

## Non-linear Regression Models for Hydraulic Geometry Relationships in Al-Abbasia Meandering Reach in Euphrates River

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### ABSTRACT

This paper is an extension and continuation of an earlier work by the authors on the phenomenon of meandering in Al-Abbasia reach located in the middle of the Euphrates river, Al-Najaf governorate in Iraq. The authors have developed several power functions and models depending on dimensional analysis and Buckingham  $\pi$ -theorem for modeling and predicting the hydraulic geometry of the selected reach. The paper employs the non-linear regression technique for developing mathematical models for computing the width and mean depth of the reach depending on its hydraulic characteristics. This paper is part of an M. Sc. thesis carried out in 2014. The developed relationships are straightforward to be applied in design and analysis with results of high acceptability; the reach width (W) model has an  $R^2$  of 0.97, while the reach mean depth ( $D_m$ ) model has an  $R^2$  of 0.93. Different statistical methods have been utilized to compare the different models. The results reveal that non-linear regression models are the best models to correlate the different characteristics of the reach.

**KEYWORDS:** Euphrates, Al-Abbasia, Najaf, Regression analysis, River geometry, River hydraulics.

### INTRODUCTION

Typical functional relationships that exist in the river morphology include cross-sectional geometry, longitudinal profile and hydraulic factors (Maddock, 1953).

The authors in previous works (Mohammed et al., 2014a and b) have focused on developing hydraulic geometry relationships for Al-Abbasia reach located in the middle of the Euphrates river, Najaf governorate, Iraq. The relationships were developed using both

power functions and dimensional analysis. The power functions' models (eight models) were developed by using independent variables including area, width, mean depth, velocity, maximum depth, mean size of bed material, main channel slope and specific gravity, whereas the dependent variable was the discharge. Statistical analysis revealed direct relationships ( $R^2=0.87-0.94$ ) between the discharge (Q) and different properties, such as reach width (w), reach mean depth ( $D_m$ ), specific gravity of reach ( $G_s$ ) and mean particle size ( $d_{50}$ ). In contrast, there were moderate relationships ( $R^2 = 0.35-0.63$ ) between the reach discharge (Q) and other reach properties, such as reach sectional area (A), reach flow velocity (V), reach main slope (S) and reach

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max. depth ( $d_{\max}$ ). Equations (1) and (2) illustrate the relationships of the reach width (W) and reach mean depth ( $D_m$ ) (Mohammed et al., 2014a):

$$W = 0.5Q^{1.3} \quad \dots (1)$$

$$D_m = 99Q^{-0.91} \quad \dots (2)$$

Using the dimensional analysis approach, two models were developed; one for the width of water surface with  $R^2$  of 0.93 and the second for reach mean depth with  $R^2$  of 0.90. Equations (3) and (4) present the relationships of the reach width (W) and reach mean depth ( $D_m$ ) (Mohammed et al., 2014 b):

$$W = 54 \times 10^6 d_{50} (v Q / 9.81 d_{50}^4)^{-0.23} (Gs)^{-0.01} (S)^{0.003} \dots (3)$$

$$D_m = 25 \times 10^{-4} d_{50} (v Q / 9.81 d_{50}^4)^{0.6} (Gs)^{2.6} (S)^{0.017} \dots (4)$$

All the relationships produced in the present work have been compared with other relationships in literature and verified for validity (Hashim, 2014).

Lee and Julien (2006) examined large database for the hydraulic geometry of alluvial channels through non-linear regression analysis. The parameters describing the hydraulic geometry are: channel width, average flow depth, mean flow velocity and channel slope. The three independent variables are: discharge, median bed particle diameter and channel slope. Regression equations were tested for channel width ranging from 0.2 to 1,100 m, flow depth from 0.01 to 16 m, flow velocity from 0.02 to 7 m/s and channel slope from 0.0001 to 0.08. Equations (5) and (6) present the relationships of flow depth ( $D_m$ ) and channel width (W) (Lee and Julien, 2006):

$$W = 3.004Q^{0.426} d_{50}^{-0.002} S^{-0.153} \quad \dots (5)$$

$$D_m = 0.201Q^{0.336} d_{50}^{-0.025} S^{-0.060} \quad \dots (6)$$

Julien and Wargadalam (1995) examined the downstream hydraulic geometry of alluvial channel in

terms of bankfull width, average flow depth, mean flow velocity and friction slope from a three-dimensional stability analysis of non-cohesive particles under two-dimensional flows. The analytical formulations were tested with a comprehensive data set consisting of 835 field channels and 45 laboratory channels. Field and laboratory observations are in very good agreement with the calculations. Equations (7) and (8) present the developed relationships for the flow depth ( $D_m$ ) and channel width (W) (Julien and Wargadalam, 1995):

$$W = 1.33Q^{0.44} d_{50}^{-0.11} S^{-0.22} \quad \dots (7)$$

$$D_m = 0.2Q^{0.33} d_{50}^{0.17} S^{-0.17} \quad \dots (8)$$

The current paper involves employing non-linear regression analysis to develop easier empirical relationships for Al-Abbasia reach. The database used includes 21 sections. The formulae correlate the reach width (W) and mean depth ( $D_m$ ) with different hydraulic properties of the selected reach, including mean particle size ( $d_{50}$ ), discharge (Q) and slope (S). These variables were selected for comparison purposes. Two statistical methods have been used in this research: root mean squared error (RMSE) and F-test, in order to evaluate the performance of each formula for width and mean depth to present the statistical comparison among the different mentioned models.

## WORK METHODOLOGY

### Selected Site

This paper is part of an M. Sc. work conducted earlier by Hashim (2014).

Al-Abbasia reach along the middle part of Euphrates river was selected to investigate the different geometry hydraulic characteristics. This region is about 6000 m in length and located between latitudes (32.04°- 32.03°) and longitudes (44.26°- 44.29°). This selected reach was divided into 21 sections to perform the field work, which included measurement of the hydraulic characteristics of

the river sections and longitudinal slopes of the stream, as well as soil sampling. The majority of the cross-section data was measured by using Acoustic Doppler Current Profiler (ADCP). Plate (1) shows the selected sections.

### Data Analysis

Non-linear regression analysis has been conducted to produce easier empirical formulae for Al-Abbasia reach. These formulae were correlating the reach width ( $W$ ) and mean depth ( $D_m$ ) with different hydraulic properties of 21 sections on the selected reach, including mean particle size ( $d_{50}$ ), discharge ( $Q$ ) and slope ( $S$ ).



Plate (1): Selected reach and sections of measured data

Table 1. Limitations of the studied characteristics in Al-Abbasia reach

No.	Characteristic	Symbol	Limitations	Unit
1	Mean Depth	$D_m$	1.7 - 4.5	m
2	Discharge	$Q$	34 - 78	$m^3/sec$
3	Width of the river	$W$	48 - 184	m
4	Main channel slopes	$S$	$2 \times 10^{-5}$ - 0.02	-
5	Mean size of bed material	$d_{50}$	0.16 - 0.34	mm

### Data Limitations

Table (1) presents the limitations of the different characteristics of the selected reach in Al-Abbasia region to perform the analysis in order to produce different models. These characteristics include discharge ( $Q$ ), width of water surface ( $W$ ), mean depth

( $D_m$ ), main channel slopes ( $S$ ) and mean size of bed material ( $d_{50}$ ).

### Statistical Comparison

The two statistical methods, root mean squared error (RMSE) and F-test, have been employed to evaluate the

performance of non-linear formulae for width and mean depth with formulae built using the power functions and dimensional analysis for the field data of Al-Abbasia reach. This is in addition to the 45° line method used to compare different non-linear formulae.

regression analysis for reach width (W) with mean particle size (d<sub>50</sub>), discharge (Q) and slope (S), with an R<sup>2</sup> value of 0.97.

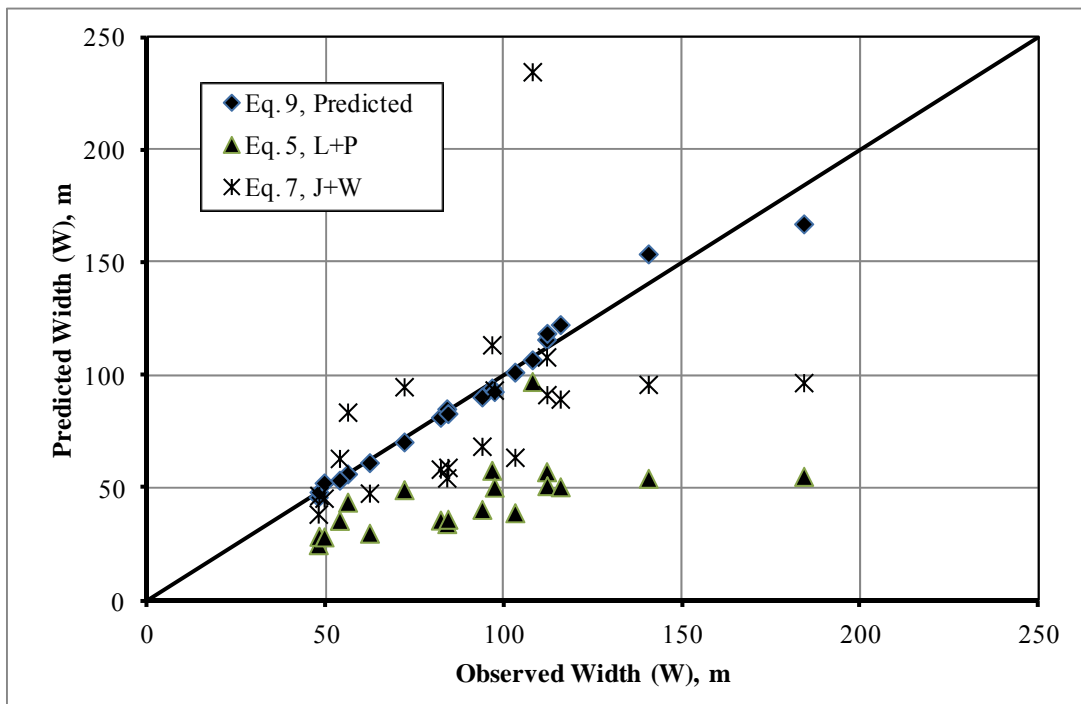
$$W = 8100 \cdot d_{50}^{0.9} \cdot Q^{0.76} \cdot S^{0.001} \quad \dots (9)$$

**RESULTS AND DISCUSSION**

**Regression Equation for Reach Width (W)**

Equation (9) demonstrates the result of non-linear

where: W is the surface width (m), Q is the reach discharge (m<sup>3</sup>/sec), d<sub>50</sub> is the mean particle size (m) and S is the main slope of the reach bed.



**Figure (1): Comparison among observed and different prediction methods for width**

Figure 1 shows a comparison among different models with respect to the observed data. It is evident that the present model is the closest one to the observed data.

**Regression for Mean Depth (D<sub>m</sub>)**

Equation (10) illustrates the result of non-linear regression analysis for mean reach depth (D<sub>m</sub>) with mean particle size (d<sub>50</sub>), discharge (Q) and slope (S), with an R<sup>2</sup> value of 0.93.

$$D_m = 0.065 \cdot d_{50}^{-0.66} \cdot Q^{-0.42} \cdot S^{0.027} \quad \dots(10)$$

where: D<sub>m</sub> is the mean reach depth (m), Q is the reach discharge (m<sup>3</sup>/sec), d<sub>50</sub> is the mean particle size (m) and S is the main slope of the reach bed.

Figure 2 shows a comparison among different models with respect to the observed data. Again, it is clear that the present model is the closest one to the observed data.

**Statistical Comparisons of Results**

Two statistical methods are used in this research; root mean squared error and F-test, to evaluate the performance of each formula for width and mean depth, through giving the extents of error and acceptance with respect to the observed values.

**Root Mean Squared Error**

The root mean squared error (RMSE) value is

commonly used for error measurement. The sum of squares gives more weight to higher error values and consequently to higher error variances. RMSE has the same units as the observed and predicted data. Smaller values indicate better agreement between observed and predicted values. Figures 3 and 4 present a statistical comparison among models according to RMSE.

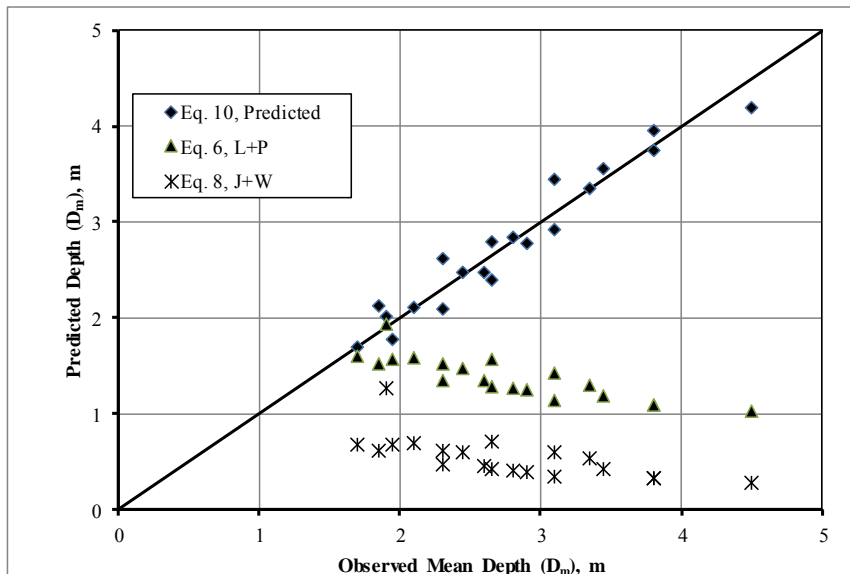


Figure (2): Comparison among observed and different prediction methods for mean depth

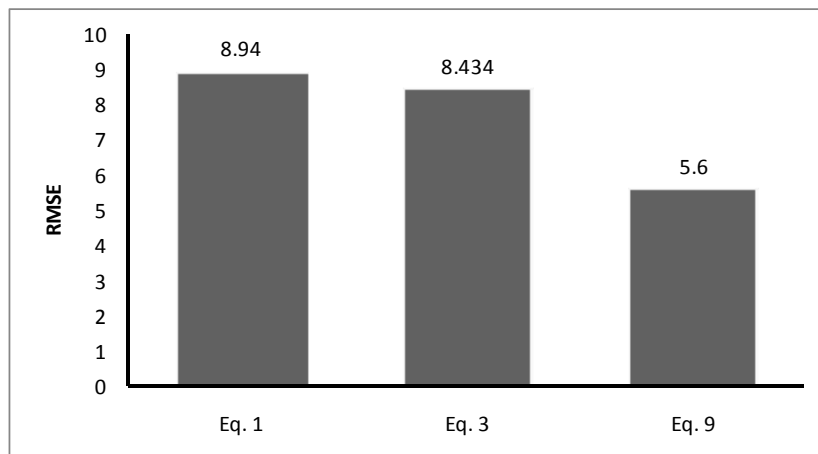


Figure (3): Statistical comparison among width models according to RMSE

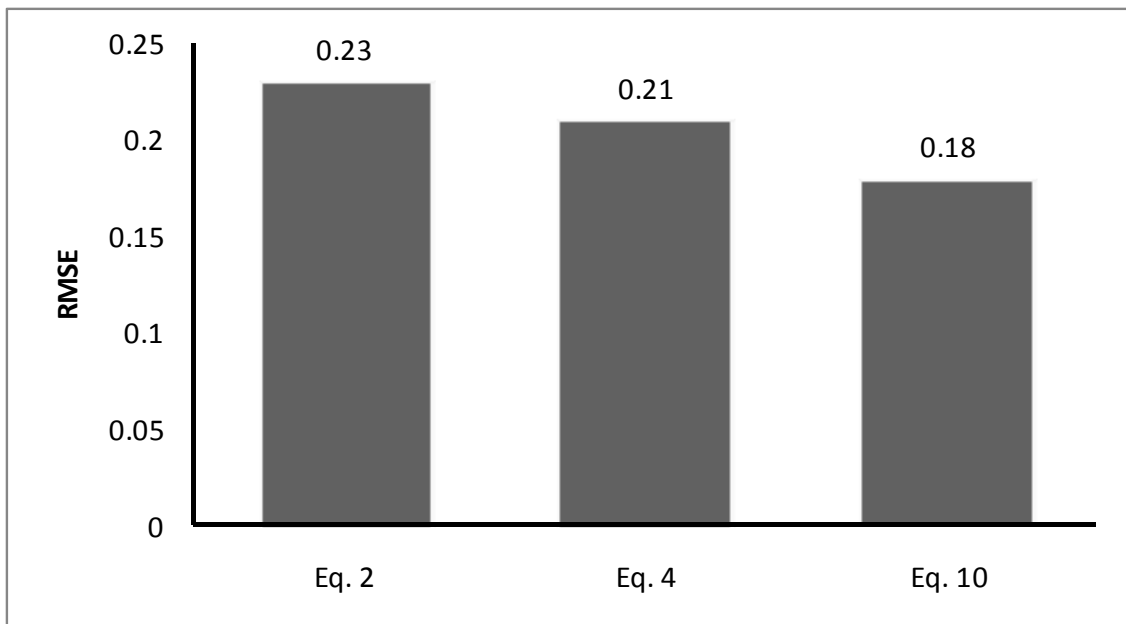


Figure (4): Statistical comparison among mean depth models according to RMSE

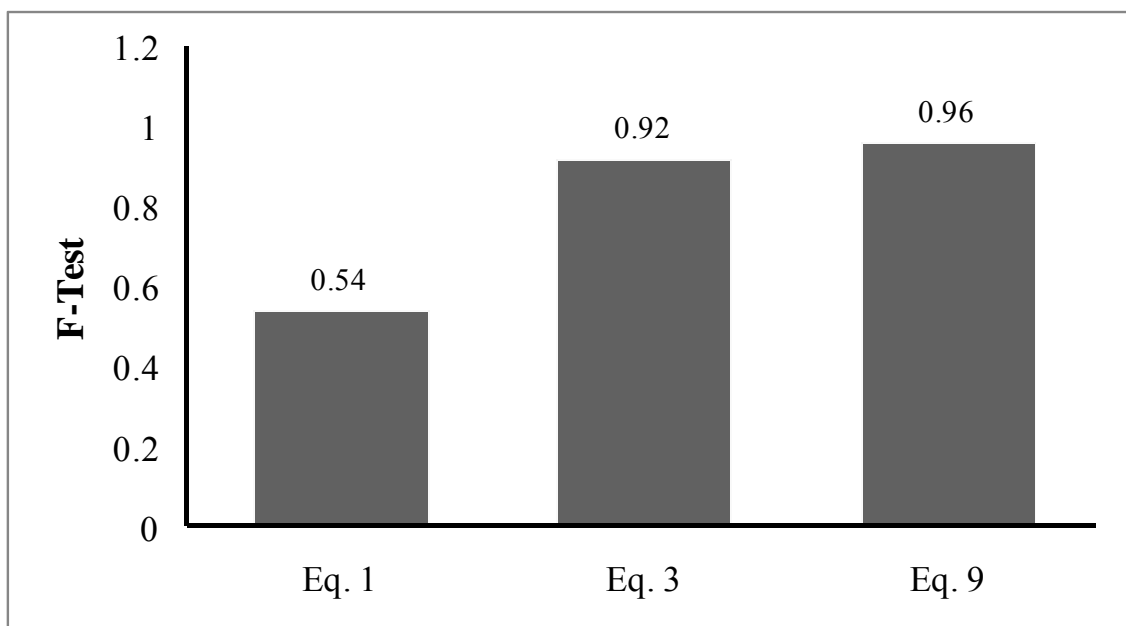


Figure (5): Comparison among width models according to F-Test

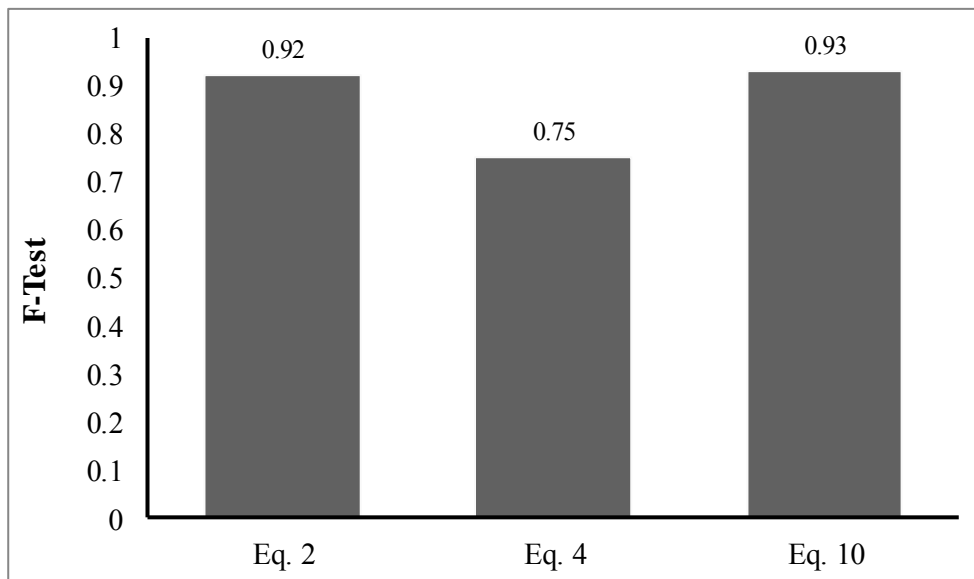


Figure (6): Comparison among mean depth models according to F-Test

### F-test

F-test is named after the famous statistician R. A. Fisher. It is also sometimes known as the Fisher F-distribution. F is the ratio of two variances. The F-distribution is most commonly used in the analysis of variance (ANOVA) and the F-test to determine whether two variances are equal or not. F has a minimum of 0, but no maximum value (all values are positive). Larger values indicate better agreement between measured and calculated values. Figures 5 and 6 present a statistical comparison among models according to F-test.

### CONCLUSIONS

Empirical analysis has been used to develop relationships that are straightforward for application in design and analysis. These relationships are with highly acceptable results as in Eq. (9) for reach width with an  $R^2$  value of 0.97 and Eq. (10) for reach mean depth with an  $R^2$  value of 0.93. The equations produced are the best

for Al-Abbasia reach among the studied models produced by other authors.

Different statistical analyses were conducted to compare the different models and the results revealed that Equations (9) and (10) are the best models to correlate the different reach properties.

### RECOMMENDATIONS

For further research, we recommend to investigate other prediction models rather than power functions, such as statistical analysis for multi-variable models.

The authors suggest selecting a suitable solution for the meandering process problems. Also, the authors recommend using the relationships developed for stream and channel design.

Finally, it is quite important to consider the use of remote sensing technology to study the migration of the river among the important issues to control water resources and hydraulic structures.

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