initial compression. The two standard $\log t$ and $\sqrt{t}$ methods and most existing methods utilize the same theoretical basis for evaluating the initial compression, but these methods differ in the way in which the EOP consolidation is identified. The $\sqrt{t}$ method generally yields lower δ_p and higher c_v values as compared to the $\log t$ method; the differences in c_v values estimated by the existing methods for a particular pressure increment could chiefly be explained by the differences in δ_p values obtained by these different methods (Al-Zoubi, 2008b, 2010).

In this paper, the coefficient of vertical consolidation c_v is expressed as a function of slope m of the initial linear portion of the δ_t - $\sqrt{t}$ curve and the EOP settlement δ_p. The graphical construction of the conventional Taylor method is extended for improving the estimation of δ_p (and thus c_v) as compared to the Casagrande method that yields almost identical δ_p values to those obtained from pore water pressure measurements (Mesri et al., 1999b; Robinson, 1999). Experimental results of oedometer tests on clayey soils are used to validate and compare the extended Taylor method (ETM) with existing methods.

ANALYTICAL BACKGROUND

The relationship between average degree of consolidation U and time factor T of the Terzaghi theory may, depending on the range of U, be given by the following two expressions (Terzaghi, 1943):

$$U = \sqrt{\frac{4}{\pi}}\sqrt{T} \quad for\ U \le 52.6\ \% \qquad (1)$$

$$\ln(1-U) = \ln\frac{8}{\pi^2} - \frac{\pi^2}{4}T \quad for\ U \ge 52.6\ \% \qquad (2)$$

In the Terzaghi theory, the consolidation time t is expressed as a function of the time factor T, longest drainage path H_m and coefficient of vertical consolidation c_v as follows:

$$t = \frac{T\,H_m^2}{c_v} \qquad (3)$$

The settlement δ_t may be given in terms of U and δ_p by the following expression:

$$\delta_t = U\ \delta_p \qquad (4)$$

where $\delta_p = R_p - R_o$; R_p is the dial reading at the EOP consolidation and δ_t is the settlement at time t during consolidation that is equal to $R_t - R_o$; R_t is the dial reading at time t and R_o is the initial compression defined as the dial reading that corresponds to 0% consolidation. The initial compression R_o may be given as follows:

$$R_o = \frac{R_2 - R_1\sqrt{t_2/t_1}}{1-\sqrt{t_2/t_1}} \qquad (5)$$

where R_1 and R_2 are the dial gauge readings at time t_1 and time t_2, respectively, and are selected such that these two points are on the initial linear portion of the R_t - $\sqrt{t}$ curve. The Casagrande and Taylor methods use the same basis for obtaining the initial compression R_o. The Casagrande method arbitrarily takes $t_2 = 4t_1$, therefore, $\Delta = R_1 - R_o = R_2 - R_1$ or $R_o = 2R_1 - R_2$ whereas, in the Taylor method, R_o is determined graphically as the intercept of the initial linear portion of the R_t - $\sqrt{t}$ curve. Hence, the Taylor, Casagrande and extended Taylor methods are similarly affected by the factors that influence the initial portion of the consolidation curve. However, these methods differ in the way by which the primary consolidation range (or EOP δ_p) is obtained as shown later.

The coefficient of consolidation may be expressed based on Eqs. (1), (3) and (4), as follows:

$$c_v = \frac{\pi}{4}\left(\frac{m\,H_m}{\delta_p}\right)^2 \quad (6)$$

where m is the slope of the initial linear portion of the experimental δ_t - $\sqrt{t}$ curve that may be expressed as follows:

$$m = \frac{R_2 - R_1}{\sqrt{t_2} - \sqrt{t_1}} \quad (7)$$

Because Eq. (6) involves three unknowns (i.e., R_0, R_p and m; where $\delta_p = R_p - R_0$), the coefficient of vertical consolidation may not be obtained from only the initial linear portion. Therefore, at least one additional data point (t_i, R_{ti}) must be selected from the later stages of consolidation beyond the initial linear portion (theoretically, at $U \geq 52.6\%$) along with the two data points (t_1, R_1) and (t_2, R_2) required for obtaining the initial compression R_0 (Eq. 5) and the slope m (Eq. 7).

Equation (6) can, however, be used for assessing the coefficient of vertical consolidation independently of the procedure in which the EOP δ_p is obtained; for example, at U of 90% (used in the Taylor method), the slope m and EOP settlement δ_p can be expressed in terms of δ_{90} and t_{90}, respectively, as follows:

$$m = 1.153\frac{\delta_{90}}{\sqrt{t_{90}}} \quad (8)$$

$$\delta_p = \frac{\delta_{90}}{0.90} \quad (9)$$

Substituting Eqs. (8) and (9) into Eq. (6), the coefficient of consolidation may be given by the following expression:

$$c_v = \frac{\pi}{4}\left(\frac{1.153\delta_{90}H_m 0.90}{\sqrt{t_{90}}\delta_{90}}\right)^2 = \frac{0.848H_m^2}{t_{90}} \quad (10)$$

Equation (10) is the same as that used by the $\sqrt{t}$ method (Taylor, 1948) to estimate c_v.

On the other hand, at U of 50% (used in the Casagrande method), the EOP settlement δ_p can be expressed as follows:

$$\delta_p = 2\ \delta_{50\%} \quad (11)$$

The slope m can also be expressed as the secant slope of the observed δ_t - $\sqrt{t}$ curve at 50% consolidation as follows:

$$m = \frac{\delta_{50\%}}{\sqrt{t_{50}}} \quad (12)$$

Substituting Eqs. (11) and (12) into Eq. (6), the coefficient of consolidation may be given by the following expression:

$$c_v = \frac{\pi}{4}\left(\frac{\delta_{50\%}H_m}{\sqrt{t_{50}}\ 2\ \delta_{50\%}}\right)^2 = \frac{0.196H_m^2}{t_{50}} \quad (13)$$

Equation (13) is the same as that used by the $\log t$ method (Casagrande and Fadum, 1940) to estimate c_v.

Equation (6) is thus valid for estimating the coefficient of consolidation c_v regardless of the procedures used to estimate δ_p. It should, however, be emphasized that the c_v value depends on these procedures that may yield different δ_p values. Hence, the differences in c_v values obtained by the various existing methods for a particular pressure increment can be explained by the differences in δ_p values obtained either explicitly or implicitly by these existing methods. Therefore, any improvements that can be made on the estimation of the EOP settlement δ_p will improve the estimation of c_v values as may be deduced from Eq. (6).

The extended Taylor method (ETM) is introduced in the following section to improve the estimation of δ_p and c_v values and compared to existing methods by using experimental results of clayey soils (Table 1).

These soils cover a wide range of liquid limit and plasticity; the testing procedures of these soils were described in detail by Al-Zoubi (2008b, 2010, 2013, 2014).

Table 1. Basic properties of the clayey soils utilized in the present study

Soil	Particle size			Liquid limit %	Plastic limit %	Specific gravity G
	Sand %	Silt %	Clay %			
Azraq Green Clay (AGC-5; AGC-6 and AGC-8)	8	23	69	108	42	2.76
Mutah Clay (Mutah-0)	15	60	25	44	26	2.73
Chicago Blue Clay [a] (CBC-3)	4	64	32	29	17	2.73
Chicago Blue Clay [b] CBC (Taylor, 1948)	------	------	------	------	------	2.73
Madaba Clay (Madaba-6)	14	41	45	55	25	2.78
Treated Madaba Clay (Mad-t1, 2% cement)	------	------	------	------	------	------

[a] This specimen of CBC was tested by the Author.
[b] This specimen of CBC was reported by Taylor (1948).

The Extended Taylor Method (ETM)

In the conventional Taylor $\sqrt{t}$ method, a factor of 1.15 is used along with the initial linear portion of the δ_t - $\sqrt{t}$ curve to compute δ_p and c_v at 90% consolidation. This factor (i.e., 1.15) can be interpreted as the ratio of the secant slope m_{50} at 50% consolidation to the secant slope m_{90} at 90% consolidation. Assuming that the experimental and theoretical ratios are the same, the following expression can be obtained for this ratio.

$$ROSS(U = 90\%) = \frac{m_{50}}{m_{90}} = \frac{\delta_{50}/\sqrt{t_{50}}}{\delta_{90}/\sqrt{t_{90}}} = \frac{M_{50}}{M_{90}} = \frac{50\sqrt{T_{90}}}{90\sqrt{T_{50}}} = 1.15 \quad (14)$$

where $ROSS$ is the ratio of secant slopes; $\delta_{50} = 0.50\delta_p$; and $\delta_{90} = 0.90\delta_p$.

In the conventional Taylor method, the 90% consolidation was selected arbitrarily such that it is as close as possible to the EOP consolidation but least affected by secondary compression. The conventional Taylor method can, therefore, be extended for obtaining δ_{pi} and c_{vi} at other U_i values than 90%. In other words, the same graphical procedure of Taylor (1948) can be used such that the third point is arbitrarily selected at different U_i values for obtaining δ_{pi} and c_{vi} at those selected U_i values. This extension of the Taylor method requires a factor at each selected U_i value to be used for obtaining δ_{pi} and c_{vi} in the same way the factor of 1.15 is used in the conventional Taylor method at U_i of 90%. The ratio of secant slopes ($ROSS_i$) at 50% and at any other U_i value can be generalized for this purpose as follows:

$$ROSS(U_i,\%) = \frac{m_{50}}{m_U} = \frac{\delta_{50}/\sqrt{t_{50}}}{\delta_U/\sqrt{t_U}} = \frac{M_{50}}{M_U} = \frac{50/\sqrt{T_{50}}}{U_i/\sqrt{T_U}} = \frac{112.65\sqrt{T_{U_i}}}{U_i} \quad (15)$$

Table 2. Results of direct analytical method (DAM) and extended Taylor method (ETM) using consolidation data from Taylor (1948) for Chicago blue clay (CBC)

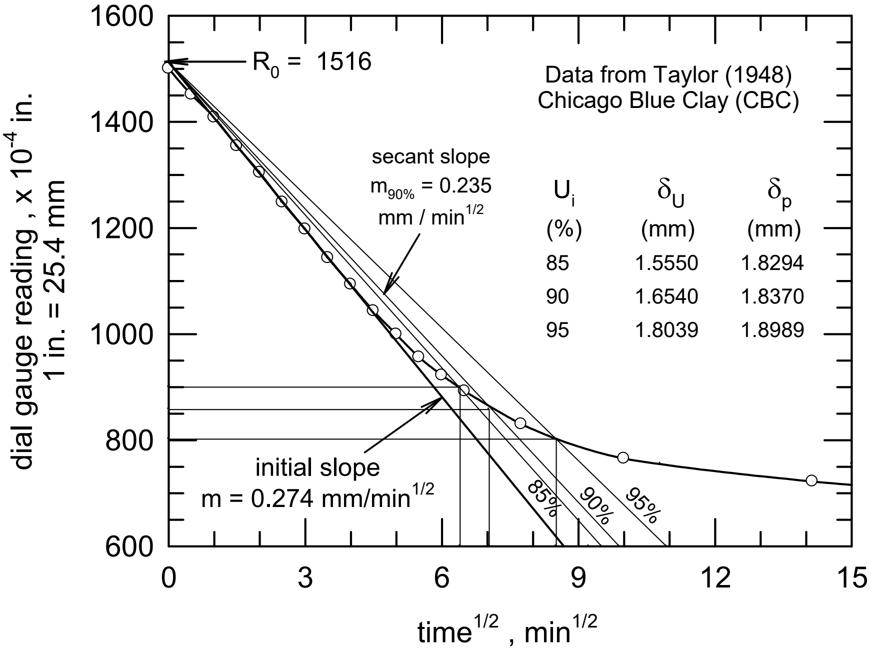
Early stages of consolidation (theoretically, $U < 52.6\%$)										
Time (min)	0	0.25	1	2.25	4	6.25	9	12.25	16	
Dial Reading, R (x 10^{-4} in) 1 in = 25.4 mm	1500	1451	1408	1354	1304	1248	1197	1143	1093	
Slope m (mm /min$^{-1/2}$)	0.274									See Fig. 1
R_0 25.4 x 10^{-4} (mm)	1516									
Later stages of consolidation (theoretically, $U > 52.6\%$)										
Time (min)	20.25	25	30.25	36	42.25	60	100	200	400	1440
R (x 10^{-4} in) 1 in = 25.4 (mm)	1043	999	956	922	892	830	765	722	693	642
Settlement δ_{ti}	1.201	1.313	1.422	1.509	1.585	1.742	1.908	2.017	2.090	2.220
Extended Taylor method (ETM)										
EOP δ_{pi} (mm)	1.665	1.717	1.777	1.792	1.804	1.863	1.940	-----	-----	-----
c_v / H_m^2 (10^{-3} min^{-1})	21.3	20.2	18.7	18.4	18.2	17.0	15.7	-----	-----	-----
Local $U_i = \frac{\delta_{ti}}{\delta_{pi}}$, %	72.1	76.5	80.0	84.2	87.9	93.5	98.4	-----	-----	-----
Global $U = \frac{\delta_{ti}}{\delta_p}$, %	61.8	67.6	73.2	77.7	81.6	89.7	98.2	-----	-----	-----
Direct analytical method (DAM)										
EOP δ_{pi} (mm)	1.674	1.717	1.780	1.791	1.806	1.864	1.911	2.018	2.092	2.220
c_v / H_m^2 (10^{-3} min^{-1})	21.1	20.1	18.7	18.4	18.1	17.0	16.2	-----	-----	-----
Local $U_i = \frac{\delta_{ti}}{\delta_{pi}}$, %	71.7	76.5	79.9	84.3	87.8	93.5	99.8	-----	-----	-----
Global $U = \frac{\delta_{ti}}{\delta_p}$, %	61.8	67.6	73.2	77.7	81.6	89.7	98.2	-----	-----	-----

The corresponding EOP settlement δ_{pi} at the arbitrarily selected U_i value can be given by the following expression:

$$\delta_{pi} = \frac{100}{U_i}\delta_U \tag{16}$$

where $\delta_{ti} = R_{ti} - R_0$ is the settlement at time t_i and $\delta_{pi} = R_{pi} - R_0$. The subscript i is added to δ_p and U

because of the dependence of δ_p on the arbitrarily selected δ_{ti} value as shown in the following analysis.



U_i (%)	δ_U (mm)	δ_p (mm)
85	1.5550	1.8294
90	1.6540	1.8370
95	1.8039	1.8989

Figure (1): Graphical construction for the extended Taylor method (ETM); data from Taylor (1948) for Chicago blue clay

The settlement δ_U ($=\delta_{ti}$ at U_i) at any arbitrarily selected U_i value can be obtained by utilizing the same graphical procedure of Taylor (1948) at that U_i value as demonstrated in Fig. 1, which shows the determination of three EOP δ_{pi} values at three arbitrarily selected U_i values. The estimated EOP δ_{pi} values are plotted against the selected $\delta_U = \delta_{ti}$ values in Fig. 2, which was plotted by using additional data points (Table 2). These results show that the estimated EOP δ_{pi} depends on the third point (t_i, δ_{ti}) selected for the analysis. Figure 2, however, shows that the estimated EOP δ_{pi} value increases linearly with the selected δ_{ti} value during primary consolidation. This linear relationship between δ_{pi} and δ_{ti} can be expressed as follows:

$$\delta_{pi} = a + b\delta_{ti} \quad (17)$$

where a and b are the intercept and slope of the linear δ_{pi} - δ_{ti} relationship, respectively.

Figure 2 and Table 2 show that as the compression-time curve approaches the EOP consolidation, the estimated δ_{pi} value approaches the arbitrarily selected δ_{ti} value; therefore, the following expression can be suggested at the EOP consolidation:

$$\delta_{pi} = \delta_{ti} \quad (18)$$

Hence, a unique EOP δ_p value can be defined for any particular pressure increment where the estimated δ_{pi} values become equal to the arbitrarily selected δ_{ti} values. Therefore, the unique EOP δ_p may be expressed by equating Eq. 17 and 18 as follows:

$$\delta_p = \frac{a}{1-b}$$
(19)

Equation 19 shows that the unique EOP δ_p value can be obtained from the linear relationship between δ_{pi} and δ_{ti} observed in the primary consolidation range by forward extrapolation without the need to use secondary compression (Fig. 2). This extrapolation requires at least two compression-time data points in the range $U \geq 52.6\%$ to obtain EOP δ_p in addition to the two data points (t_1, R_1) and (t_2, R_2) required for back-calculating the initial compression R_0 and the slope m of the initial linear portion; the coefficient of vertical consolidation can then be computed by Eq. 6. Similar trend is observed in Fig. 3 for another specimen of treated Madaba clay (Mad-t1).

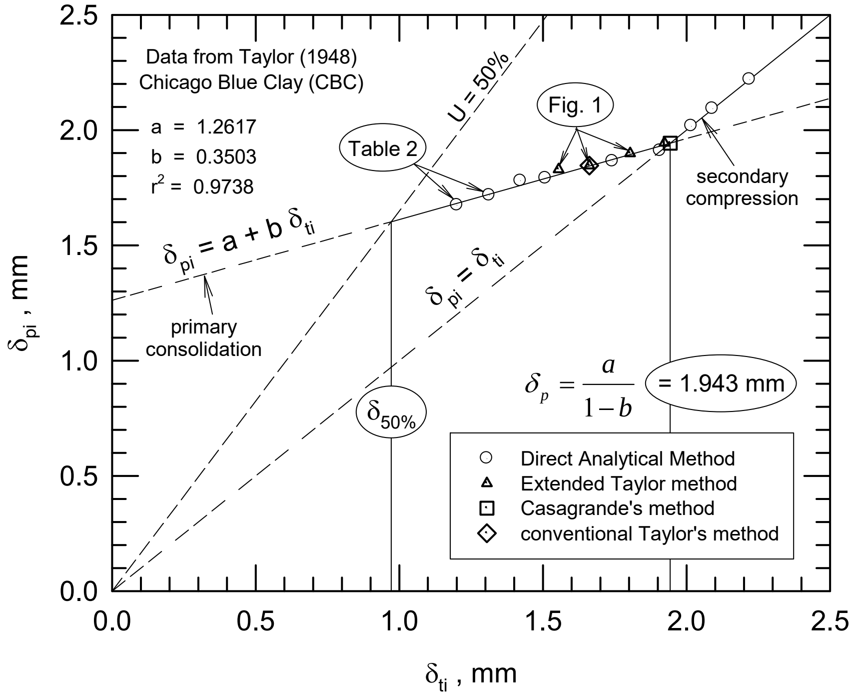


Figure (2): The δ_{pi} - δ_{ti} relationships obtained by the extended Taylor method (ETM) and direct analytical method (DAM); data from Taylor (1948) for Chicago blue clay

Based on Figs. 2 and 3, two types of end-of-primary (EOP) settlement are distinguished. The first type is called the local EOP settlement δ_{pi} (Eq. 16), which is the settlement determined by matching the observed compression-time curve to the Terzaghi theory at a single U_i value by using three consolidation data points (two points at $U \leq 52.6\%$ and one point at $U \geq 52.6\%$) as is the case in the Taylor method at U_i of 90%. The second type is called the global EOP settlement δ_p (Eq. 19), which is the settlement determined by forward extrapolation using four or more consolidation data points (two points at $U \leq 52.6\%$ and two points at $U \geq 52.6\%$). The extended Taylor method (ETM) yields both local and global EOP settlements.

Validity of the Proposed ETM Method

Comparison with Experimental Compression-Time Curves

The validity of the extended Taylor method (ETM) is verified by comparing the experimental results with the Terzaghi theory throughout the entire primary consolidation stage using c_v and δ_p values obtained by the extended Taylor method. Excellent agreement exists between the experimental and theoretical U - T curves in the primary consolidation stage as shown in

Fig. 4 for three clays (Azraq green, Madaba and Chicago blue clays). It should be pointed out that the experimental curves deviate from each other and from the Terzaghi theoretical relationship in the secondary compression stage.

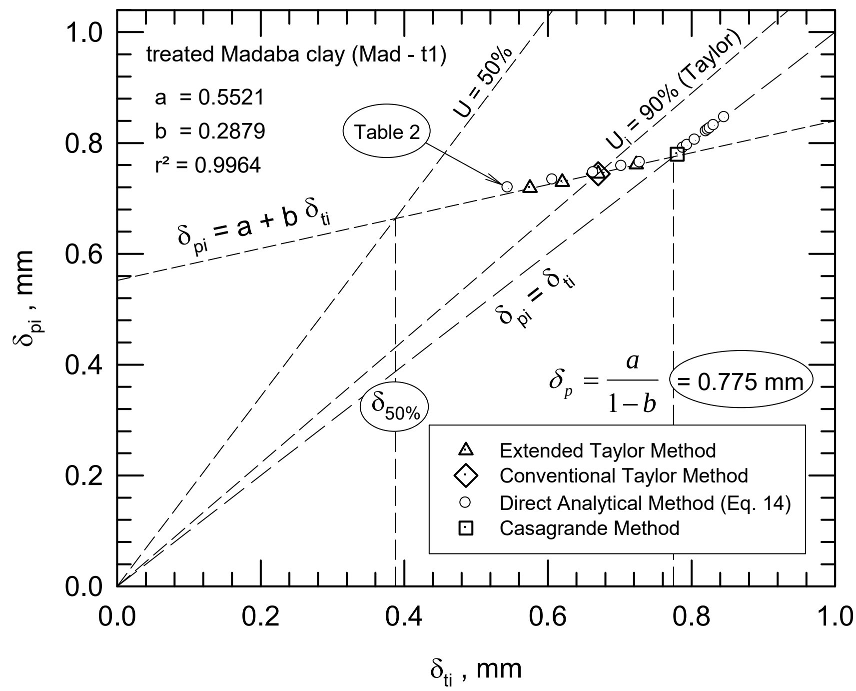


Figure (3): The δ_{pi} - δ_{ti} relationships obtained by the extended Taylor method (ETM) and direct analytical method (DAM); treated Madaba clay (Mad-t1)

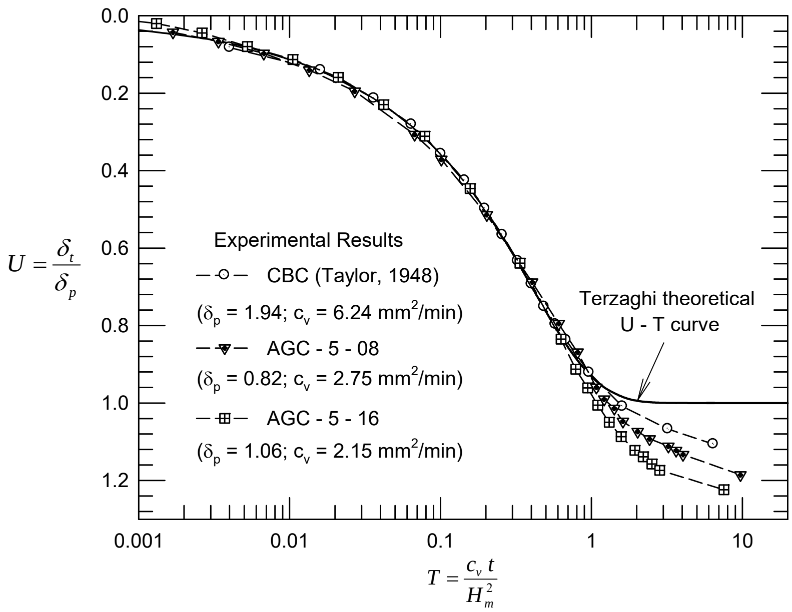


Figure (4): Comparison of the experimental U - T curves obtained using the extended

Taylor method (ETM) for three soil specimens with the Terzaghi theoretical relationship

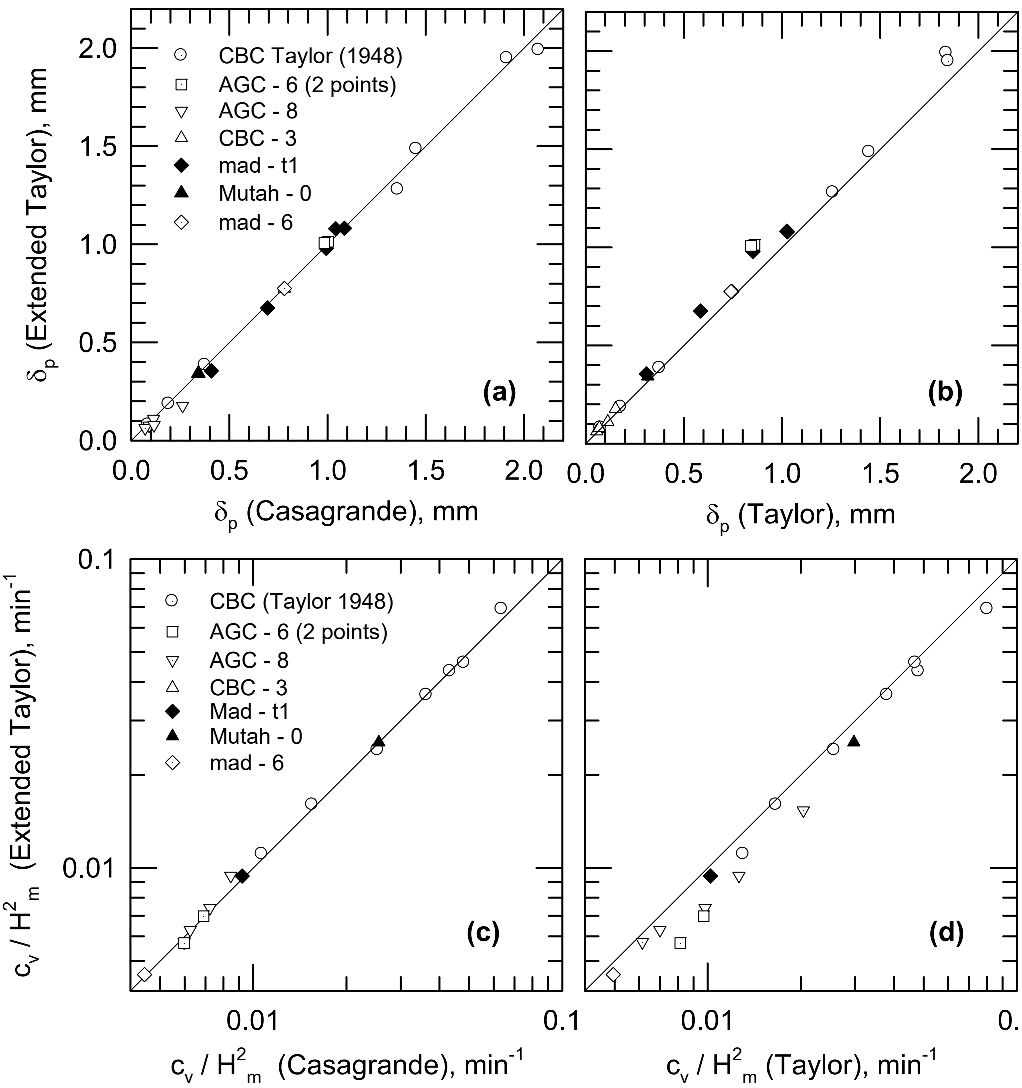


Figure (5): (a) and (b) Comparison of δ_p values of the extended Taylor method with those of the Casagrande and Taylor methods; (c) and (d) Comparison of c_v values of the extended Taylor method with those of the Casagrande and Taylor methods; for the cases where c_v is practically constant

Comparison with the Direct Analytical Method (Al-Zoubi, 2008a)

Al-Zoubi (2008a) showed that the EOP settlement δ_{pi} can be given, based on Eqs. 2 to 6, by the following rigorous expression (Eq. 20):

$$f\left(\delta_{pi},\delta_{ti},t_i\right)=\ln\left(1-\frac{\delta_{ti}}{\delta_{pi}}\right)-\ln\frac{8}{\pi^2}+1.94\frac{m^2}{\delta_{pi}^2}t_i=0 \quad (20)$$

In order to solve Eq. 20 for δ_{pi}, three data points {i.e., (t_1, R_1), (t_2, R_2) and (t_i, R_{ti})} must be selected from the consolidation data. The first two data points

(t_1, R_1) and (t_2, R_2) are required for obtaining the initial compression R_0 (Eq. 5) and the slope m of the initial linear portion of the δ_t - $\sqrt{t}$ curve (Eq. 7). The third data point (t_i, R_{ti}) can be taken at any time beyond the initial linear portion (theoretically, at $U \geq 52.6\%$).

The solution of Eq. 20 using the selected three data points requires iterations for obtaining the EOP δ_{pi} (and then c_{vi} by Eq. 6); this solution can be obtained graphically or numerically by using any method for finding the roots of an equation (Al-Zoubi, 2008a). It should, however, be mentioned that Al-Zoubi (2014) developed a non-iterative procedure for solving Eq. 20.

Figures 2 and 3 show that the linear relationships between EOP δ_{pi} and δ_{ti} are practically the same for both the direct analytical method (DAM) and the extended Taylor method (ETM). Consequently, the global EOP δ_p and c_v values obtained by the extended Taylor method (ETM) are almost identical to those of the direct analytical method (DAM).

The conventional Taylor procedure can be shown to be a graphical solution of the direct analytical method (DAM). Substituting $\delta_{ti} = \delta_{90}$ and $t_i = t_{90}$ into Eq. 20 yields a value of 1.15 for the ratio of the secant slope at 50% to that at 90% used by the Taylor method, where $\delta_{90} = 0.90\delta_{pi}$. The EOP δ_{pi} values obtained by the Taylor method in which the third point (t_i, δ_{ti}) is taken at $U = 90\%$ are shown in Figs. 2 and 3. These δ_{pi} values of the Taylor method (open diamond) are on the linear relationships between δ_{pi} and δ_{ti} in primary consolidation obtained by the direct analytical and extended Taylor methods.

Hence, the conventional Taylor $\sqrt{t}$ method (Taylor, 1948) is theoretically correct when only three data points are used. However, the Taylor method inherently includes limitation due to the fitting of the experimental compression-time curve in which the actual time to EOP consolidation exhibits a definite value (i.e., t_p) to the Terzaghi theory in which the theoretical time to EOP consolidation is infinity (Al-Zoubi, 2014). This limitation of the standard Taylor method is overcome in this study by introducing the extended Taylor method (ETM) that uses forward extrapolation to obtain the global EOP δ_p.

On the other hand, the EOP δ_p values of the standard $\log t$ method (open square) are quite comparable to those (global EOP δ_p values) of the extended Taylor and direct analytical methods (Figs. 2 and 3). The $\log t$ method yields global EOP settlement δ_p as it is obtained by extrapolating the primary consolidation data to the secondary compression range.

Comparison of the Proposed Extended Taylor Method (ETM) with the Taylor and Casagrande Methods

The conventional Taylor $\sqrt{t}$ method (Taylor, 1948) generally yields lower δ_P and higher c_v values than those of the $\log t$ method (Casagrande and Fadum, 1940) as reported in the geotechnical engineering literature (Lambe and Whitman, 1969; Hossain, 1995; Sridharan and Prakash, 1995; Robinson, 1999; Al-Zoubi, 2010, 2013). The δ_P values of the Taylor method may range from 0.5 to 1 of those of the $\log t$ method, whereas the c_v values of the Taylor method may range from 1 to 4 times those of the $\log t$ method (Al-Zoubi, 2008b).

The differences in c_v values estimated by the existing methods for a particular pressure increment may chiefly be attributed to two main reasons. Firstly, the different procedures used to estimate the EOP settlement may yield different δ_p values and thus different c_v values are obtained. Secondly, the use of pressure increments in which c_v is not constant, contrary to the assumption of the Terzaghi theory, also contributes to the differences observed in the computed c_v values. The extended Taylor method (ETM) was developed for improving the procedure of estimating the EOP settlement δ_p using the $\sqrt{t}$ method as compared to the $\log t$ method that yields almost identical δ_p values to those obtained from pore water pressure measurements (Mesri et al., 1999b; Robinson, 1999). Moreover, only cases where c_v is practically constant are considered in this study to validate and compare the ETM with the standard $\log t$ and $\sqrt{t}$

methods to obtain more reliable results for evaluating and comparing these methods developed based on the Terzaghi theory that assumes constant c_v.

Experimental results of oedometer tests show that the δ_p and c_v values obtained by the extended Taylor method (ETM) are quite similar to those of the standard $\log t$ method as demonstrated in Figs. 5(a) and (c) for cases where c_v is practically constant (data points in this case mostly correspond to the normally consolidated range). These results also show that the EOP δ_p values of the conventional Taylor method are lower than those of the extended Taylor and Casagrande methods as shown in Fig. 5(b), whereas the c_v values of the Taylor method are higher than those of the extended Taylor and Casagrande methods as shown in Fig. 5(d).

Graphical Procedure for the Extended Taylor Method (ETM)

The extended Taylor method (ETM) can be performed by using the experimental δ_t - $\sqrt{t}$ curve in the same graphical procedure as the conventional Taylor method (Fig. 1), but the procedure is repeated twice using any two different U_i values instead of the one used in the Taylor method at 90%. The procedure of ETM can be summarized as follows:

A. Plot the compression - root time (R_t - $\sqrt{t}$) curve.

B. Obtain the initial compression R_0 and slope m either graphically as shown in Fig. 1 or select two data points [(t_1, R_1), (t_2, R_2)] such that these two points are on the initial linear portion of the δ_t - $\sqrt{t}$ curve, and then compute R_0 from Eq. 5 and m from Eq. 7.

C. Arbitrarily select U value (say, U_1) from the later stages of consolidation beyond the initial linear portion (e.g., 80%, 85%, 90%,… etc.) and compute the settlement δ_{t1} ($=\delta_{U1}$) that corresponds to the selected U_1 value using Fig. 1 and the ratio ($ROSS_i$) by Eq. 15 in the same way the 90% consolidation of the conventional Taylor method is obtained.

D. Compute a first local EOP settlement $\delta_{p1} = \delta_{t1} / U_1$.

E. Repeat C and D; using another U value (say, U_2).

F. Compute a second local EOP settlement $\delta_{p2} = \delta_{t2} / U_2$.

G. Repeat E and F using other U values if needed, for example, to plot Fig. 2.

H. Calculate a and b using the linear δ_{pi} - δ_{ti} relationship (Fig. 2) or using the two points computed in D and F [i.e., (δ_{p1}, δ_{t1}), (δ_{p2}, δ_{t2})] as follows:

$$b = \frac{\delta_{p2} - \delta_{p1}}{\delta_{t2} - \delta_{t1}} \tag{21}$$

$$a = \delta_{p1} - b\delta_{t1} \tag{22}$$

I. Calculate the global EOP settlement δ_p by Eq. 19.

J. Calculate c_v by Eq. 5.

An illustrative example is provided in the **Appendix** for the pressure increment of Fig. 1.

SUMMARY AND CONCLUSIONS

The Taylor graphical procedure is extended for evaluating the coefficient of vertical consolidation c_v and EOP settlement δ_p. The extended Taylor method (ETM) utilizes the same graphical construction as the conventional Taylor method, but computes c_v and δ_p by using at least two arbitrarily selected U_i values instead of the single value used in the conventional Taylor method at $U_i = 90\,\%$. The extended Taylor method (ETM) can be applied by using a minimum of four compression-time data points for estimating the coefficient of vertical consolidation; at least two data points are required from the early stages of consolidation ($U \le 52.6\,\%$) for back-calculating the initial compression (by backward extrapolation) and the initial slope of the δ_t - $\sqrt{t}$ curve and at least two data points from the later stages of consolidation ($U \ge 52.6\,\%$) for computing the EOP settlement (by forward extrapolation).

In this study, two types of EOP settlement are distinguished; the first type is called the local EOP settlement δ_{pi} obtained by matching the compression-time curve to the Terzaghi theory at a specific U_i value as is the case in the conventional Taylor method; the second type is called the global EOP settlement δ_p obtained by matching the compression-time curve to the Terzaghi theory at two or more U_i values as is the case in the extended Taylor method (ETM) and direct analytical method (DAM). The local EOP settlement δ_{pi} is generally lower than the global EOP δ_p. Therefore, the available methods that yield local EOP δ_{pi} (e.g., Taylor method) generally give higher c_v values than those methods that yield global EOP δ_p (e.g., Casagrande method, direct analytical method and extended Taylor method).

The extended Taylor method computes c_v and δ_p values without the need to use secondary compression. Hence, the extended Taylor method requires as much testing time as the conventional Taylor method but yields c_v and δ_p values quite similar to those of the Casagrande method for the case of constant c_v. The Casagrande method, however, requires longer testing time than both the Taylor and extended Taylor methods. Therefore, the proposed method has the advantages of both the Taylor and Casagrande methods.

Appendix (Example: Graphical Approach)

The graphical procedure of the extended Taylor method (ETM) can be summarized in a few steps. Firstly, the initial compression R_0 and the slope m are estimated from the initial linear portion of the δ_t - $\sqrt{t}$ curve (from Fig. 1, R_0 = 1516 and m = 0.274). Secondly, an arbitrarily selected U_i value is used to obtain a first local EOP settlement in a way similar to the graphical procedure used in the Taylor method at U_i = 90% (from Fig. 1, for U_1 = 85%, $ROSS$ = 1.096 by Eq. 15, δ_{t1} = 1.5550 mm, and thus δ_{p1} = 1.8294 mm by Eq. 16). Thirdly, another arbitrarily selected U_2 value is used to obtain a second local EOP settlement (from Fig. 1, for U_2 = 95%, $ROSS$ = 1.260, δ_{t2} = 1.8039 mm, δ_{pi} = 1.8989 mm). This step can be repeated using additional U_i values (U_3, U_4,… etc.) if needed to plot Fig. 2. Fourthly, the a and b values can be calculated by Eq. 16 if only two U_i values are used or can be obtained graphically from a plot similar to Fig. 2 if more than two U_i values are used. For the pressure increment of Fig. 1, $a = 1.2617$ and b = 0.3503 (Fig. 2) and therefore the global EOP settlement δ_p can be obtained by Eq. 19 as $\delta_p = a/(1-b)$ = 1.942 mm. Finally, the c_v / H_m^2 value can be computed by Eq. 6 as $\pi(0.274/1.942)^2/4 = 0.0156$ min^{-1}. The corresponding values of the Casagrande and Taylor methods for this particular pressure increment (Fig. 1) are 0.0155 and 0.0173, respectively; the c_v value computed by the ETM is quite similar to that of the Casagrande method.

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