



Digital-image Correlation and Cohesive Fracture Analysis of Fibre-reinforced Concrete Experiments: The Sergipe Test

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ARTICLE INFO

Article History:

Received: 28/9/2024

Accepted: 6/12/2024

ABSTRACT

Fibres are added to the concrete mix to improve its tensile strength and fracture properties, and they are used in several engineering applications. Therefore, fracture energy is a well-known quantity worth measuring. Some experiments in the technical literature are designed to quantify fracture energy. This paper proposes a new experiment to analyze the mechanical properties of fibre-reinforced concrete, named the Sergipe test. Such experiment consists of a square hollowed specimen submitted to a flexural test, where tensile cracks appear in different locations. Digital image correlation was used to analyze the cracking behaviour and crack opening displacement (COD). Regarding the experimental results of the Sergipe test, both specimens reached similar force-displacement and force-COD results despite different cracking locations at the specimens' bottom. Such characteristics might indicate the suitability of the proposed experiment for fracture energy measurement. Regardless, the proposed experiment may be a benchmark for future non-linear models. To analyze such conditions, the well-established cohesive fracture model implemented in ABAQUS® is used to reproduce the experimental observations numerically, achieving good accuracy.

Keywords: Cohesive fracture, Digital image correlation, Fracture energy, Fibre-reinforced concrete, Sergipe test.

INTRODUCTION

Concrete is susceptible to cracking, mainly due to its relatively low tensile strength. Cracks formed in concrete represent damage to the material and can decrease the structure's life service. Some researchers studied repairing methods, such as using Ferro-Cement (Aldarf et al., 2024) and fibre-reinforced polymers (Ma'en et al., 2007, Cunha et al., 2021, Naserifard & Yazdian, 2022). In this last method, some papers analyzed the bond strength of polymers (Haddad &

Haddad, 2019).

The response of the concrete has been analyzed for different conditions, in which several approaches suggest new methods for diminishing cracking, such as adding fibres to the concrete composition (Saravana & Sumathi, 2017; Jalil & Sultan, 2021; Janaki & Shafabakhsh, 2022). The fibres can be made of several materials, such as steel, carbon, glass and plants (Wu et al., 2023). Fibre-reinforced concrete (FRC) is a current building technology with several studies and applications, such as tunnel linings (Wang et al., 2021;

Oliveira et al., 2023, Zhang et al., 2024), airfield pavements (Yan et al., 2023) and bridges (Ruiz et al., 2023). Zhang et al. (2023) reviewed works concerning the effect of steel fibres on the fracture properties of concrete.

Some parameters evaluated for fracture properties of concrete are fracture energy, load-deflection curve and load-crack opening. Fracture energy (G_f) is a well-known variable from fracture mechanics, corresponding to the work needed for a complete rupture of a cross-section. Experimentally, fracture energy can be determined from the load *versus* displacement curve measured from a three-point bending test (Yan et al., 2023). Numerical analysis also estimates fracture energy (Rashidi et al., 2024; Zhang et al., 2021; Zheng et al., 2023). Zeng et al. (2023) developed a mesoscopic numerical method for evaluating the fracture energy of concrete, considering the interface between mortar and coarse aggregate, evaluating aspects, such as aggregate location and size, as well as crack-propagation path.

Digital Image Correlation (DIC) is a widely used non-destructive measurement technique for analyzing strains and displacements in materials and structures. Among others, Rosenfeld and Kak (1982) and Peters et al. (1983) postulated the foundations of DIC. In DIC current form, Besnard et al. (2006) developed a methodology to evaluate continuous displacement fields from pairs of images. They achieved significant progress concerning DIC techniques regarding spatial resolution and uncertainty. DIC can also be applied to evaluate the parameters related to the fracture properties of concrete (Enfedaque et al., 2015; Chen et al., 2021). Finally, He et al. (2024) presented a recent review of DIC-research progress and applications.

Several methods simulate concrete fracture. Experimental validation of these methods is crucial to verifying the ability of numerical models to handle complex geometries and loading conditions. Mukhtar and El-Tohfa (2023) presented a compilation of numerical models and experimental concrete fracture tests, such as the double-edge notched direct tension test and the three-point bending test that may be used for such purposes.

In this paper, a steel fibre-reinforced concrete experiment is proposed. This experiment tends to reach symmetric responses, and it might be helpful as an alternative for fracture energy measurement in the future. One of its advantages is that notches are

unnecessary to predict crack locations due to the test's geometry. Moreover, the specimens presented the complete cracking at three cross-sections. Since fracture energy is the work required to rupture a cross-section, the proposed experiment may yield an average value of G_f due to cracks occurring in more than one cross-section. In contrast, when analyzing G_f by beams, concrete heterogeneity can lead to significantly different responses from one beam to another, increasing the uncertainty of the results. Even notched beams may present such variability, since the notch does not present a pointed tip; i.e., the main crack can be located anywhere at the top of the notch.

In addition, due to its non-linear response, the proposed experiment may serve as a benchmark for validating and applying new non-linear theories, such as those based on fracture and damage mechanics (Kurumatani et al., 2016; Sun & Xu et al., 2022; Arruda et al., 2022; Campos et al., 2022; Pereira Junior et al., 2023; Teles et al., 2023; Amorim et al., 2024; Nardi & Leonel, 2024, Tran & Bui, 2024) or based on the phase-field theory (Voyiadjis & Mozaffari, 2013; Teichtmeister et al., 2017; Fang et al., 2019; Tournet et al., 2022; Wang et al., 2024; Shajan et al., 2024).

EXPERIMENTAL PROGRAM

Concrete Dosage, Specimens Cast and Characterization Tests

To cast the specimens to be evaluated, the ratio 1:2.35:2.60 was adopted (Table 1), with a water/cement ratio (w/c) equal to 0.585. The type of cement adopted was CP-V-ARI (high-early-strength Portland cement) from a Brazilian brand, where the consumption was equal to 367.56kg/m³. In addition, a plasticizing additive was used, with a rate of 0.4% to the cement mass. Furthermore, the experiments were carried out in steel fibre-reinforced concrete, with a fibre dosage of 39.25kg/m³ (0.5% in volume). The fibres were of the type FF4, with anchoring at the ends, the diameter, length and aspect ratio of which were equal to 0.75mm, 60mm, and 80, respectively. During the experiments, it was noticed that the fibres are pulled out regardless of their length compared to the specimens' dimensions. The slump test was conducted before the addition of the steel fibres in the concrete mix, where 5cm of slump was observed.

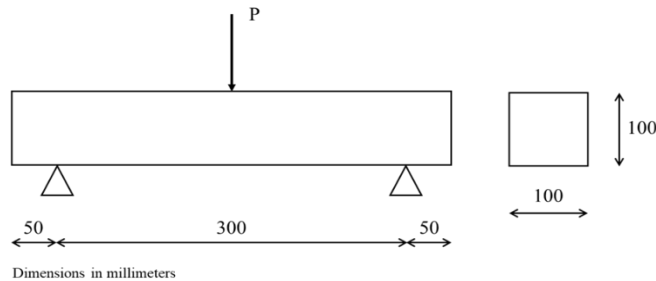
Table 1. Concrete mix proportions

Material	Composition (kg/m ³)
Cement	367.56
Fine Aggregate	863.77
Coarse Aggregate 9.5mm	955.66
Steel Fibres	39.25
Water/Cement ratio	0.585

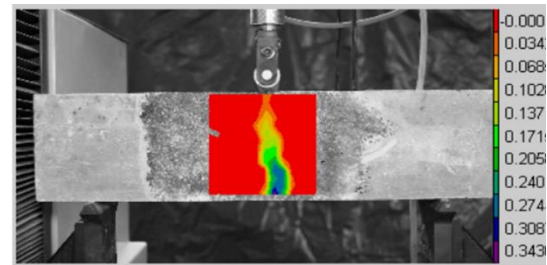
All nine cylinder specimens (100mm×200mm), five beams (100×100×400mm³) (Fig. 1(a)) and two Square Hollowed (SH) specimens (external area: 400×400mm², internal area: 200×200mm²; width: 100mm) (Fig. 2(a)) were cast from the same batch of concrete. After 24 hours, the specimens were demoulded and cured in a water tank, and tests were conducted after 7 days. The cylinder specimens were used to define the mean (f_{cm}) and characteristic (f_{ck}) compressive strength, as well as the elasticity modulus (E), presented in Table 2.

Table 2. Physical properties of the fibre-reinforced concrete

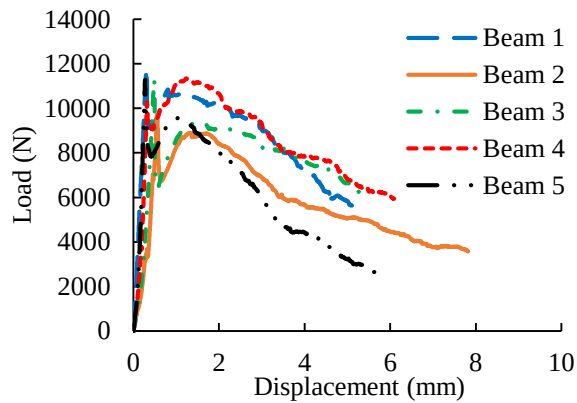
Specimen	Compressive strength (MPa)	Elasticity modulus (GPa)	Fracture energy (N/mm)
1	25.73	-	4.60
2	30.86	-	4.59
3	26.51	-	4.24
4	29.14	-	5.22
5	28.31	-	4.95
6	28.58	-	-
7	-	31.37	-
8	-	28.65	-
9	-	32.49	-
Mean value	28.19	30.84	4.72
Characteristic value	25.16	-	-
Standard deviation	1.85	1.98	0.373



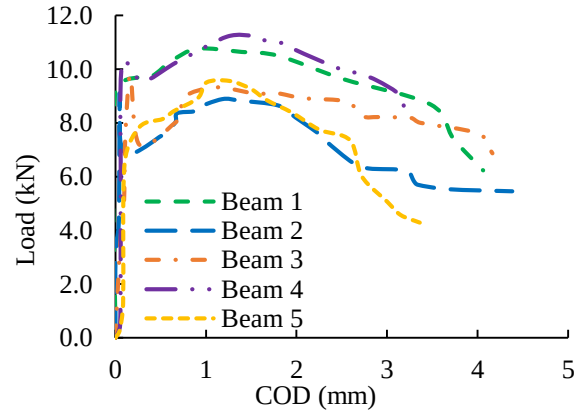
(a)



(b)



(c)



(d)

Figure (1): (a) Geometry, (b) DIC analysis of the beam, (c) Load vs. displacement and (d) COD experimental results of the beams

Fracture Energy Measurement

The beams were experimented with a three-point bending test to quantify the fracture energy (Rilem, 1985). The effective span of the beams is 300mm. The crack opening displacement of the beams was measured by the DIC technique (Fig. 1(b)). Such technique identifies the movement of pixels and, therefore, allows the calculation of displacement, strain and/or crack opening. To ensure greater accuracy of the results, the regions of the specimens where cracks were expected to open were painted randomly in black, white and blue. The recording frames of the tests were used in the DIC analysis, where the values of the opening cracks related to each load level were obtained.

Fig. 1(c) presents the experimental results of load vs. displacement of the beams. The maximum load values of the beams and the shape of the curves during the propagation of the cracks are similar. Furthermore, all curves are in the format expected from the technical literature of fibre-reinforced concrete (see, e.g. Trindade et al., 2020 and the references therein). The crack opening displacement (COD) values of the beams are shown in Fig. 1(d).

According to RILEM TC-50 FMC (1985), the fracture energy can be calculated by:

$$G_f = \frac{1}{(h-a)b} \int P d\delta \Rightarrow G_f = \frac{W_f + mg\delta_f}{(h-a)b} \quad (1)$$

W_f being the work of external loads, defined as the area under the load vs. displacement curve of the three-point bending test, h and b the height and width of the beam (100mm x 100mm), respectively, a the notch depth ($a = 0$), g gravity acceleration and m and δ_f the mass and the maximum vertical displacement of the beam, respectively. The fracture energy values are presented in Table 2.

Square Hollowed Specimens: The Sergipe Test

Two square hollowed specimens were tested to propose a new experiment, named here as the Sergipe test. The experiment consists of a square hollowed specimen, the base of which is totally supported, and a concentrated load is applied at its top. The base of the specimen is not attached to the testing machine; i.e., it is free to lift, as depicted in Fig. 2(a). The specimens were tested until they presented considerable cracking. The load vs. displacement curves are presented in Fig. 2(b).

The cracks appeared predominantly at the centre of the top of the specimens and at the sides. Due to the possibility of the base lifting, cracks also appeared at this location (Fig. 2(c)). Considering the nature of the problem, the base of the specimens is submitted to constant bending moment, and, therefore, cracks can appear anywhere on this part. The bottom crack on specimen 1 occurred approximately 4.33cm from the left internal face, while on specimen 2, cracks occurred close to the internal faces (Fig. 2(d)). The test of the specimens was recorded with a digital camera in order to verify the cracking patterns during the application of the load. The frames of the recording were extracted and used on a digital image correlation software to verify the cracking opening (Fig. 2(e)-(f)).

The first crack of the SH specimen 1 appeared at the centre of its top, when the load level and the imposed vertical displacement were approximately 26.54kN and 0.718mm, respectively (39.42s of testing). Shortly thereafter, an upper left crack appeared at load level equal to 25.46kN and a displacement of 0.744mm (43.13s of testing). Finally, an upper right and an inferior crack appeared simultaneously when the load level was 25.09kN, and the vertical displacement at the centre was 0.786mm (44.72s of testing). The cracks evolved differently over time due to different positions (Fig. 3(a)). The central crack in the upper part of the specimen was the fastest to evolve, while the one located at the lower part evolved the least. Furthermore, as the upper left crack started before the upper right one, a more differentiated propagation occurred, despite the symmetry of the problem. As the crack in the lower part propagated, the upper left one slowed down its propagation and presented, in the final stage of the test, similar propagation to the upper right crack. Thus, it can be stated that at the end of the test, the specimen's performance approached symmetrical behaviour.

The same analysis was made for the SH specimen 2. The central crack was the first to appear, at a load level of 35.44kN and a vertical displacement of 0.851mm (51.15s of testing). Then, the upper left crack started at a load level equal to 35.31kN and a vertical displacement of 0.878mm (52.78s of testing). The upper right crack was first observed at a load level of 31.25kN and a vertical displacement of 0.913mm (54.85s of testing). The lower left crack appeared at a load level and vertical displacement of 30.04kN and 0.947mm,

respectively (56.92s of testing). Finally, the last crack that appeared was the lower right crack, at a load level of 21.40kN and a vertical displacement of 3.027mm

(181.68s of testing). Although that was the last crack to appear, it was the one that evolved the fastest, as shown in Fig. 3(b).

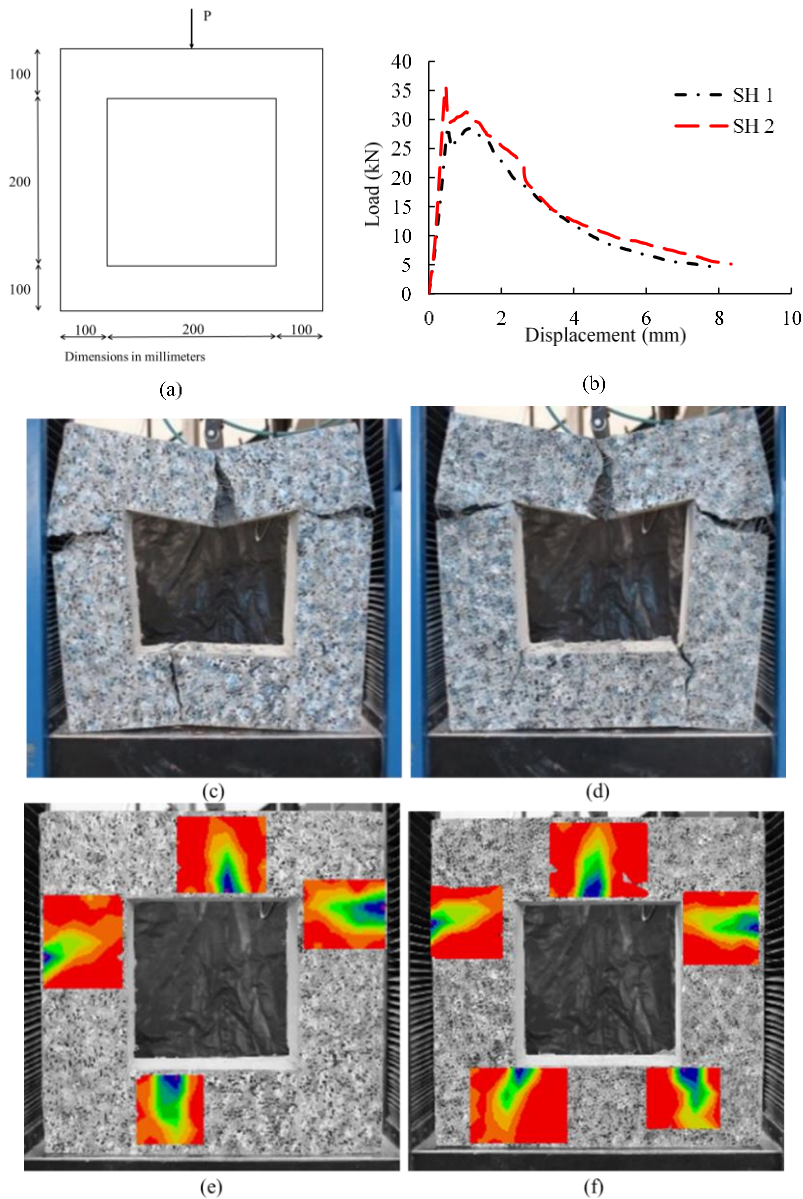


Figure (2): (a) Square hollowed geometry, (b) Experimental load vs. displacement results and (c)-(f) cracked condition of the square hollowed specimens

The envelopes of crack opening displacements for each position of the specimens were created with the crack opening data (Fig. 3(c)-(f)).

RESULTS AND DISCUSSION

The behaviours of the beams and the square hollowed specimens were simulated with the finite element analysis software ABAQUS® (2013), using

cohesive elements to verify the cracking propagation. The models may be created by considering different damage initiation criteria: maximum stress, maximum specific strain, nominal quadratic stress or nominal quadratic strain, and damage evolution criteria: maximum displacement or fracture energy.

The numerical modeling developed in this paper added cohesive fracture zones in the regions where the cracking was experimentally observed. For these zones,

the COH2D4 of the Abaqus library was set, a four-node, two-dimensional cohesive element with two degrees of freedom per node. The remaining area was simulated with the CPS4 finite element, also from the Abaqus library, a four-node bilinear plane stress quadrilateral finite element without reduced integration and with two degrees of freedom per node. The constitutive model for the cohesive zones was defined by the elastic stage, and damage initiation and evolution. The analysis type was set as traction; for the second one, the quadratic nominal stress (*quads damage*) criterion was selected. This

criterion evaluates the interaction among the stresses, in which maximum values equal the tensile strength. Finally, the damage evolution was defined based on the energy criteria, with linear softening behaviour, in which the fracture energy was set. The remaining areas of the examples; i.e., the other finite elements, were defined with a linear-elastic response and analysis type set as isotropic. This characteristic may define the ABAQUS® model as an inter-element cohesive cracking model (Amorim et al., 2024). Further details on the simulation are given as follows.

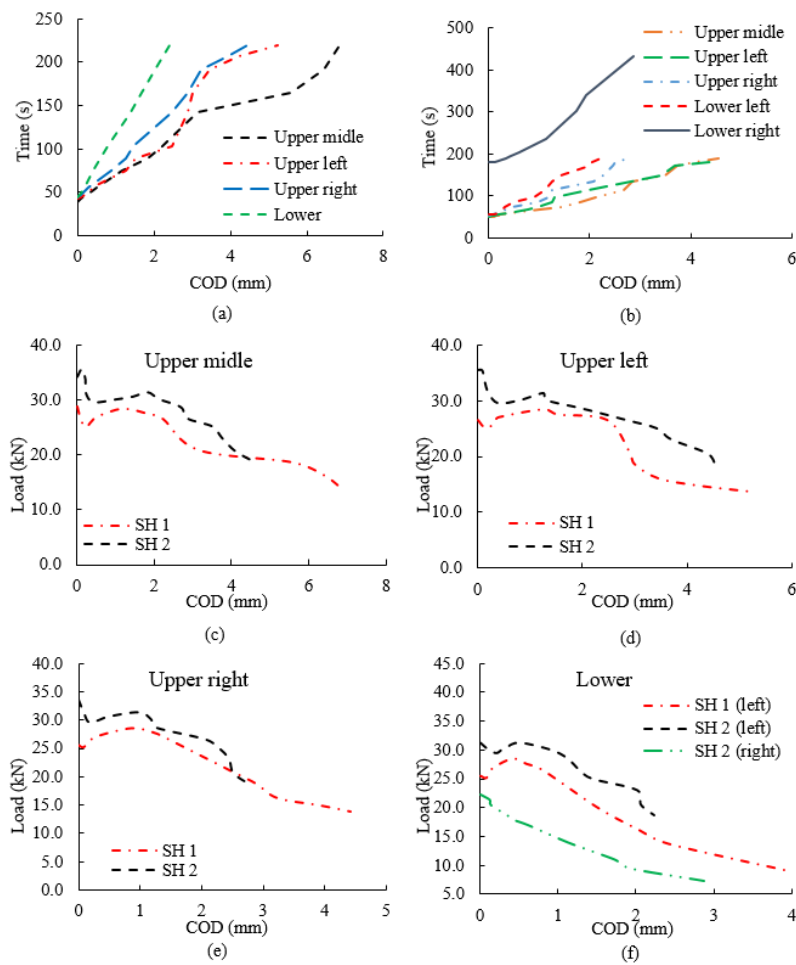


Figure (3): COD experimental results of the square hollowed specimens

L-shaped Plate Benchmark Test (Winkler et al., 2004)

The first example aimed to check the accuracy of the numerical procedure developed in this paper by validating the computational modeling using a benchmark example experimentally tested by Winkler et al. (2004). It consists of an L-shaped plate concrete specimen with dimensions equal to 500 mm x 250 mm,

100 mm thick and the boundary conditions, where a 0.6-mm displacement was imposed at 30 mm from the edge and clamped at the base (Fig. 4(a)). According to the authors, the elasticity modulus, Poisson's ratio, tensile strength, and fracture energy are equal to 25.85 GPa, 0.18, 2.7 MPa and 0.065 N/mm, respectively. The simulation was carried out as previously described, and the results were compared to the experimental results

and the ones obtained by Teles et al. (2023) in terms of load vs. displacement curves (Fig. 4(b)) and the cracked condition (Fig. 4(c)-(d)). It is possible to note that the numerical results obtained in this paper are quite close to the experimental envelope; i.e., the maximum and minimum values experimentally observed by Winkler et al. (2004) during the experimental results campaign.

Note that these results are also close to the ones obtained by Teles et al. (2023), applying the lumped damage mechanics non-linear theory. The cracked condition by both numerical models agrees with the experimental observations. Therefore, this numerical modeling is feasible to be extended to the experimental results obtained in this paper.

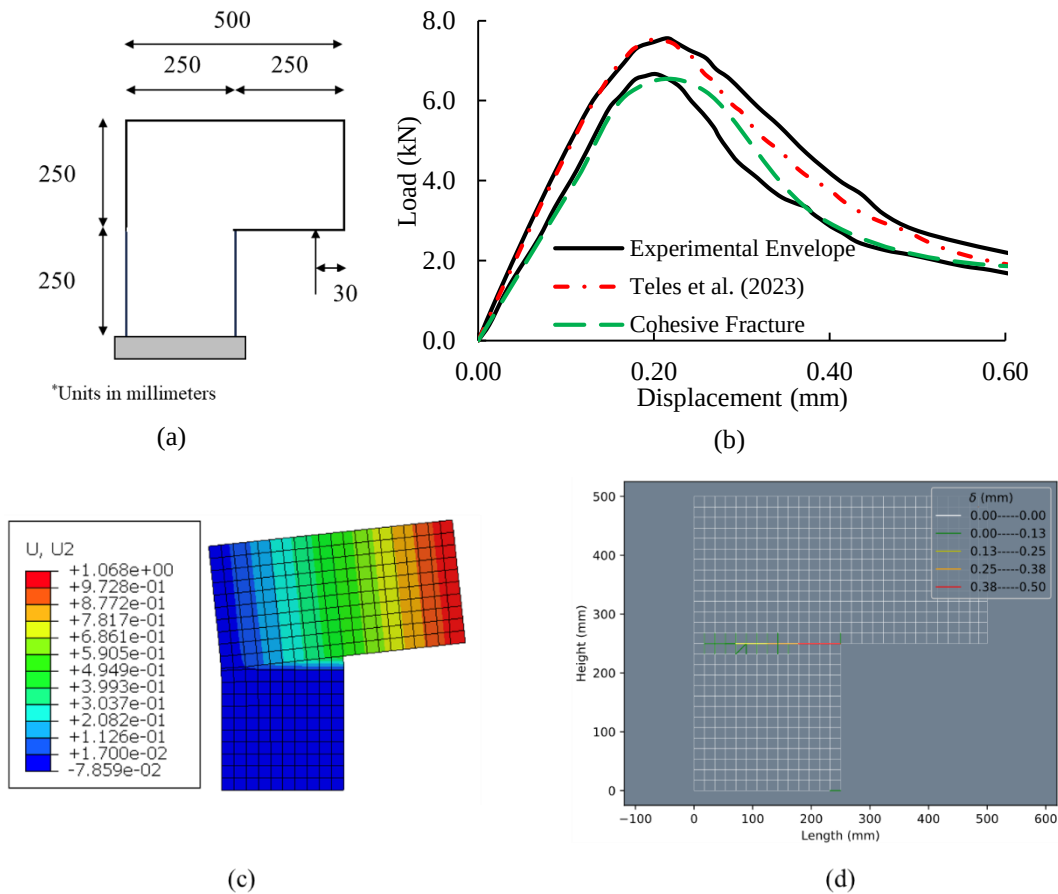


Figure (4): (a) Geometry, (b) Load vs. displacement, (c) Deformed shape and (d) Numerical results by Teles et al. (2023)

Beams under a Three-point Bending Test

Fig. (1) shows the beams' geometry. To use ABAQUS®, the cohesive zone is adopted at the mid-span, where its elements are 1.0 mm thick (Fig. 5(a)). Firstly, a mesh-convergence study was conducted, in which the regular mesh with 475 finite elements (Fig. 5(a)) was defined for all beam analyses. The tensile strength and Poisson's ratio were set equal to 1.9 MPa and 0.2, respectively. The slope of the cohesive fracture model was set equal to 10 times the elasticity modulus and the fracture energy was set as the value obtained for each beam (Table 2).

The numerical analyses aimed to verify the response

for each beam's fracture energy (Table 2) and the response for the mean value of fracture energy. Six simulations were conducted, and the results in load vs. displacement and Load vs. COD curves were compared (Fig. 5).

The experimental results were summarized by the experimental envelopes; i.e., the maximum and minimum load values for the displacement end COD results, aiming to visualise the numerical ones better. The experimental envelope of load vs. displacement of the beams is shown in Fig. 5(b), as well as the numerical responses. It is possible to note that the cohesive fracture model can recover the response of the beams, in which

all responses of the beams are within the experimental envelope. The elastic stage is quite close, the difference being due to the heterogeneity of the concrete, which was not taken into account in the model. Regarding the post-peak stage, the model cannot describe the load reduction before the resistance gain. This is explained by the model's hypothesis that the steel fibres are not modelled. After this resistance gain, the model led to results that agreed with the experimental ones, in which

the curve for the average value of fracture energy is in the middle of the experimental envelope. The numerical responses for the crack opening displacements of the beams are depicted in Fig. 5(c) and present a similar behaviour, with values within the experimental envelope satisfactorily close to the references after the resistance gain. Therefore, it is possible to confirm that the cohesive fracture strategy, based on the fracture energy, led to accurate results.

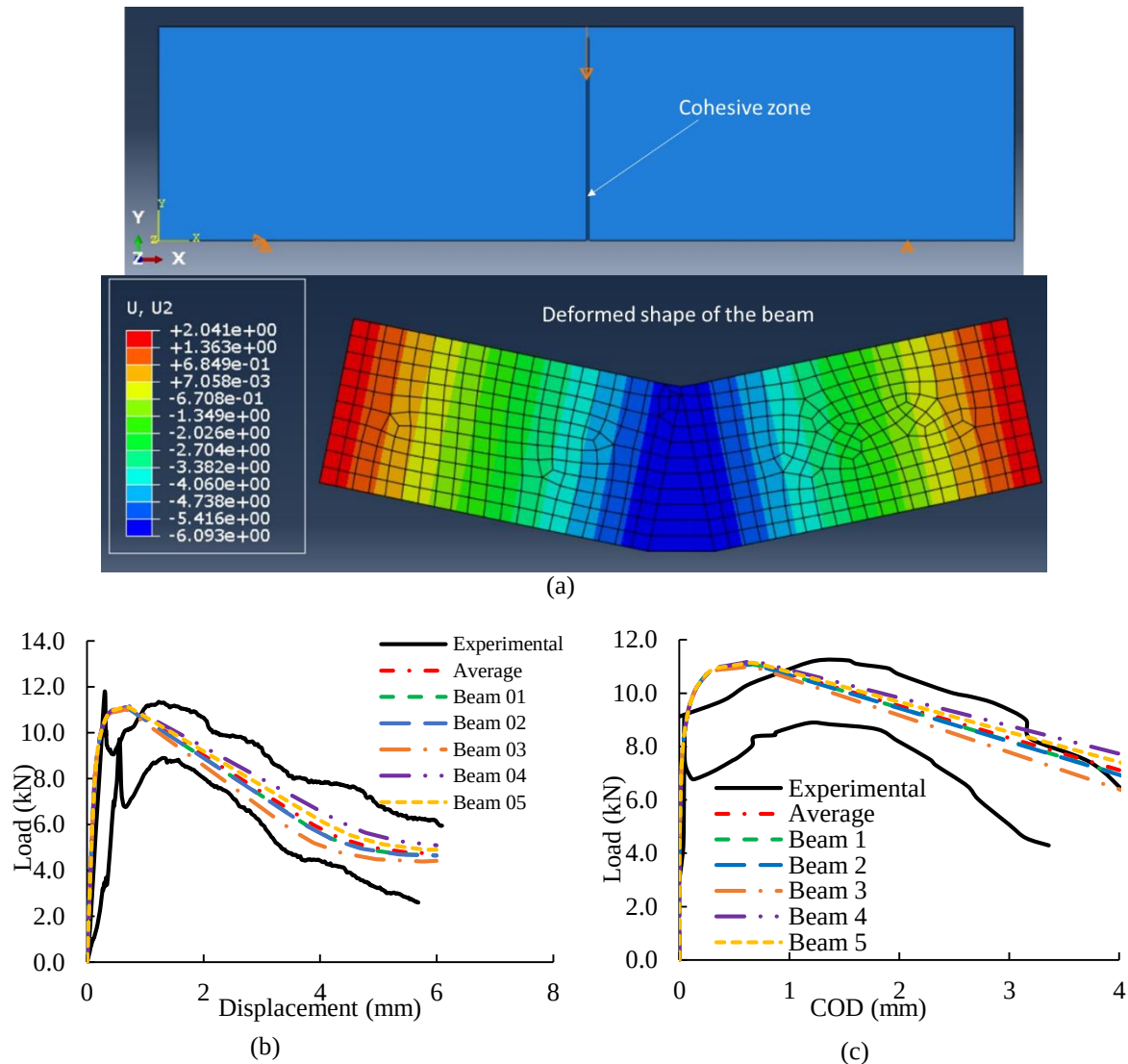


Figure (5): (a) ABAQUS® simulation, (b) Load vs. displacement and (c) COD results of the beams

Square Hollowed Specimens: The Sergipe Test

The experimental beams, the values of which were validated in the numerical analysis of the previous section, obtained the fracture energy of the concrete. The numerical modeling using the cohesive fracture model of the ABAQUS was also validated according to the

results presented in the two previous sections. Thus, the ABAQUS model is applied in the experimental results of the square hollowed specimens, named *the Sergipe test*. Similar to the model of the beams, the cohesive zones present 1.0mm of thickness and were located at the positions where the experimental cracking was

observed (Fig. 6(a)). The tensile strength and Poisson's ratio were equal to 1.05 MPa and 0.2, respectively. For the cohesive model, the slope was equal to 10 times the elasticity modulus. The fracture energy adopted in the model was equal to the average value obtained from the beams (Table 2), equal to 4.72 N/mm. Since the base of the specimen lifted during the experiment, a simply supported scheme was chosen for the numerical analysis, with supports located in the middle of the vertical regions (Fig. 6(a)).

Firstly, a new linear-elastic mesh-convergence study was conducted, adopting a regular mesh with 1417 finite

elements. Fig. 6(a) depicts the deformed shape at the end of the numerical analysis using the ABAQUS cohesive fracture model. Despite the symmetry of the problem, the numerical analysis presented an asymmetric response once the cohesive zone at the right-bottom location was not activated (Fig. 6(a)). This behaviour was also experimentally observed (Fig. 2). The results were compared in terms of load vs. displacement and load vs. COD curves, where the first one is shown in Fig. 6(c), for the two specimens, as well as the numerical results.

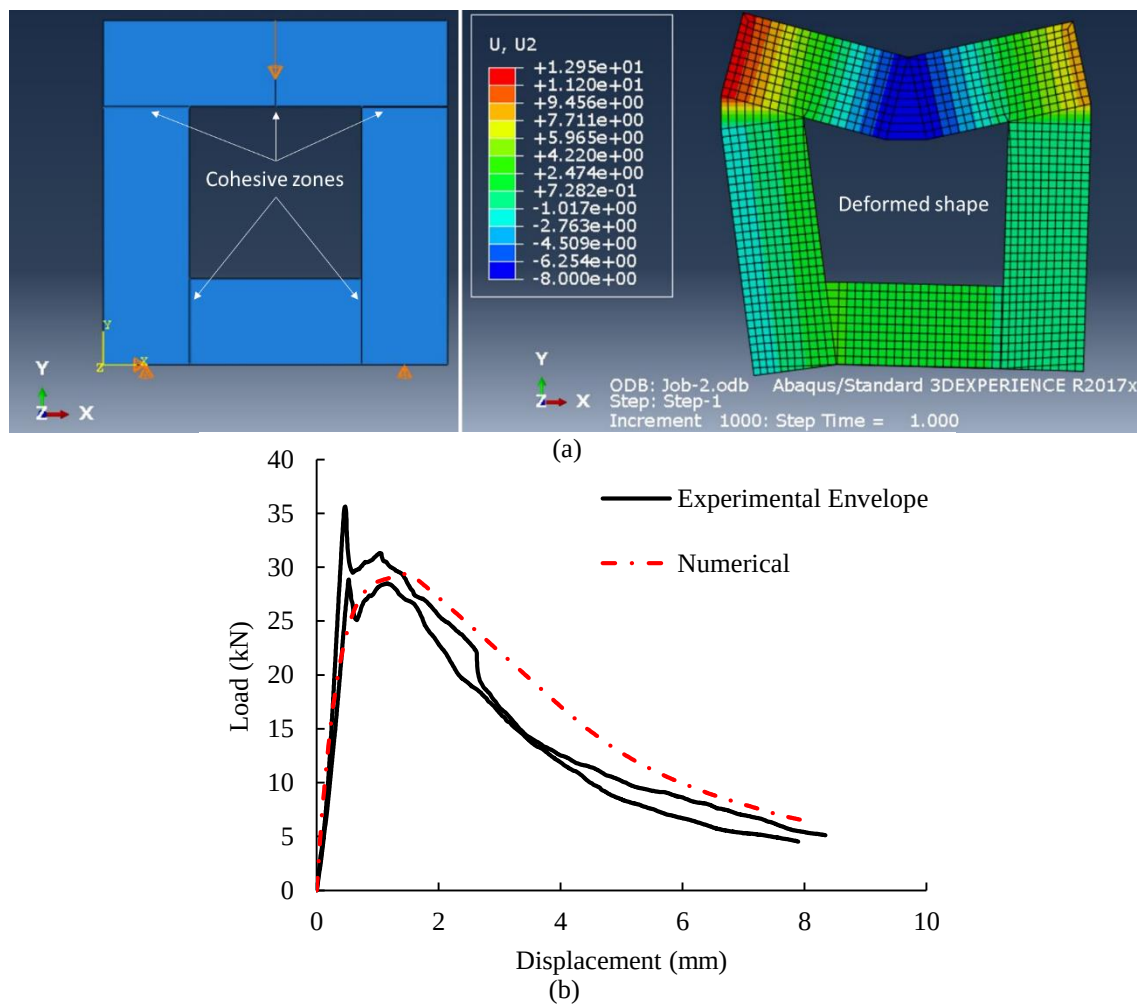


Figure (6): (a) ABAQUS® simulation and (b) Load vs. displacement results of the SH specimens

It is possible to note that the numerical response agrees with the experimental observation, with the elastic stage quite close to the experimental envelope. The post-peak stage is also close to the experimental envelope, in which the curves present a similar decay shape. The numerical results regarding the COD results, depicted in

Fig. 7, were also evaluated for all cracking zones of the SH specimens. It is possible to note that all results are quite close to the experimental ones, with values within the experimental envelopes for all positions. Therefore, the numerical results confirm that the cohesive model accurately described the Sergipe test.

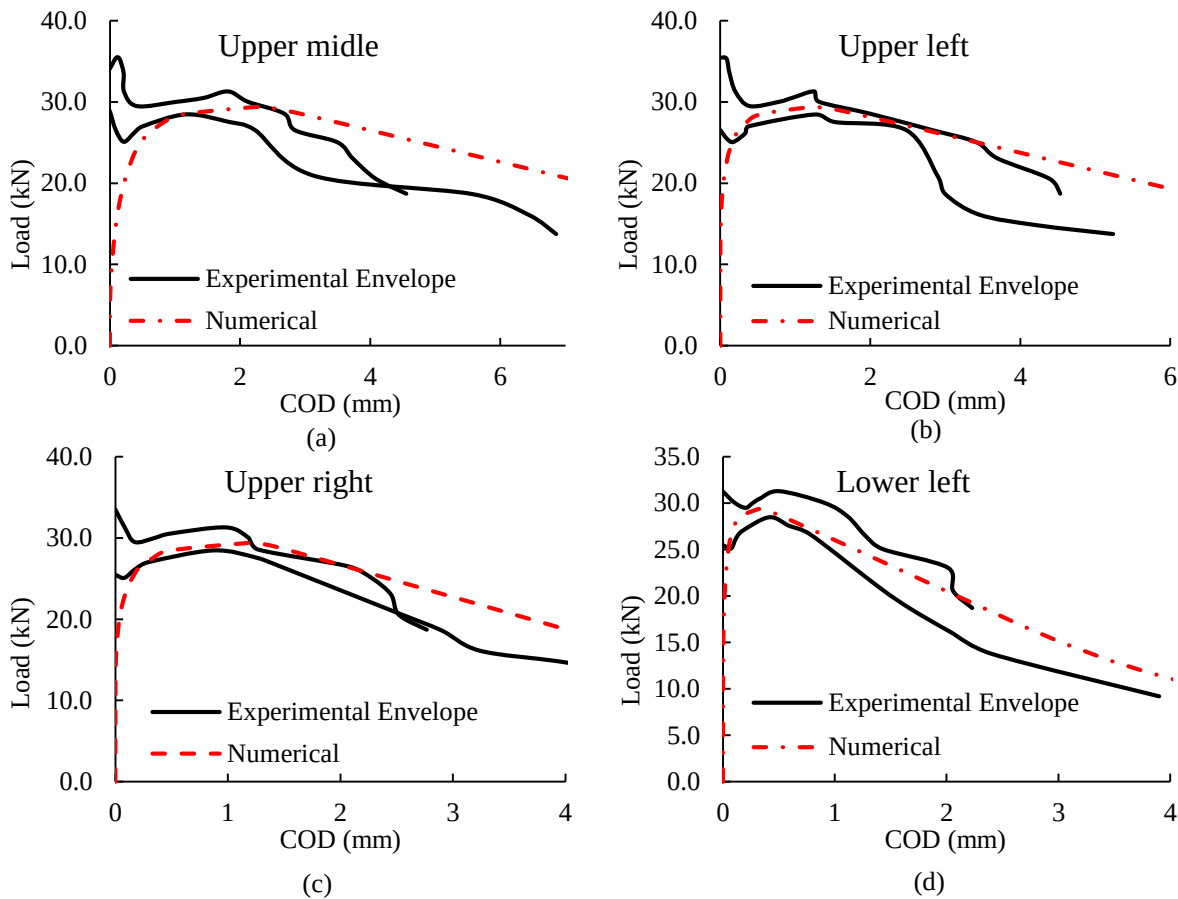


Figure (7): Load vs. COD results of the square hollowed specimens

CONCLUSIONS

- This paper proposes a new experiment, the Sergipe test, as an alternative to estimating the mechanical properties of steel fibre-reinforced concrete (SFRC), such as the fracture energy.
- The proposed experiment consists of a bending test of squared hollowed specimens, where its corners induce the location of some cracks.
- Since the experiment is quite different from the established ones and presents a unique behaviour, there is no need to insert notches.
- An experimental campaign was developed to obtain the fracture energy of the SFRC and the experimental results of the square hollowed specimens (the Sergipe test).
- The specimens were analyzed using the digital image correlation technique. For both specimens, the experimental responses are quite close to each other, even when considering that they present different numbers of cracks.
- The specimens presented similar areas under the load

vs. displacement curves. If an extensive experimental campaign can prove it, that characteristic may be used to measure fracture energy and even fracture tenacity.

- The well-established cohesive fracture model of ABAQUS® was applied to show the reproducibility of the proposed experiment, and the numerical results are quite close to the experimental ones.
- Finally, the proposed experiment may be used as a *benchmark* to validate and apply new non-linear numerical methods.

Acknowledgements

The second author acknowledges the Scientific Computing and Visualization Laboratory of the Federal University of Alagoas (LCCV/UFAL) for access to an ABAQUS® license. The second author acknowledges CAPES (*Coordenação de Aperfeiçoamento de Pessoal de Nível Superior*) for the financial support of his D.Sc. studies. The third author acknowledges CNPq (*Conselho Nacional de Desenvolvimento Científico e Tecnológico*), Brazil, for the grant 436523/2018-3 (*Chamada Universal CNPq MCTIC*). The third

author acknowledges the PROAP / CAPES grants 23113.058556/2019-31, 23113.064080/2019-78, 23113.025644/2020-58 and 23113.031287/2022-78 provided by the Post-graduate Programme in Civil Engineering of the Federal University of Sergipe (PROEC/UFS). The authors acknowledge the

Laboratory of Building Materials and Structures and the Laboratory of Mathematical Modeling in Civil Engineering of the Disaster Research Institute of the Federal University of Sergipe (LAMCE/IPD/UFS and LAMEC/IPD/UFS) for the physical support during this investigation.

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