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A Novel Oppositional Modified Adaptive Weight-Based Arithmetic Optimization for Time-Cost Trade-off Problems in Construction Projects

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ABSTRACT

Time-Cost Trade-off Problems (TCTPs) represent a critical class of multi-objective optimization challenges in construction project management, requiring the simultaneous minimization of project duration and total cost. To address these conflicting objectives more efficiently, this study proposes a novel hybrid multi-objective optimization framework, termed Oppositional Modified Adaptive Weight-Based Arithmetic Optimization Algorithm (Oppositional MAWA-AOA). The main novelty of the proposed approach lies in the first integration of Opposition-Based Learning (OBL) within the MAWA-AOA framework, where OBL is applied exclusively during the iteration-level updating phase to enhance exploration and maintain population diversity. The MAWA mechanism dynamically adjusts the relative importance of time and cost objectives throughout the search process, enabling adaptive traversal of different regions of the Pareto-optimal front. The performance of the proposed algorithm is evaluated using three benchmark construction project networks comprising 63, 81, and 146 activities. The results demonstrate that Oppositional MAWA-AOA consistently produces high-quality and well-distributed Pareto-optimal solutions with strong convergence characteristics. In terms of solution quality, the proposed method achieves competitive hypervolume (HV) values across all cases and attains the highest HV value of 0.650 for the medium-scale 146-activity project, highlighting its effectiveness in handling complex optimization landscapes. Furthermore, Oppositional MAWA-AOA exhibits exceptional computational efficiency, achieving a reduction of up to 99.27% in the number of function evaluations (NFEs) compared with the hybrid heuristic metaheuristic (HHMH). Comparative analyses against established multi-objective algorithms, including NDSII-TLBO, NDSII-PSO, Aquila Optimizer, Hybrid Genetic Algorithm, and NDS-AOA, confirm the robustness and scalability of the proposed approach. Overall, the results indicate that integrating OBL with MAWA-AOA significantly enhances performance for medium-scale TCTPs, offering a practical and efficient decision-support tool for construction project scheduling.

Keywords: Opposition-based learning, Arithmetic optimization algorithm, Modified adaptive weight approach, Time-cost optimization.

INTRODUCTION

The Critical Path Method reduces project duration by allocating additional resources to critical activities, which often lowers indirect costs, but increases direct expenses. The balance between time reduction and cost increase is defined as the time-cost trade-off problem, the objective of which is to minimize total project cost while considering both factors. As the number of activities and execution options grows, solving this problem becomes increasingly complex. The significance of TCTP has been recognized since the early development of CPM (Tran, 2020; Pham et al., 2024). Early studies assumed a continuous relationship between time and cost, where duration changes directly affected project cost (Siemens, 1971; Feng et al., 2000).

Discrete time-cost trade-off problems have attracted considerable attention because of their broad practical applications (Afshar et al., 2009; Eirgash et al., 2019; Agarwal et al., 2024; Kumar et al., 2024). Unlike the continuous form, the discrete TCTP represents each activity using a finite set of execution modes, each with specific time and cost values. This structure better reflects real construction practice, where activities are performed using predefined alternatives. As a result, discrete TCTP provides a more realistic and flexible framework for project optimization (Vanhoucke & Debels, 2007).

In recent years, metaheuristic algorithms have become widely used for multiobjective optimization because of their adaptability, robustness, and ability to escape local optima (Deb et al., 2002; Toğan & Eirgash, 2019a; Kumar et al., 2024; Maurya et al., 2025). Population-based methods, such as Genetic Algorithms, Particle Swarm Optimization, Teaching Learning-Based Optimization, and Jaya, are frequently applied in project scheduling and resource allocation (Zheng et al., 2005; Rao, 2016; Aminbakhsh & Sonmez, 2017; Eirgash & Toğan, 2023b). These techniques effectively explore complex search spaces and produce high-quality solutions for challenging engineering problems.

Despite progress, solving time-cost trade-off problems remains difficult. Traditional approaches include mathematical programming and heuristic methods (Afshar et al., 2009), but mathematical models become computationally expensive for large projects, while heuristics often yield inconsistent or near-optimal results (Aminbakhsh & Ahmed, 2023; Eirgash & Toğan, 2024). Moreover, these methods struggle with multiobjective tradeoffs. Consequently, metaheuristic algorithms have emerged as flexible and efficient alternatives for handling complex, large-scale, and multiobjective TCTPs (Aminbakhsh & Sonmez, 2016).

Researchers increasingly employ advanced and hybrid optimization strategies to handle the complexity of time-cost trade-off problems and obtain more robust solutions (Panwar & Jha, 2019; Eirgash & Toğan, 2023b; Bettemir & Yücel, 2023; Patil et al., 2024). By combining the strengths of different algorithms, hybrid models enhance solution quality and mitigate issues, such as premature convergence and weak global search (Hu & He, 2014). For instance, Banihashemi and Khalilzadeh (2021) developed a GA PSO hybrid for sustainable TCTPs, while recent studies have hybridized NSGA II with PSO and Ant Colony Optimization to improve convergence and maintain Pareto diversity (Chauhan & Kumar, 2023). These approaches strengthen the exploration exploitation balance and are well suited for complex construction scheduling and resource-optimization problems.

Recently, Abualigah et al. (2021) introduced the Arithmetic Optimization Algorithm (AOA) as a new metaheuristic optimization technique. The AOA is based on fundamental arithmetic operations, such as multiplication, division, subtraction, and addition, and is designed to navigate complex search spaces effectively. Its mathematical underpinning allows it to balance exploration with exploitation, making it useful for a variety of optimization problems. In their original study, they demonstrated that the AOA outperformed 11 well-known metaheuristic algorithms in solving various complex optimization problems.

AOA has been successfully applied to various engineering design problems, including welded beam, spring, pressure vessel, truss, and speed reducer optimization (Nurmuhammed et al., 2024), and has also been used for time-cost trade-off problems in construction planning (Sulub et al., 2024). However, it faces limitations in large and complex problems, such as premature convergence and imbalance between exploration and exploitation (Kaveh & Hamedani, 2022). To address these issues, improved AOA variants have been developed using adaptive parameter control, learning strategies, hybrid models, and enhanced initialization methods (Aminbakhsh & Sönmez, 2017; Bettemir & Sönmez, 2012; Eirgash et al., 2023a).

Despite these advancements, AOA-based algorithms may still face difficulties in handling large-scale, complex engineering problems. In this context, Opposition-Based Learning (OBL), first introduced by Tizhoosh (2005), has emerged as a notable approach and has garnered significant attention over the last decade (Mahdavi et al., 2018; Xu et al., 2020; Huang & Hu, 2024). OBL assesses both a candidate solution and its opposite, which enables the expedited convergence and improvement of the search process in complex problem domains.

Gen and Cheng (2000) first applied the Adaptive Weight Approach for time-cost optimization by converting multiobjective problems into a single objective form. Zheng et al. (2004) later developed MAWA, which combines weighted objectives into one function. However, Toğan et al. (2022) reported reduced performance of MAWA on medium and large problems due to limited exploration. To address this issue, the present study introduces an oppositional MAWA approach that improves efficiency for complex TCTPs while maintaining solution quality comparable to NDS in small and medium cases. More to the point, the non-dominated sorting (NDS) method is widely applied to solve TCTPs due to its effectiveness in ranking solutions based on Pareto dominance in multiobjective optimization. However, when dealing with large populations and multiple objectives, it often leads to increased computational time (Sulub et al., 2024).

The contribution of this study to the existing literature on the TCTP in construction management (Afshar et al., 2009; Eirgash et al., 2019; Toğan et al., 2022; Sulub et al., 2024) is twofold: (i) introducing an opposition-based learning (OBL) concept to enhance the balance between exploration and exploitation; and (ii) proposing a multi-objective optimization approach that integrates the AOA with the MAWA, and applying it to large-scale TCTP cases to demonstrate the capability of AOA in generating high-quality Pareto front solutions.

Accordingly, the primary aim of this research is to develop a novel oppositional adaptive weight strategy as an alternative to the conventional MAWA for solving multi-objective TCTPs. Notably, despite the extensive application of oppositional-based learning and MAWA-based optimization techniques in construction scheduling (Zheng et al. 2004; Eirgash & Dede, 2018; Sulub et al., 2024), no previous study has integrated oppositional-based learning with the MAWA framework. This study addresses this gap by proposing, for the first time, an OBL-enhanced MAWA-based optimization approach for solving time-cost trade-off problems.

To validate the performance of the proposed oppositional MAWA-AOA algorithm, three benchmark case studies from the construction literature were examined, involving 63, 81, and 146 activities, respectively. The results were compared with those of established models that rely on approximation-based Pareto fronts or heuristic-based near-optimal solutions (Feng et al., 2000; Hegazy & Ayed, 1998; Zheng et al. 2004). The performance was benchmarked against recent algorithms, including the TLBO, Aquila optimizer, and NDS-AOA (Sulub et al., 2024), PSO (Aminbakhsh & Sonmez, 2016), and HGA (Sonmez & Bettemir, 2012). The results show that the Oppositional MAWA-AOA obtained satisfactory solutions, demonstrating its applicability and robustness in addressing multiobjective scheduling problems in construction engineering and management. As a result, this study provides the following significant contributions to the domains of building project optimization and metaheuristic algorithm development:

- A hybrid multiobjective model combining AOA, MAWA, and OBL is proposed to improve solution quality for complex time-cost trade-off problems.
- OBL increases population diversity and speeds up convergence by evaluating solutions together with their opposites.
- MAWA adaptively balances multiple objectives, improving the trade-off between exploration and exploitation.
- The method is validated on construction projects with 63, 81, and 146 activities, with the latter two examined for the first time.
- Performance comparisons using metrics, such as HV and Sp, show competitive or superior results in diversity and optimality.

Table 1 summarizes the parameter settings used for the applied methods. The algorithm parameters were selected based on their original studies, where these values showed strong performance.

Table 1. Parameter setting for the proposed algorithm

Algorithm[*]	**Setting**	**Description**
TLBO (Rao et al., 2010)	TF = round [1 + rand(0,1) {2 −1}]	Teaching factor
AO (Abualigah et al., 2021b)	α =0.10 δ =0.10	The exploitation adjustment parameters
AOA (Abualigah et al., 2021a)	1. μ =0. α =5	Control parameter to adjust the search process Sensitive parameter defining the exploitation accuracy over the iterations
OBL (Tizhoosh, 2005)	Jr =0.3	Jumping condition
[*] TLBO: Teaching learning-based optimization, AO: Aquila optimizer, AOA: Arithmetic optimization algorithm.		

Table 2 presents a comparison of studies that applied the MAWA and NDS approaches to solve TCTPs, along with the scale of the problems addressed.

Table 2. A review of NDS and MAWA approaches for trade-off problems

Author (s)	**Algorithm**	**Approach**	**Activities**	**Project size**
Zheng et al. (2004)	GA	MAWA	7	Small
Zheng et al. (2005)	GA	MAWA	7	Small
Afshar et al. (2009)	ACO	NDS	18	Small
Aminbakhsh and Sönmez (2017)	PSO	NDS	18, 180, 360, 720	Small and medium
Luong, Tran, and Nguyen (2018)	DE	NDS	18	Small
Eirgash and Dede (2018)	TLBO	MAWA	18, 63	Small
Toğan and Eirgash (2019b)	TLBO	NDS	18, 63 and 630	Small, medium and large
Albayrak (2020)	PSO	NDS	37	Small
Toğan and Eirgash (2019a)	TLBO	MAWA	7, 18, and 63	Small and medium
Toğan et al. (2022)	GA, TLBO, Jaya	MAWA	18, 63 and 630	Small, medium and large
Bettemir 2023	GA	Linear scheduling via LOB	19	Small
Agarwal et al. (2024)	PSO	NDS	19	Small
Pham et al. (2024)	MVO	NDS	18, 19, 69, and 290	Small, medium and large
Sulub et al. (2024)	AOA	NDS	63, 81, and 146	Medium and large
Eirgash et al. (2026)	MGO	NDS	18, 37	Small
Toğan et al. (2025)	R-AOA	NDS	81, 146, 208, and 291	Medium and large
This study	AOA	OBL MAWA	63, 81, and 146	Medium and large

The study begins with the formulation of the time-cost optimization model, followed by a description of the MAWA method and the AOA and OBL optimizers used for solving TCTPs. It then presents numerical results and comparisons to demonstrate the effectiveness of the proposed approach. Figure 1 provides a flowchart of the Oppositional MAWA-AOA procedure.

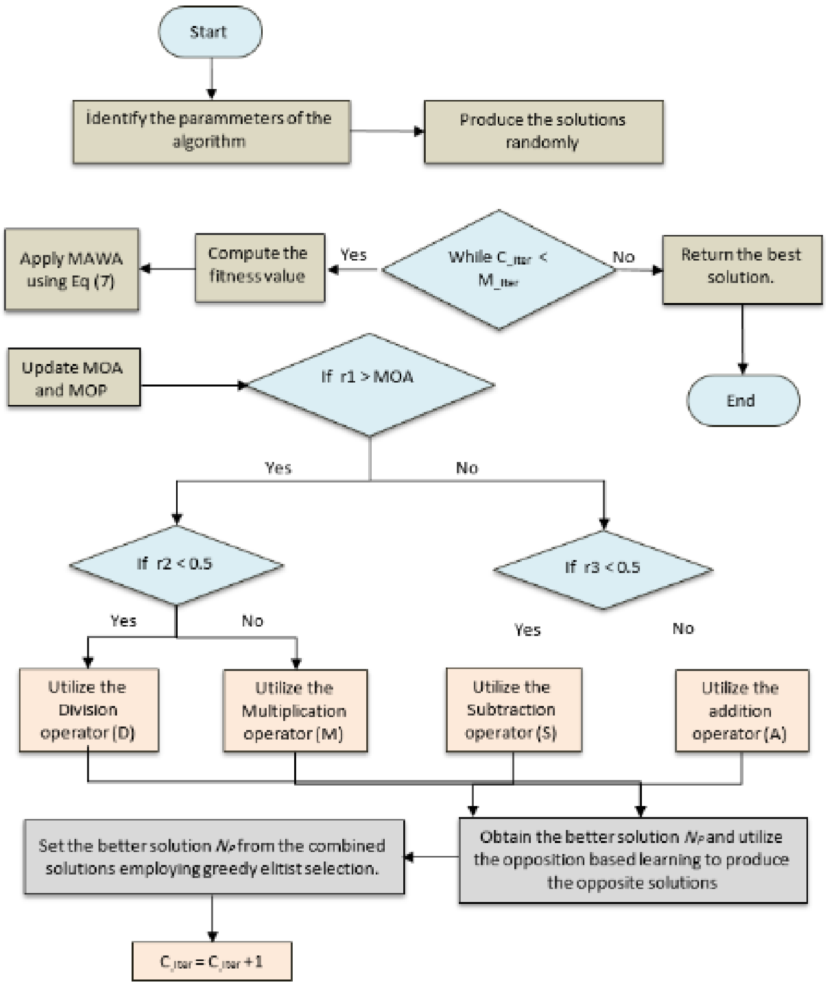


Figure 1. The flow chart of oppositional MAWA-AOA

BACKGROUND KNOWLEDGE

In this section, we provide a fundamental overview of the TCTP, AOA and OBL techniques, specifically aiming to introduce essential concepts necessary for understanding the rest of the paper.

Problem Formulation of Trade-off Problems

The trade-off optimization process aims to reduce both project duration and cost at the same time, identifying optimal solutions that can be applied across all activities in construction projects.

Objective 1: Minimizing of project time (PT): The minimization of project duration is formulated as follows:

$$PT = \sum_{i=1}^{n} T_i = Max(ES_i + d_i)$$

$$ES_i = \underset{all\ predecessors\ j\ of\ i}{Maximum}(ES_j + d_j) \tag{1}$$

where T_i represents the time required for activity i $\{i = 1, 2\ldots n\}$, n being the total number of critical activities on the given critical path; ES_i denotes the earliest start time of activity i; and d_i indicates the duration of activity i.

Objective 2: Minimizing of project cost (PC): The minimization of project cost is formulated as follows:

$$PC = \sum_{i=1}^{n} \cos t_i\ , \tag{2}$$

where $cost_i$ represents the cost associated with activity i based on a specific execution method, and n denotes the total number of activities.

Modified Adaptive Weight Approach (MAWA) in Multi-objective Optimization

To guide the search and balance tradeoffs, MAWA uses a solution quality-based weighting technique. According to Zheng et al. (2004), the adaptive weighting process in MAWA follows four critical conditions, which are described as follows:

1. $S_c^{max} \neq S_c^{min}$ ve $S_t^{max} \neq S_t^{min}$

$$V_c = {S_c}\Big/\left(S_c^{max} - S_c^{min}\right) \qquad Wc = V_c / \left(V_c + V_t\right) \tag{3}$$

$$V_t = {S_t}\Big/\left(S_t^{max} - S_t^{min}\right) \qquad Wt = V_t / \left(V_c + V_t\right)$$

2. $S_c^{max} = S_c^{min}$ and $S_t^{max} = S_t^{min}$

$$W_c = 0{,}5 \qquad W_t = 0{,}5 \tag{4}$$

3. $S_c^{max} \neq S_c^{min}$ and $S_t^{max} = S_t^{min}$

$$W_c = 0{,}1 \qquad W_t = 0{,}9 \tag{5}$$

4. $S_c^{max} = S_c^{min}$ and $S_t^{max} \neq S_t^{min}$

$$W_c = 0{,}9 \qquad W_t = 0{,}1 \tag{6}$$

Let S_t^{max} and S_t^{min} represent the project duration goal function's maximum and minimum values in the current iteration. Similarly, let S_c^{max} and S_c^{min} denote the highest and lowest values of the total direct cost objective function in the same iteration. The parameters v_t and v_c are defined as the ratios between the minimum values and the ranges for project duration and total cost, respectively, and are as follows:

$$F(x) = W_t * \left(\frac{S_t - S_t^{min} + m)}{S_t^{max} - S_t^{min} + m)}\right) + W_c * \left(\frac{S_c - S_c^{min} + m}{S_c^{max} - S_c^{min} + m}\right) \quad m = 0{,}001 \tag{7}$$

Here, X denotes a candidate solution with fitness $F(X)$); S_c and S_t represent its total cost and duration. The parameter $m \in (0,1)$ is a small random value, and w_c and w_t are adaptive cost and time weights. The term mmm is added to Eq. (7) to avoid $S_c^{max} = S_c^{min}$ or $S_t^{max} = S_t^{min}$, following Zheng et al. (2004).

Arithmetic Optimization Algorithm (AOA)

Abualigah et al. (2021) introduced the Arithmetic Optimization Algorithm, a population-based method that uses basic arithmetic operators to guide the search. Starting from random solutions, it iteratively balances global exploration and local exploitation. Figure 2 illustrates these operations.

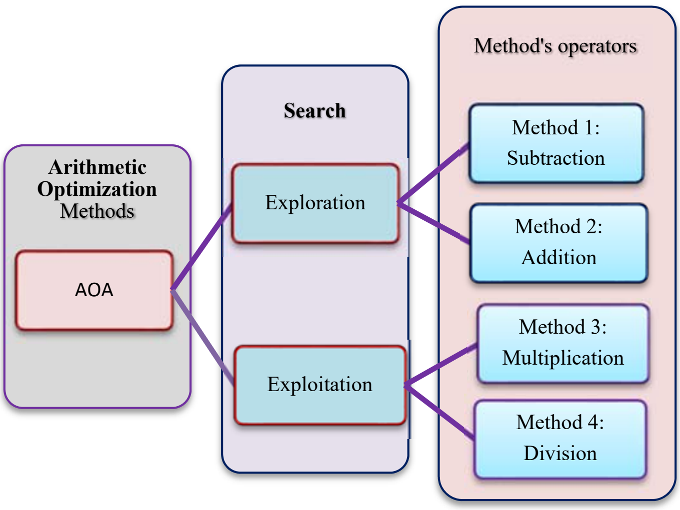


Figure 2. Mathematical operations in AOA

AOA starts with the initialization phase, during which a matrix (**X**) of candidate solutions is generated, as shown in Equation (8). This matrix has dimensions ($m \times n$) denoting the population size (i =1, …, m) and the number of design variables (j=1, …, n). Each element in this matrix corresponds to a potential value for a specific variable in a given solution, forming the initial search space for the optimization process.

$$\mathbf{X} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,n-1} & x_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n-1} & x_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{m-1,1} & x_{m-1,2} & \cdots & x_{n-1,n-1} & x_{m-1,n-1} \\ x_{m,1} & x_{m,2} & \cdots & x_{m,n-1} & x_{m,n} \end{bmatrix} \quad (8)$$

Following initialization, AOA selects the best solution in the population, referred to as X_{best}, based on objective function values. This solution acts as a reference for guiding the search. To determine whether the next step will involve exploration or exploitation, a coefficient called the Math Optimizer Accelerator (MOA) is computed using Equation (9).

$$MOA\ (C_{Iter}) = Min + C_{Iter} \times \left(\frac{Max - Min}{M_{Iter}}\right) \quad (9)$$

MOA (C_{Iter}) represents the MOA value at iteration C_{Iter}, where M_{Iter} is the maximum number of iterations and *Min* and *Max* are its bounds. A random number r1 in [0, 1] determines the search phase. If r1 exceeds MOA, the algorithm performs exploration using division and multiplication operators, as defined in Equation (10).

$$X_i(C_{Iter} + 1) = \begin{cases} X^{best} \div (MOP + \varepsilon) \times \big((UB - LB) \times \mu + LB\big), & r2 < 0.5 \\ X^{best} \times MOP \times \big((UB - LB) \times \mu + LB\big), & \text{otherwise} \end{cases} \quad (10)$$

In Equation (10), $X_i(\mathrm{C_{Iter}} + 1)$ represents the updated position of the i^th solution in the next iteration. This update is guided by the best solution found so far X^{best}, with parameters including a small constant ε, the variable bounds (UB and LB), a fixed control parameter μ=0.5, and a random number r2∈(0,1). Additionally, the MOP, calculated using Equation (11), plays a role in adjusting the search dynamics during the exploration phase.

$$MOP\ (C_{Iter}) = 1 - (C_{Iter}/M_{Iter})^{\frac{1}{\alpha}} \quad (11)$$

At iteration MOP ($\mathrm{C_{Iter}}$), the function value MOP ($\mathrm{C_{Iter}}$) is evaluated. A constant parameter α =5 controls the exploitation accuracy. When a random number r1 is less than or equal to MOA, the algorithm performs exploitation by updating the solution using either addition or subtraction, as defined in Equation (12).

$$X_i(C_{Iter} + 1) = \begin{cases} X^{best} - MOP \times \big((UB - LB) \times \mu + LB\big), & r3 < 0.5 \\ X^{best} + MOP \times \big((UB - LB) \times \mu + LB\big), & \text{otherwise} \end{cases} \quad (12)$$

All variables and parameters in Equation (12) retain their previously defined meanings, with r3 being a random number. The algorithm repeats these update procedures until it reaches the predefined termination criterion, typically the maximum number of iterations.

Opposition-Based Learning (OBL)

OBL, introduced by Tizhoosh (2005), improves search performance by evaluating a solution together with its opposite. Rahnamayan et al. (2007) reported that opposite solutions are more likely to be closer to the global optimum than random ones. In Oppositional MAWA-AOA, this strategy is applied only during the iteration phase. After position updates, opposite solutions are generated and the better candidate is retained. Figure 3 illustrates opposite points in a D-dimensional space.

$$X_j^{\wedge} = ub_j + lb_j - X_j, \quad j = 1, \cdots, D \quad (13)$$

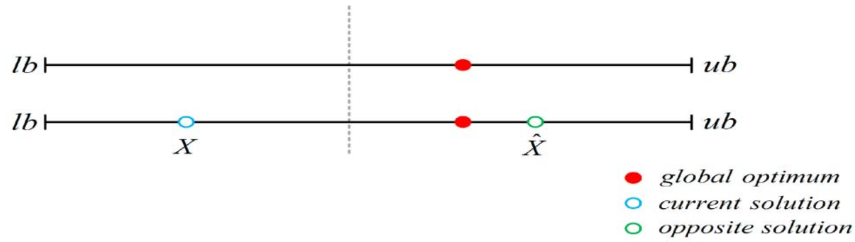


Figure 3. The classic OBL approach

Oppositional MAWA-AOA for TCTP Problems

The process of incorporating the AOA with the opposition jumping rate-based population to solve TCTP problems is demonstrated in Figure 4.

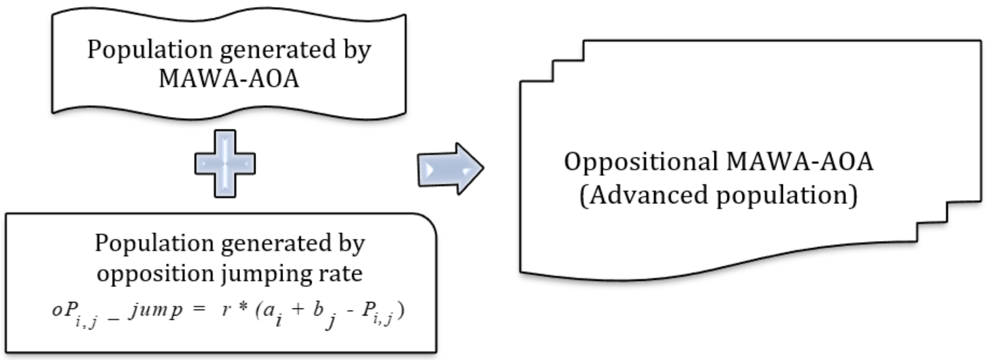


Figure 4. OBL-based population updating

NUMERICAL EXAMPLES

This study evaluates the Oppositional MAWA-AOA model for generating Pareto-optimal solutions to TCTPs using three benchmark cases. Simulations in MATLAB (R2024b) were run 10 times per case to ensure consistent results.

Case Study of 63-activity Project

A 63-activity construction project from Sonmez and Bettemir (2012) is used to test the proposed model. Table 3 (supplementary data) lists execution modes with durations (Dur) in days and total costs in US dollars.

The performance of oppositional MAWA-AOA is compared with hybrid GA (Sonmez & Bettemir, 2012), PSO (Aminbakhsh & Sonmez, 2016), and NDS-AOA (Sulub et al. 2024) for 63 -activity projects, and the simulation results are presented in Table 4 (supplementary data). The comparative results of the 63-activity TCTP problem demonstrate MAWA-AOA's effectiveness in balancing solution quality and computational efficiency. While producing comparable project durations (537-570 days) to other methods, MAWA-AOA achieves these results with significantly fewer computational resources - requiring only 65,250 function evaluations compared to 250,000 for HGA and 162,180 for TLBO. The algorithm's performance is particularly notable in its solution diversity, offering a broader range of time-cost alternatives (537-570 days) than NDS-AOA's more clustered solutions (629-666 days). This suggests that oppositional MAWA-AOA's adaptive weighting mechanism successfully explores the solution space while maintaining computational efficiency. The method's ability to match NDS-AOA's performance with identical NFE counts (65,250), but producing more distributed solutions, highlights its practical advantage for construction scheduling. These results validate oppositional MAWA-AOA as a balanced approach that doesn't sacrifice solution quality for speed, making it particularly suitable for medium-scale projects where the optimization of both performance and computational practicality is an important consideration. The slightly higher project costs observed ($5.77-5.94 million versus $5.41-5.69 million for other methods) represent reasonable tradeoffs for the achieved time reductions and computational savings.

Figure 5 illustrates the graphical representation of the overall average Pareto front solutions obtained by the comparison algorithms for the 63-activity project.

This visualization supports decision-makers in making informed choices during the early stages of project implementation.

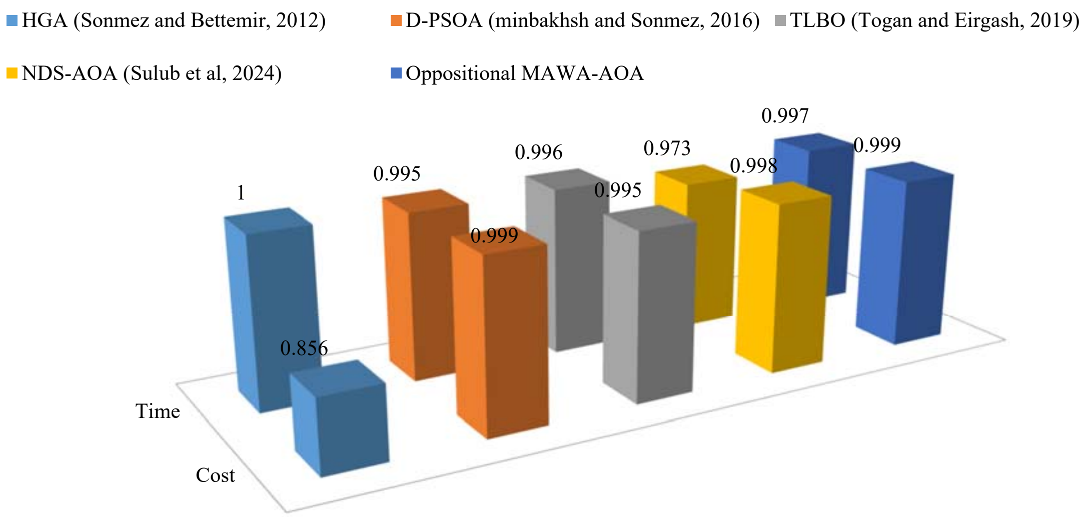


Figure 5. Pareto front solutions of the comparison algorithms for 63-activity problem

Numerical Example of 81-activity Project

A complex construction project involving 81 activities, originally examined by Bettemir and Birgonul (2023), is utilized to assess the performance of the proposed algorithm. Each activity is associated with six distinct execution modes, which significantly increase the scheduling complexity. For the purpose of cost analysis based on project duration, an indirect cost rate of $2,000 per day is applied. Given the combinatorial characteristics of the problem, the total solution space encompasses approximately 1.072×10^{63} possible scheduling configurations.

To evaluate the effectiveness of the proposed Oppositional MAWA-AOA algorithm, its performance is compared with the benchmark results provided by Bettemir and Birgonul (2023), who employed a hybrid heuristic-metaheuristic (HHMH) approach. A summary of the trade-off outcomes is presented in Table 5 (supplementary data). The results clearly show that the Oppositional MAWA-AOA consistently achieves outcomes that are either comparable to or better than existing solutions in terms of time and cost efficiency. For example, at a project duration of 345 days, the proposed algorithm delivers a total cost of $3,491,150, which improves upon the $3,498,200 produced by the classical MAWA-AOA at the same duration. Similar trends are observed for other durations (e.g., 346, 322, and 327 days), where the proposed method remains competitive with the HHMH approach.

In terms of computational efficiency, Oppositional MAWA-AOA offers a significant advantage. While using the same population size (NOP = 200), it requires only 30,000 function evaluations (NFEs) to generate results, compared to the 1,250,000 NFEs needed by the HHMH method. Oppositional MAWA-AOA reduces the number of function evaluations by approximately 99.17% compared to HHMH. Overall, the proposed algorithm emerges as a powerful and time-efficient technique for solving TCTPs in construction planning, providing a suitable alternative to more computationally demanding metaheuristic approaches.

Table 6 (supplementary data) provides a comparative analysis of the proposed Oppositional MAWA-AOA algorithm with three leading multiobjective optimization models reported by Sulub et al. (2024): NDSII-TLBO, NDSII-AO, and NDSII-AOA. All algorithms are applied to the same 81-activity TCTP project to generate efficient Pareto front solutions. A direct comparison with the results of Bettemir and Birgonul (2023) in terms of scheduling performance is not possible, as their study only presented selected trade-off outcomes rather than a complete project schedule or a full set of Pareto-optimal solutions. To ensure a fair and consistent evaluation, the performance of the Oppositional MAWA-AOA algorithm is assessed in comparison with the NDSII-based methods. The results demonstrate that Oppositional MAWA-AOA delivers competitive and high-quality solutions, confirming its effectiveness and suitability as a reliable approach for addressing TCTPs in the field of construction project management.

The comparative analysis shows that NDSII-AOA (Sulub et al., 2024) achieves the shortest project duration (326 days), outperforming other NDSII- and AOA-based methods. In contrast, the HHMH approach (Bettemir & Birgonul, 2023) yields the lowest cost ($3,321,550), but results in the longest duration (389 days). NDSII-TLBO offers a balanced trade-off with 357 days and a cost of $3,371,950. Although Oppositional MAWA-AOA exhibits average cost performance, it significantly improves project duration compared to its classical version, mainly due to opposition-based learning, which enhances diversity and global search ability (Figure 6).

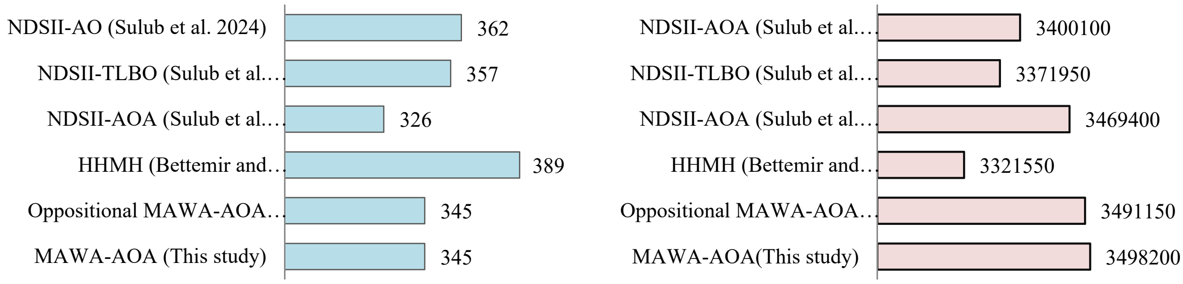


Figure 6. Pareto front solutions of TCTP for 81-activity project

As illustrated in Figure 6, the Oppositional MAWA-AOA performs competitively in terms of duration. Therefore, when minimizing project duration is prioritized over cost, the proposed algorithm offers viable Pareto front solutions comparable to or better than other state-of-the-art approaches.

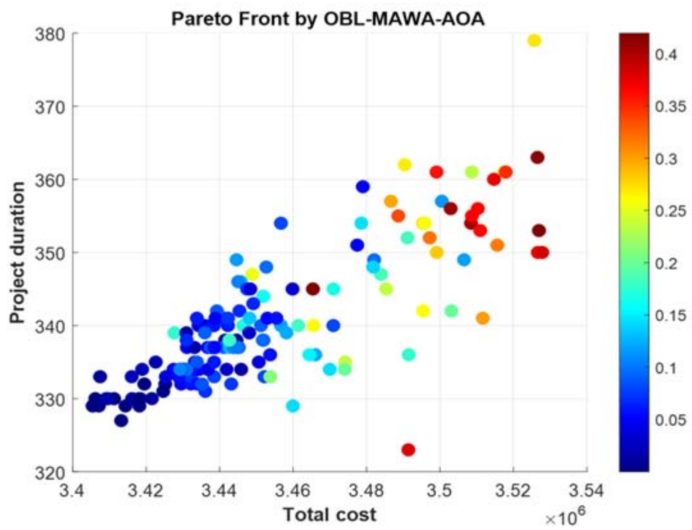


Figure 7. Pareto front solutions for 81-activity project by Oppositional MAWA-AOA

A graphical illustration of the overall average Pareto front solutions generated by the compared algorithms for the 81-activity project is presented in Figure 7. This visualization serves as a valuable decision-support tool, enabling project planners and stakeholders to make well-informed choices during the early phases of project execution.

Case Study of 146-activity Project

A large-scale construction project comprising 146 activities, initially examined by Bettemir and Birgonul (2023), is employed as a benchmark case to further assess the effectiveness of the proposed optimization models. For this analysis, an indirect cost rate of $4000 per day is applied. Due to the combinatorial nature of the problem, the scheduling complexity is substantial, encompassing approximately 1.12×10^{102} possible combinations.

While the original study by Bettemir and Birgonul (2023) presented only a single preferred solution, the Oppositional MAWA-AOA algorithm generates a comprehensive Pareto front, offering a diverse set of trade-off solutions between project cost and duration. These results, presented in both tabular and graphical forms, provide project managers with deeper insights, enabling more informed and flexible decision-making during the planning process. Oppositional MAWA-AOA reduces the number of function evaluations by approximately 99.23% compared to HHMH algorithm.

Table 7 (supplementary data) displays the trade-off outcomes produced by the comparison algorithms, illustrating the cost-duration relationships along the Pareto front. As the original study by Bettemir and Birgonul (2023) focused solely on a single optimal outcome without offering Pareto front solutions, a direct comparison can not be made. To establish a more reliable performance benchmark, the results from the proposed Oppositional MAWA-AOA are also compared against several state-of-the-art multiobjective algorithms, including NDSII-TLBO, NDSII-AOA, and NDSII-AO, reinforcing its competitiveness and practical applicability for complex TCTPs in construction project management.

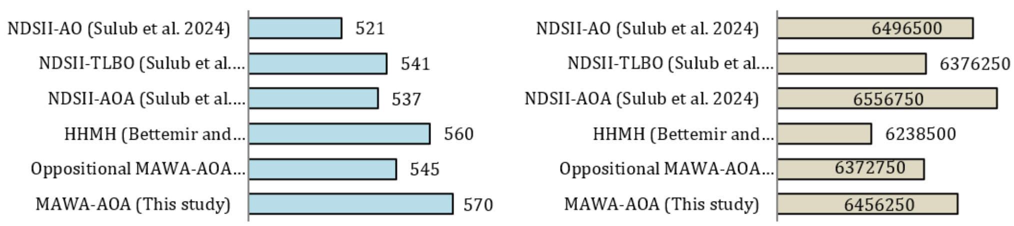
The findings summarized in Table 8 (supplementary data) present a detailed comparative evaluation of the proposed Oppositional MAWA-AOA algorithm against three cutting-edge multiobjective optimization techniques, such as NDSII-TLBO, NDSII-AO, and NDSII-AOA applied to a 146-activity TCTP. From a solution quality perspective, the Oppositional MAWA-AOA demonstrates an impressive balance between PCT and PCC, generating Pareto-optimal solutions ranging from 530 to 594 days for PCT and $6,237,250 to $ 6,591,250 for PCC.

Although NDSII-AOA yields faster schedules with PCTs as low as 487 days, these solutions incur substantially higher costs, reaching up to $6,755,750. In contrast, the Oppositional MAWA-AOA achieves a more balanced tradeoff, offering competitive completion durations without excessive overheads. This balance makes it particularly effective for real-world construction applications where cost control is a primary constraint.

While NDSII-AOA excels in minimizing duration and NDSII-AO offers slight enhancements in schedule time, both methods are less efficient in terms of cost and computational resources. Notably, the normalized NFEs for the Oppositional MAWA-AOA represent only about 97% of those required by the HHMH algorithm. This highlights the computational efficiency of the proposed algorithm, reinforcing its practicality for large-scale, complex TCTP scenarios.

A direct evaluation of scheduling efficiency with the previously developed model by Bettemir and Birgonul (2023) is not possible, as their study provided only a single preferred solution rather than a complete schedule or tabulated set of alternatives. To facilitate a fair and meaningful comparison, the performance of the Oppositional MAWA-AOA algorithm is assessed against three contemporary multiobjective optimization methods: NDSII-TLBO, NDSII-AOA, and NDSII-AO.

Figure 8, derived from the data presented in Table 8 (supplementary data), graphically illustrates the effectiveness of the Oppositional MAWA-AOA in achieving shorter project durations. In essence, when the primary objective is to minimize project duration, the approximate Pareto front solutions generated by the Oppositional MAWA-AOA are highly competitive, aligning well with or even outperforming those produced by the other benchmark algorithms.



(a) Project cost ($) (b) Project time (days)

Figure 8. Desired approximate Pareto front solutions of TCTP for 146-activity project

Figure 9 illustrates the distribution of non-dominated solutions obtained using the Oppositional MAWA-AOA algorithm, showcasing the trade-off between total project cost and project duration. The color bar represents the normalized fitness or performance value of each solution. A noticeable concentration of points in the lower-left region indicates that the algorithm successfully identified a set of solutions that balance low cost with shorter durations. Additionally, the broad dispersion of points across the cost-duration space highlights the algorithm's ability to maintain solution diversity along the Pareto front, offering project managers a range of viable scheduling alternatives based on their priorities.

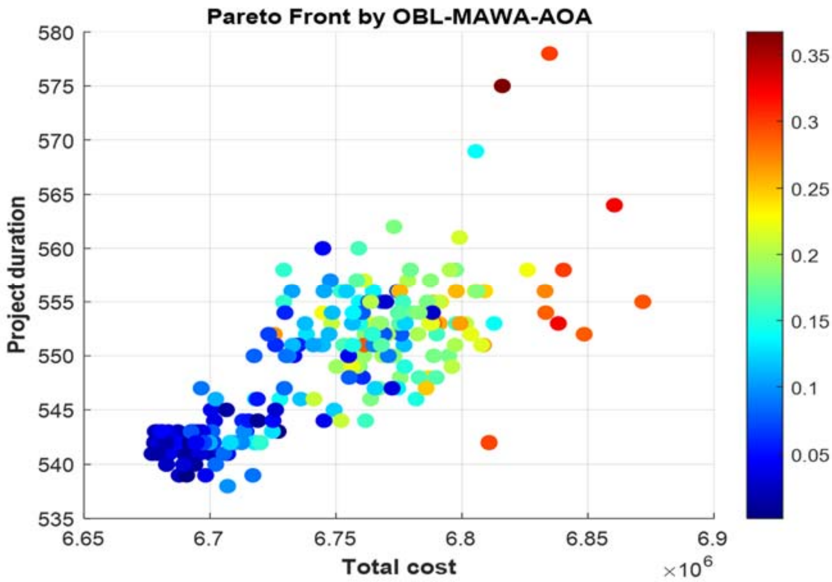


Figure 9. Pareto front solutions for 146-activity project by Oppositional MAWA-AOA

Performance Metrics for TCTP Problems

A standardized performance metric for evaluating multiobjective optimization algorithms has yet to be established. Assessing the performance of multiobjective problems is generally more challenging than that of single-objective problems. Numerous quality indicators have been developed to measure convergence and diversity in these algorithms (Habibi et al., 2017). In this thesis, the performance metrics NFE, Sp, and HV are used. Each of these metrics reflects different aspects of algorithm performance, as described below.

- ***Hyper-volume (HV)***: measures the dominance and coverage of the Pareto front, indicating how well the algorithm balances multiple objectives.
- ***Spread (Sp):*** assesses the uniformity and distribution of solutions, where lower Sp value means better solution diversity.

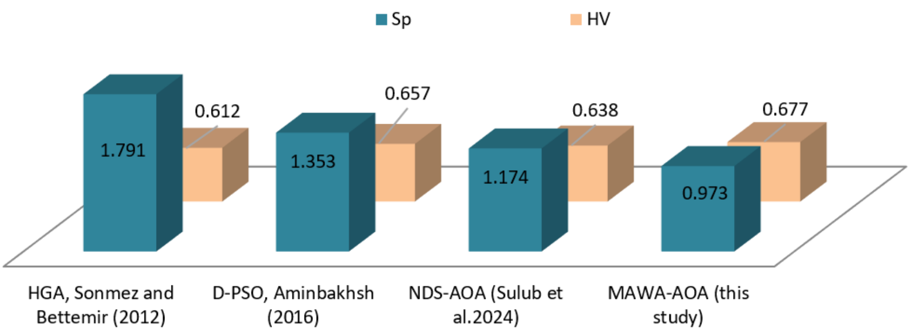


Figure 10. Graphical representation of the HV and Sp comparisons for 63- activity project

The graphical representation of the HV and Sp metrics comparisons of the proposed algorithm with other algorithms for the 63-activity project is depicted in Figure 10. The comparative analysis of algorithms, like HGA, D-PSO, and NDS-AOA, indicates the suitabality of the MAWA-AOA in order to obtain diversity in the search space.

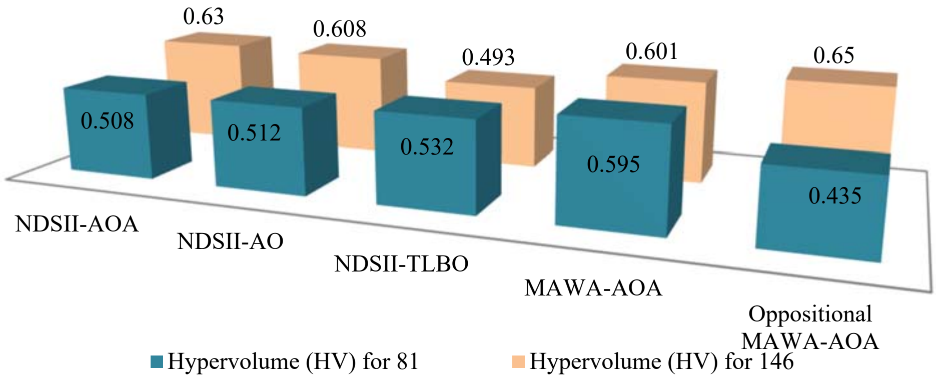


Figure 11. HV performance comparison between the compared algorithms

Similarly, a comparative analysis of algorithms applied to projects with 81 and 146 activities is shown in Figure 11 which includes NDSII-TLBO, NDSII-AOA, NDSII-AO, and the plain MAWA-AOA algorithms.

In Figure 11, the HV metric, which reflects both the convergence and diversity of Pareto front solutions, is used to assess algorithmic performance. As shown, the Oppositional MAWA-AOA algorithm outperforms all others for the 146-activity project, achieving the highest HV value of 0.650, indicating a broader and more optimal Pareto front. Conversely, for the 81-activity project, it yields a lower HV (0.435), while the MAWA-AOA algorithm shows strong performance in both cases (0.595 and 0.601). These results suggest that the Oppositional MAWA-AOA is particularly effective for large-scale, complex problems, offering superior exploration capabilities and solution diversity.

DISCUSSION

The results demonstrate that the proposed Oppositional MAWA-AOA algorithm is highly effective in identifying high-quality Pareto-optimal solutions for complex project scheduling problems involving 63, 81, and 146 activities. For the 63-activity project with five execution modes, the proposed method successfully generated well-distributed Pareto fronts while achieving a substantial reduction in the number of function evaluations (NFEs), confirming its strong computational efficiency. Across the tested cases, Oppositional MAWA-AOA achieved up to a 99.27% reduction in NFEs compared with the hybrid heuristic metaheuristic (HHMH), highlighting its advantage in medium-scale optimization scenarios.

For the 81-activity project with six execution modes, a slight decline in solution quality was observed, as reflected by a lower hyper-volume (HV) value of 0.435, which can be attributed to increased mode complexity and higher daily indirect costs. In contrast, the algorithm exhibited its strongest performance for the 146-activity project, achieving the highest HV value of 0.650, indicating superior convergence and diversity of Pareto-optimal solutions. These results suggest that the effectiveness of the proposed oppositional strategy becomes more pronounced as problem scale and complexity increase.

The conventional MAWA-AOA algorithm showed limitations due to premature convergence toward local optima, particularly for larger problem instances. By integrating OBL at the iteration level, the proposed approach enables the algorithm to escape local traps through the evaluation of opposite solutions, significantly enhancing exploration. This improvement is closely linked to the opposition jumping mechanism, which expands the search space and strengthens global search capability.

A key contribution of this study is the generation of diverse Pareto front solutions for the 81- and 146-activity projects, providing greater flexibility for decision-makers. This represents a clear advantage over the HHMH approach reported by Bettemir and Birgonul

(2023), which yields only a single optimal solution. Moreover, comparative analyses against NDS-AOA, NDS-TLBO, and NDS-AO confirm that Oppositional MAWA-AOA delivers competitive or superior trade-off solutions in terms of both project duration and total cost. Overall, the integration of OBL significantly enhances robustness, scalability, and solution diversity, making the proposed method well-suited for real-world construction project scheduling.

Despite the promising results, this study has several limitations. First, the performance of the proposed Oppositional MAWA-AOA algorithm was evaluated using a limited number of benchmark TCTP instances, and its effectiveness for ultra-large or highly constrained real-world projects may require further investigation. Second, although the integration of OBL significantly enhances exploration, the sensitivity of the algorithm to OBL-related control parameters was not systematically analyzed in this study. A comprehensive sensitivity analysis could provide deeper insights into parameter robustness and optimal settings. Finally, the computational experiments were conducted under fixed parameter configurations, and adaptive or self-tuning strategies may further improve performance and scalability. Addressing these limitations constitutes an important direction for future research.

CONCLUSION

The following conclusions are drawn regarding the performance and contributions of the proposed Oppositional MAWA-AOA algorithm:

- Methodological contribution: Introduced Oppositional MAWA-AOA, the first integration of OBL into the MAWA-AOA framework for multiobjective time-cost trade-off optimization, enhancing global search capability.
- Robustness and scalability: Demonstrated stable performance on projects with up to 146 activities, effectively handling high combinatorial complexity and multiple execution modes.
- Computational efficiency: Achieved up to 99.27 percent fewer function evaluations than HHMH while maintaining competitive or superior optimization results.
- Solution quality and practical relevance: Generated well distributed, high quality Pareto fronts, including the best HV value of 0.650 for large-scale cases, supporting its use as a decision support tool in construction scheduling.

Data Availability

The datasets used and/or analyzed during the current study are available from the corresponding author upon reasonable request.

Numerical examples can be found using the following link

https://drive.google.com/drive/folders/1hA5cjI3SlwhGzWlZIq8daaAYpCM37RzC?usp=drive_link

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Conflict of Interests

1. The authors declare that they have no competing financial interests or personal relationships that could have influenced the work reported in this paper.

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