

## **Parametric Study of Continuous Concrete Beam Prestressed with External Tendon**

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### **ABSTRACT**

This paper presents the results of a parametric study of the flexural behavior of continuous concrete members prestressed with external tendons. The behavior at ultimate limit states is evaluated. A nonlinear analysis model, based on the three dimensional finite element method, ANSYS computer program (10.0) was used. The nonlinear material and geometrical analysis were adopted. The results obtained show good agreement with experimental results. This research was carried out to study the effects of several factors on the overall behavior of externally prestressed beams in terms of the values of compressive concrete strength and effective prestressing stress. That behavior was slightly affected compared with experimental results. The undeviated external tendons mobilized lower nominal flexural resistance and inelastic deflection than deviated tendons did. The increase in the beam capacity in the beam subjected to loads at the third span is greater than in the beam with a single load at the mid – span. The result of increasing the effective depth is that the ultimate load capacity is essentially increased.

**KEYWORDS:** ANSYS, Flexural behavior, Concrete continuous members, External prestressing, Tendon.

### **INTRODUCTION**

In the past 2 decades, there has been an increasing need around the world to strengthen reinforced concrete bridges due to heavier traffic loads, progressive structural aging and corrosion of steel reinforcement. One of the preferred strengthening methods is external prestressing because of its speed and the possibility of monitoring, future retensioning and replacing the tendons (Tan and Tjandra, 2007). Increased ultimate capacity and smaller deflection under service loads reduced beam depth and weight (Dall' Asta et al., 2007).

External prestressing is a post- tensioning method becoming popular not only in new bridge construction

but also as a strengthening method for existing concrete structures. The addition of external tendons, however, leads to a different structural system of which there has been little documentation (Tan and Ng, 1997).

In an externally prestressed structure, the tendons are placed on the outside of concrete sections to which they are in contact only at deviators and anchorages. Prestressing force is introduced to the concrete section through end anchorages and the tendons assume a rectilinear profile in between the anchorages.

Under load, the tendons are free to move relative to the concrete section in between the anchorages and/or deviators, and the resulting change in tendon eccentricity leads to what is known as second order effects (Tan and Ng, 1997). Keeping tendons outside the structure has several advantages with the development

of very strong cables with effective corrosion protection systems, and due to other technical advantages, structures with external prestressing have again come into vogue in the recent past (Rao and Mathew, 1996). To strengthen a beam by external prestressing, concrete blocks are commonly cast into both sides of the beam web to serve as end anchorages or deviators (Tan and Tjandra, 2007).

Extensive analytical and experimental works have been devoted to the application of external prestressing in structures. Most works were related to the flexural behavior of externally prestressed beams.

The analysis of externally prestressed members is the same as that of internally prestressed members with unbounded tendons. However, there are two additional points to be considered in the case of external cables; namely the possible slip at a deviation point and the shift of tendon eccentricity due to the deformation of the structure. Even though these effects may not be so significant at the service load stage, they can have a considerable effect at the ultimate load stage (Rao and Mathew, 1996).

An experimental investigation was carried out to study the flexural behavior of continuous concrete beams prestressed using external tendons. Aravinthan et al. (2005) carried out an experimental study on two – span continuous beams with highly eccentric external tendons. The results of this investigation revealed the effects of tendon layout, loading pattern, casting method and confinement reinforcement on the flexural strength and stress in external tendons. Harajli et al. (2006) examined the behavior of continuous externally prestressed members experimentally, and developed an analytical approach to predict their ultimate flexural response, taking into consideration the influence of the second- order effects and rotation capacity in the plastic region. Tan and Tjandra (2003) tested four two –span continuous externally prestressed T-beams. The results demonstrated that the strengthened beam could fail in shear, particularly at the high –shear region near the interior support, due to the limited increase in shear capacity. An analytical study is presented to explain and

further examine the phenomenon in beams strengthened with eight variations of tendon profiles.

Ghallab and Beeby (2005) studied the effects of several factors on the increase in ultimate stress in external parafil ropes as well as external steel tendons. These factors were related to the external prestressed and ordinary bonded steel, beam geometry and material properties. The experimental and the analytical results showed that the studied factors have the same effect on both steel (up to yield) and parafil ropes, though this effect is greater in the case of steel tendons.

This study was carried out to investigate the behavior of continuous externally prestressed members using the FEM to predict the ultimate flexural response, taking into consideration the influence of the second-order effects using the computer program ANSYS (10.0). This paper aims also to study the effects of several factors, such as concrete strength, effective prestressing stress (fpe), effective depth of the external prestressing force (dp), load type and tendon profile on the behavior of externally prestressed members.

## FINITE ELEMENT MODEL

### Finite Element Model of Concrete

The three dimensional 8- node brick element (solid 65 reinforced concrete solids) is used as a model of concrete (Wu and Lu, 2003). The element has eight corner nodes, and each node has three degrees of freedom translation in the (X, Y, Z directions) (ANSYS Manual).

The concrete is assumed to be homogeneous and initially isotropic. The compressive uniaxial stress-strain relationship for the concrete model is obtained by using the following equations to compute the multilinear isotropic stress- strain curve for the concrete as shown in Figure (1).

$$f_c = \varepsilon E_c \quad \text{for} \quad 0 \leq \varepsilon \leq \varepsilon_1 \quad (1)$$

$$f_c = \frac{\varepsilon E_c}{1 + \left(\frac{\varepsilon}{\varepsilon_0}\right)^2} \quad \text{for} \quad \varepsilon_1 \leq \varepsilon \leq \varepsilon_0 \quad (2)$$

$$f_c = f'_c \quad \text{for} \quad \varepsilon_o \leq \varepsilon \leq \varepsilon_{cu} \quad (3)$$

$$\varepsilon_o = \frac{2f'_c}{E_c} \quad (4)$$

where

$f_c$  = stress at any strain  $\varepsilon$ , N/mm<sup>2</sup>.

$\varepsilon_o$  = strain at the ultimate compressive strength  $f'_c$ .

$\varepsilon_{cu}$  = ultimate compressive strain.

$\varepsilon_l$  = strain corresponding to  $(0.3f'_c)$ .

The multilinear isotropic stress- strain implemented requires the first point of the curve to be defined by the user. It must satisfy Hooke's law

$$E = \sigma/\varepsilon \quad (5)$$

$$\sigma = 0.3f'_c \quad (6)$$

The multilinear curves were used to help with the convergence of the nonlinear solution algorithm. The crack modeling depends on smeared cracking modeling.

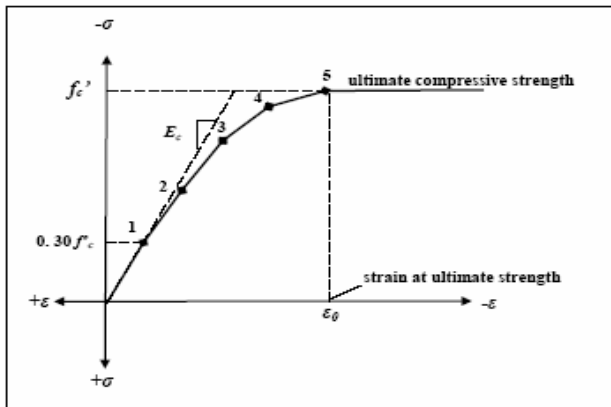


Figure 1: Simplified Compressive Uniaxial Stress-Strain Curve for Concrete

### Finite Element Model of Reinforcement

In the present study, the discrete representation is used to model the steel reinforcement by using the (3D spar Link8 element) (ANSYS Manual). The reinforcement in the discrete model uses bar or beam elements that are connected to concrete mesh nodes. The stress-strain relationship for ordinary reinforcing

steel is a bilinear relationship assumed to be elastic-perfectly plastic (Ng and Tan, 2006; Lou and Xiang, 2006) as shown in Fig. 2.

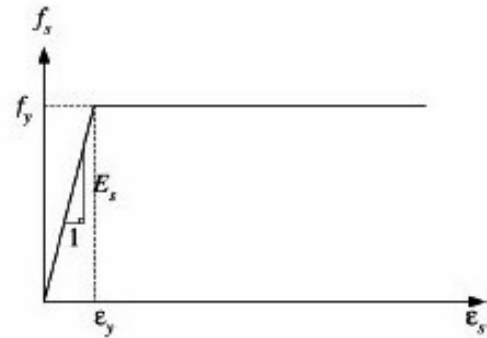


Figure 2: Constitutive Law for Steel Reinforcement (Ng and Tan, 2006)

### External Prestressed Tendon

The prestress in a finite element is applying as real prestress to the tendon with initial conditions. The prestressing stress was taken as the initial value and equal to the effective stress, and it appears in the analysis as the initial strain in the link element. For prestressing tendons, bilinear elastic-plastic with hardening is the relationship of stress-strain, as shown in Fig. 3. The modulus of the strain-hardening portion is assumed to be (2%) of the modulus of elasticity of steel.

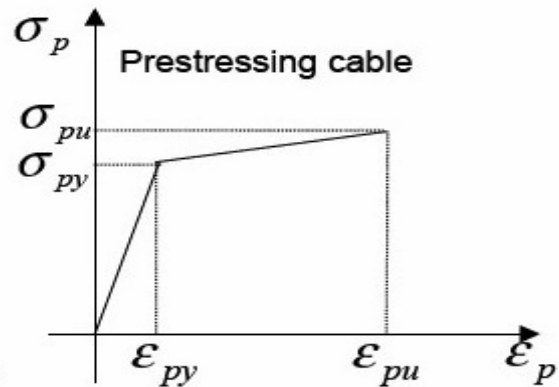


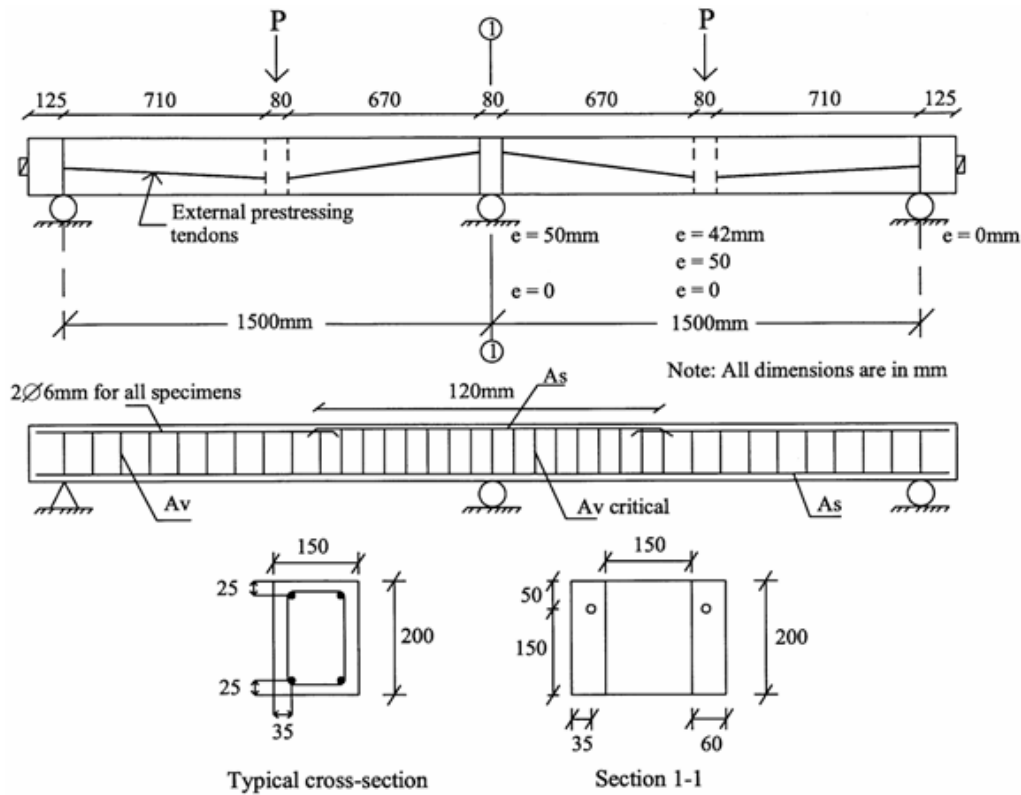
Figure 3: Constitutive Law for Prestressing Steel

### Anchorage and Steel Plates

The anchorage zone used the steel plate with the tendon element connected to it. Also, steel plates are

added at the support and loading locations to avoid stress concentration problems. The solid element (solid 45) was used for the steel plates. The element is defined

with 8-nodes, and each node has three degrees of freedom (translations in X, Y and Z directions) (ANSYS Manual).



**Figure 4: Typical Dimensions and Reinforcement Details of Selected Beams (Harajli et al., 2002)**  
(1 in. = 25.4 mm)

**PARAMETRIC STUDY**

Harajli et al. (2002) tested nine specimens, continuous over two spans. In this study, two specimens were selected to investigate the behavior of continuous externally prestressed members by developing an analytical model to predict the ultimate flexural response, taking into consideration the influence of the second – order effects.

A continuous rectangular beam over two spans with section dimensions, span length and reinforcement details was used as shown in Fig. (4) for B6D1 and B10S1B. Small concrete blocks were cast monolithically with the specimen to serve as deviators

for the external tendons. The prestressed steel consisted of 5mm wires. The ultimate strength, yield strength and modulus of elasticity were 1607, 1400 and 206850 N/mm<sup>2</sup>, respectively. The effective prestress  $f_{pe}$  was taken 932; 921 N/mm<sup>2</sup> and the concrete strength  $f_c$  was taken equal to 38.4; 42 N/mm<sup>2</sup>. The area of bonded reinforcement was taken 2(6mm) and 2(10mm) and the depth of prestressing  $d_p$  at the mid-span was equal to 142 mm and 100 mm and the internal support was equal to 150 mm and 100 mm for B6D1 and B10S1B, respectively. The ordinary reinforcement in the outer shear spans ( $A_v$  in Fig. (4)) consisted of 6mm stirrups spaced at 100mm. The specimens B6D1 are with

deviators at mid-spans, and the specimens B10S1B are without deviators at their midspans. The specimens were loaded with two concentrated point loads applied simultaneously at the middle of the spans. Only a half of

the beam was used in the analysis. The finite element mesh, the boundary conditions and the loading arrangement of B6D1 beam are shown in Figure (5).

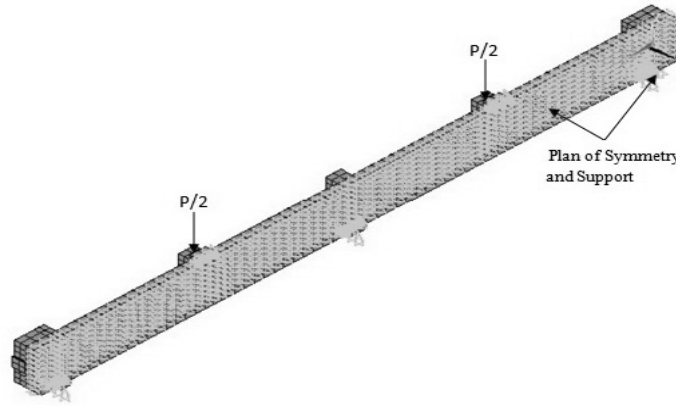


Figure 5: Finite Element Mesh, Boundary Conditions and Loading Arrangement

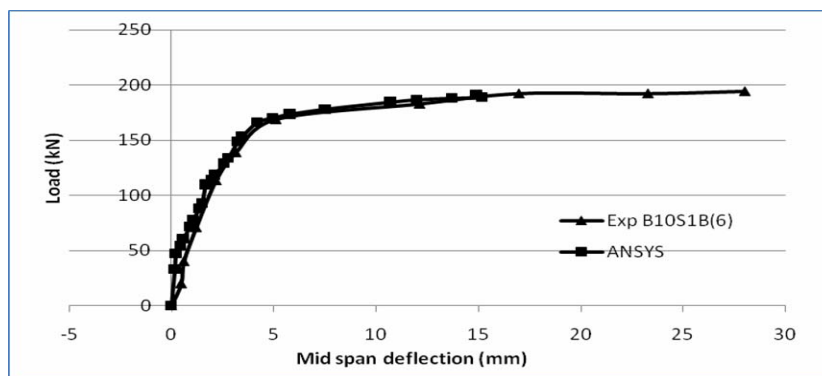
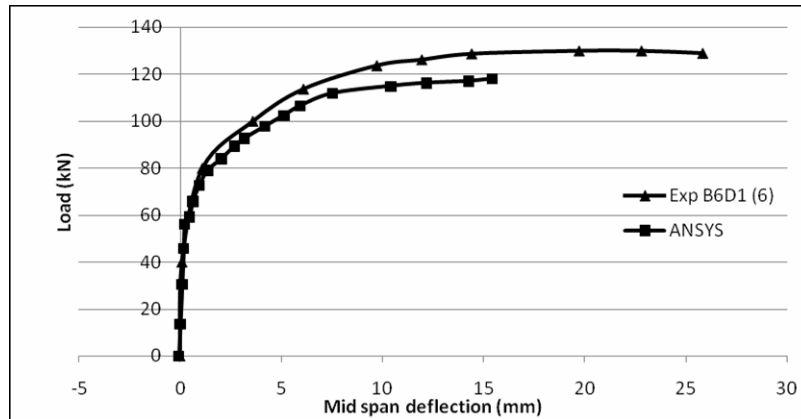


Figure 6: Load- Deflection Curves for B6D1 and B10S1B Beams (1 in. = 25.4 mm; and 1 kip = 4.46kN)

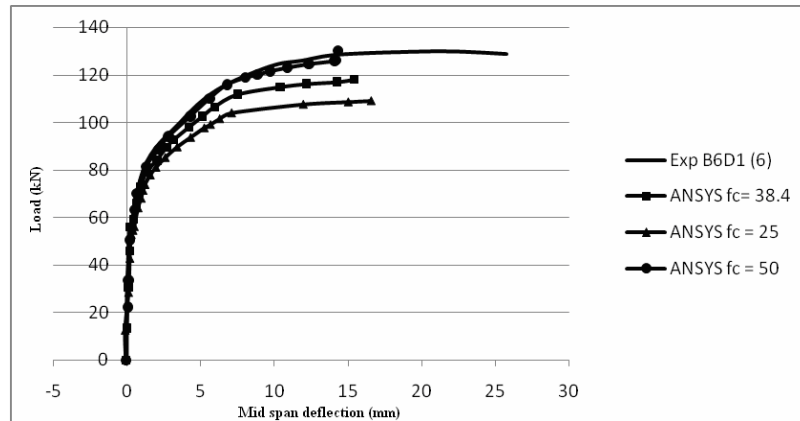


Figure 7: Effect of Grade of Concrete on the Load –Deflection Behavior for Beam B6D1

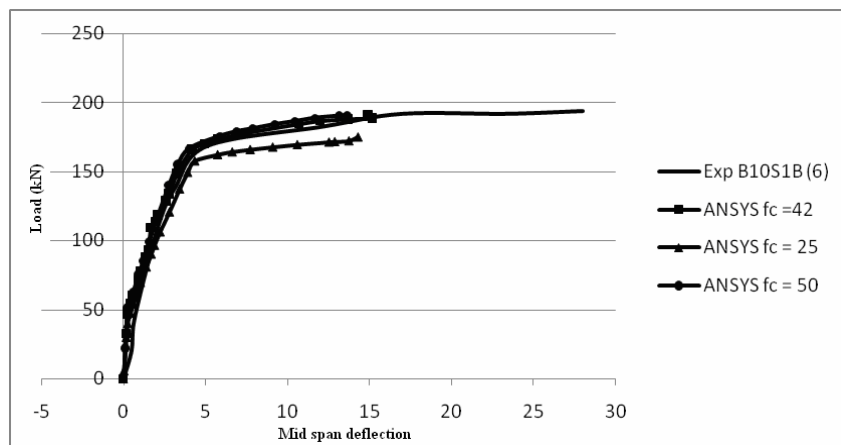


Figure 8: Effect of Grade of Concrete on the Load –Deflection Behavior for Beam B10S1B

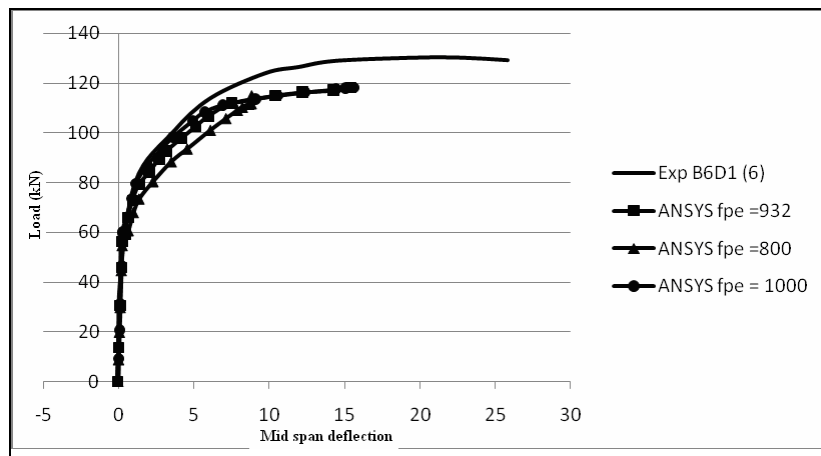


Figure 9: Effect of (fpe) on Load –Deflection Behavior for Beam B6D1

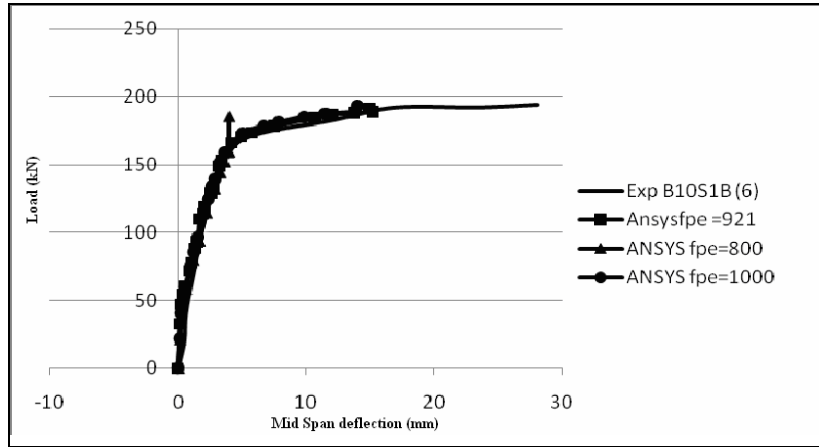


Figure 10: Effect of (fpe) on Load –Deflection Behavior for Beam B10S1B

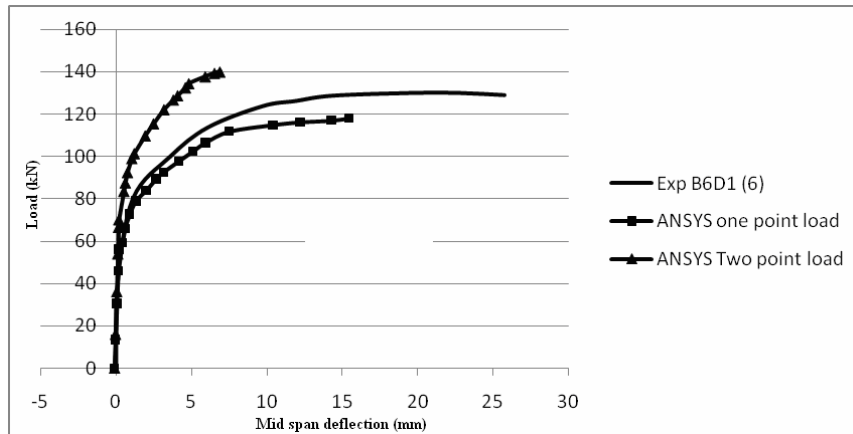


Figure 11: Effect of Type of Loading on Load –Deflection Behavior for Beam B6D1

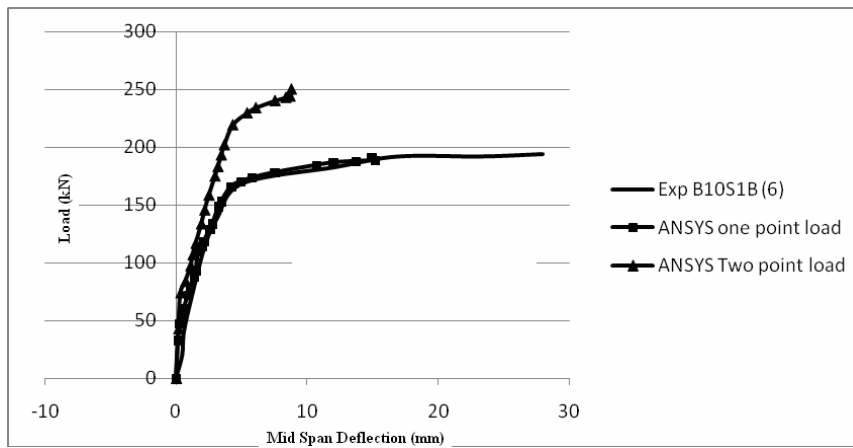
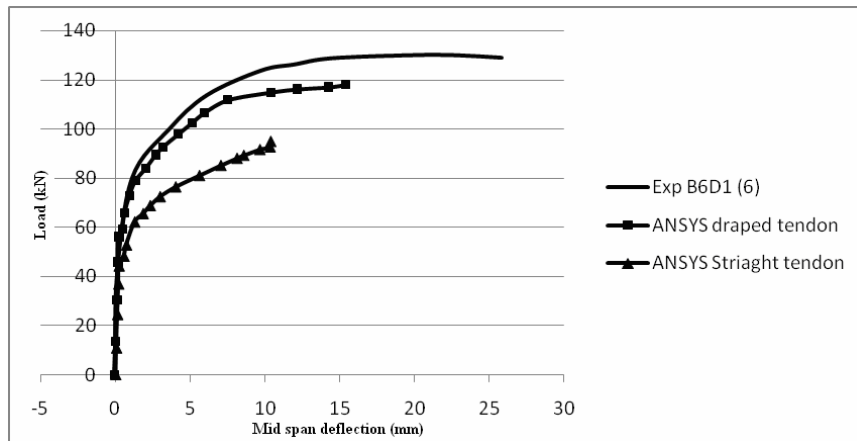


Figure 12: Effect of Type of Loading on Load –Deflection Behavior for Beam B10S1B



**Figure 13: Effect of Tendon Profile on Load –Deflection Behavior for Beam B6D1**

The experimental and numerical load- deflection curves obtained for beam B6D1 and beam B10S1B are shown in Figure (6). The Figure shows good agreement in finite element analysis with the experimental results throughout the entire range of behavior and failure mode. Good agreement in the load –deflection relation exists prior to cracking load. After the appearance of flexural cracks, the beam stiffness was reduced and the linear load –deflection behavior ended when the internal steel reinforcement began to yield. But at ultimate, the numerical loads for beams B6D1 and B10S1B are slightly smaller than the experimental loads.

In this section, beams B6D1 and B10S1B are chosen to study the effect of several important parameters on the behavior of continuous prestressed concrete beams with external tendons.

#### **Effect of Compressive Strength of Concrete**

The effect of concrete compressive strength ( $f_c'$ ) on the load – deflection behavior of beam B6D1 and beam B10S1B can be noted from Figs. (7) and (8). Different values were selected to investigate this effect. The selected values were (25, 50) MPa with the experimental load – deflection curve ( $f_c' = 38.4$  and 42) MPa for B6D1 and B10S1B, respectively. For higher values of  $f_c'$ , the behavior is slightly stiff compared with the experimental results (Harajli et al., 2002). Also, it is clear that the maximum predicated deflection is

slightly decreased as the concrete compressive strength is increased. The same effect can be noted for a beam with deviators at the mid-span and for the specimen without deviators at the mid-span.

#### **Effect of Effective Prestressing Stress**

To discuss the effect of effective prestressing stress ( $f_{pe}$ ) on the behavior and ultimate load capacity of continuous externally prestressed concrete beams, different values of prestressing stress have been selected. Tendons have the same ultimate strength. The chosen values of prestressing stress were (800, 932 and 1000 MPa) for B6D1 and (800, 921 and 1000 MPa) for B10S1B. In this analysis, only the value of effective prestressing stress was varied, while all other parameters were kept constant. The effect of prestressing stress on the load- deflection response of beams (B6D1 and B10S1B) can be noted from Figs. (9) and (10). The Figures reveal that for higher values of ( $f_{pe}$ ), the ultimate load is slightly increased, while the values of ultimate deflection are slightly decreased. This can be attributed to the effect of the external prestressing force that tends to prevent the cracks from extending and thus improves the stiffness. It can be noted that no significant effect exists on the two types of beams, this means that no effect for the deviator on the prestressing stress is found.



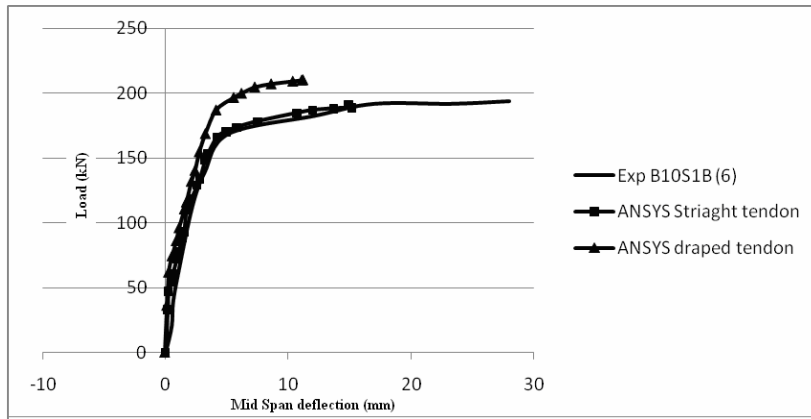


Figure 14: Effect of Tendon Profile on Load –Deflection Behavior for Beam B10S1B

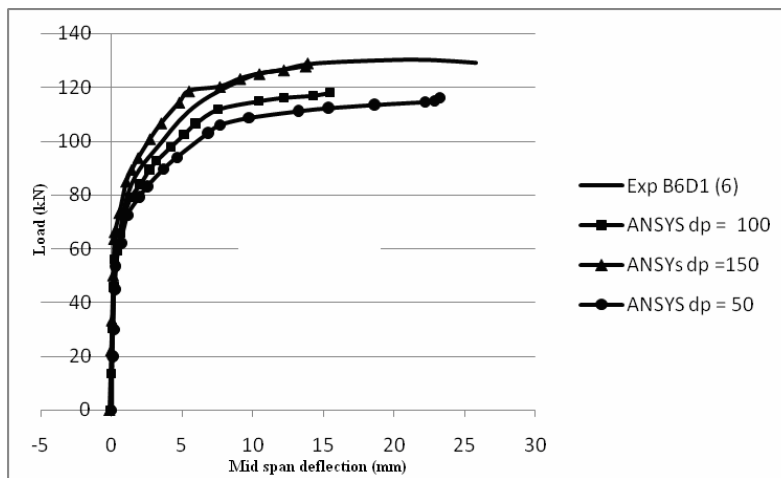


Figure 15: Effect of Effective Depth of External Prestressing Tendon on Load –Deflection Behavior for Beam B6D1

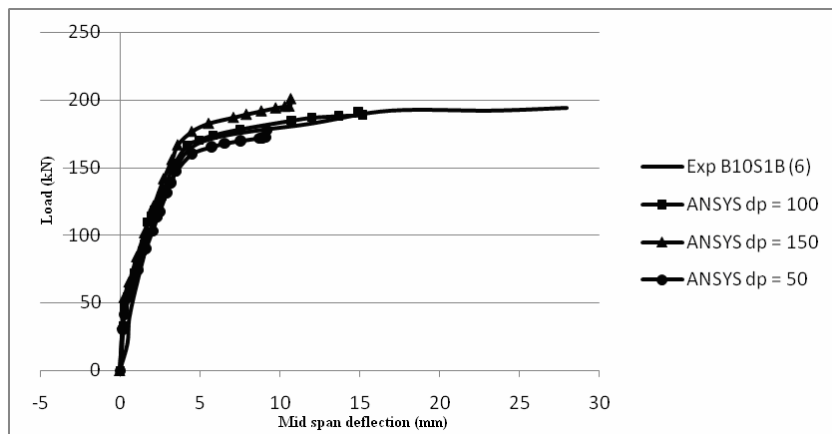


Figure 16: Effect of Effective Depth of External Prestressing Tendon on Load –Deflection Behavior for Beam B10S1B

### Effect of Loading Pattern

Two load types were examined in this study; a concentrated load at the mid-span and two concentrated loads at the third span. The increase in the beam capacity in the beam subjected to the loads at the third span is greater than that in the beam with a single load at the mid-span as shown in Figs. (11) and (12). This is because during loading cracks started to appear on the beam surface and spread as the load increased. This continued up to the formation of plastic hinge, where the strain concentrated and the stress increased up to failure.

### Effect of Tendon Profile

The load – deflection characteristics of beams B6D1 and B10S1B are shown in Figs. (13) and (14). Beams B6D1 and B10S1B were used with continuous tendons over two spans. Beam B6D1 had a draped tendon profile in the first case and a straight tendon profile in the second case, and beam B10S1B had a straight profile which is assumed to be a draped profile in the case study. It is seen that for the selected beam not only the ultimate load increased but also the result was obtaining better failure characteristics. The beams with a draped tendon profile showed a higher ultimate load and a stiffer response compared to beams with a straight tendon profile. This is attributed to the fact that failure occurs where the effective depth of the tendon is largest.

### Effective Depth of the External Prestressing Tendon

The effect of the effective depth of the external prestressing tendon can be studied taking different values for beams B6D1 and B10S1B. The values chosen were (50, 100, 150) mm. Typical numerical load deflection curves for different values of effective depth were compared to the experimental load – deflection curve ( $d_p = 100$  mm) as shown in Figs. (15) and (16). These figures reveal that as a result of increased effective depth of externally prestressed tendon, the cracking load, post cracking stiffness and ultimate load capacity are essentially increased. This increase is very clear in a beam without deviators at the mid-span and a

straight tendon profile (B10S1B) and greater than that for a beam with deviated tendon (B6D1) at the mid-span. This means that deviators reduce the second – order effects and the change in eccentricity of the tendon. This may be attributed to the increase in eccentricity of the tendon which causes an increase in its distance from the neutral axis; the greater the distance the higher the stress.

### CONCLUSIONS

A numerical nonlinear analysis model, based on the Finite Element Method (FEM) using ANSYS computer program, was developed to predict the entire load – deflection response of continuous concrete members with external prestressing. The accuracy of the analysis was verified by means of comparisons with experimental results. The numerical model reproduced the experimental results of load–deflection response; the results predicted by the model were in very good agreement with experimental data.

A parametric study was carried out using the nonlinear analysis method to evaluate the response of continuous concrete members with external prestressing tendons.

This paper investigates the effects of some factors on the flexural behavior at ultimate related to the external prestressing system, concrete strength, initial prestressing stress, effective depth of the external tendons, loading arrangement and tendon profile. It was found that as the compressive strength of concrete increased, the ultimate load capacity increased. For different values of effective prestressing stress, the ultimate load increased substantially when the effective stress increased. Tendons without deviators produce lower nominal flexural strength than tendons with deviators. Single concentrated loads produce less significant second – order effects because they mobilize low post – elastic deflection in comparison with two – third point loads. The ultimate load in the beam subjected to loads at the third span is greater than in the beam with a single load at the mid- span. The effective depth of the external prestressing tendon has a

significant effect on the ultimate load. It was found that the tendon profile has a clear effect on the ultimate load

capacity. The ultimate load increased with different draped tendon profiles compared to undraped profiles.

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